



MODELING THE DYNAMICS OF RANDOM PROCESSES IN NEXT-GENERATION WIRED/WIRELESS NETWORKS AND SMART SPACES

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ABSTRACT

This article examines the complex interaction between random processes in developing communication systems. As wired and wireless networks undergo radical changes and smart spaces become an integral part of our daily environment, understanding processes' stochastic nature is paramount. The study examines the theoretical foundations of random processes, which link mathematical science and the unpredictable phenomena inherent in the development of modern networks. It examines the fundamental characteristics, models, and representations of random processes and highlights their impact on the performance, adaptability, and effectiveness of IP-based wireless networks and smart spaces. Methodologically, the study uses mathematical models and analytical tools to determine the dynamics of random processes in each context. Understanding stochastic complexities, this study aims to provide practical recommendations to operators and stakeholders navigating an evolving communication system. In conclusion, the detailed study of random processes, offering a roadmap for the efficient operation of next-generation wired/wireless networks and smart spaces. The acquired knowledge contributes to the ongoing dialogue about adaptability and resilience necessary to meet the requirements of the modern communication environment.

KEYWORDS: random processes; wired/wireless networks.

1.INTRODUCTION

Against the background of rapid technological progress in wired and wireless networks and intelligent spaces is a comprehensive study of the theory of random processes in their dynamics. The evolution of modern networks from simple data transmitters to complex ecosystems poses the challenge of understanding random processes and their impact on network efficiency, especially in the context of IP-based wireless networks and smart spaces. Networks are turning not just into information carriers but also into complex ecosystems in which advanced technologies such as 5G and others are integrated. These changes create an environment where communication becomes integral to our daily lives. Smart spaces, ranging from homes to cities, are becoming living laboratories of innovation. Intelligent sensors, Internet of Things (IoT) devices, and advanced analytics allow these spaces to respond in real-time to the needs of their inhabitants. The digital economics testifies to the new generation's insatiable demand for multimedia services. Users expect to be immersed in real-time content - from high-definition video streaming to virtual reality. This surge challenges traditional communication systems, requiring high bandwidth, minimal latency, and high reliability. In the context of these rapid changes, conventional communication systems face significant problems — the volume and variety of data generated by next-generation multimedia services create a load on existing networks. Latency-sensitive applications require instant response, which requires a fundamental change in the perception and management of the communication infrastructure. In this context, the interaction of random processes becomes critical to ensuring the efficient and reliable operation of IP-based wireless networks. This document aims to study this issue to identify theoretical approaches that can provide the necessary harmony in such complex and rapidly changing network scenarios.

2.RESEARCH METHODOLOGY

Stochastic processes and communication systems are two related fields that use probability theory and random variables to model, analyze, and design systems that deal with uncertain or noisy data [1]. A stochastic process is a mathematical object that consists of a collection of random variables indexed by time or space. Modulation and demodulation are deterministic, but the information and noise in transmission are stochastic. These phenomena follow predictable characteristics summarized in a random process model and heavily influence digital communication system design [3]. A communication system is a system that enables the transmission and reception of information between two or more parties. A communication system typically consists of a transmitter, channel, receiver, source, and destination of information. Some researchers



describe random processes from an engineering perspective and use examples from the field of communications to explain concepts. This is a valuable approach for modeling, synthesis, and numerical simulation of random processes with applications in communications and related fields. The random processes in relation to the modeling of phenomena such as interference and fading in communications [4]. A communication system can be affected by various sources of noise and interference, such as thermal noise, atmospheric noise, or jamming signals [5]. Random processes can be used to model the behavior and characteristics of various components and factors in IP-based wireless networks, such as information source, channel noise, receiver output data, network topology, traffic scheme, mobility scheme, interference level, and quality of service [6], for analyzing and evaluating the performance and reliability of wireless networks based on IP in various scenarios and conditions [6,7], for the design and optimization of IP-based wireless networks, for example, using stochastic optimization methods, stochastic control methods or stochastic coding schemas, to improve the efficiency and reliability of IP-based wireless networks by finding optimal or near-optimal solutions for various tasks, such as resource allocation, energy management, routing, scheduling, congestion control and error control [7]. Random processes can also enhance the security and privacy of IP-based wireless networks, for example, through random key distribution, random encryption, random authentication, or random jamming, these methods can help protect the confidentiality, integrity, and availability of information transmitted and received in IP-based wireless networks by preventing or mitigating various attacks, such as eavesdropping, spoofing, replay, or denial of service [8]. Assuming the system is conceptualized as a stochastic process, its performance can be assessed through metrics indicative of its average or anticipated behavior. Examples of fidelity criteria encompass average mean squared error, the ratio of average signal power to average error power (signal-to-noise ratio or SNR), and the average probability of symbol error. The language employed in signal coding is founded on a fusion of stochastic processes and linear systems. Despite the inherent nonlinearity of signal coding systems, it is imperative to employ the tools and methodologies of linear systems for their analysis. Within a communication system, a mathematical model posits that transmitted data originates from a stochastic process, and the system may involve stochastic elements, such as the introduction of random noise or digital errors. Evaluating system performance is predicated on its average characteristics, such as root-mean-square error, signal-to-noise ratio (SNR), or average probability of symbol error. Temporal averages are computed by aggregating or integrating sample values acquired during the process and normalizing them over time. Although the short-term behavior of temporal averages exhibits randomness, in many instances, long-term behavior tends to converge towards a non-random state. Mathematical expectations prove most beneficial when computing averages within the framework of a stochastic process model. Determining the expected values of critical variables in a mathematical system model facilitates the examination of structural transformations within the system. However, in practical scenarios, average values are measured over time. Hence, it is crucial to delineate the conditions under which these two types of averages coincide and formulate management tools for systems in which they diverge. The comparison between expected values and long-term temporal averages necessitates a comprehension of the stationarity and ergodic properties of stochastic processes. Stationarity may be perturbed by the introduction of a transient element into an otherwise stationary process, yet sample averages can still converge meaningfully. Notably, the debate in the literature on the stationarity or ergodicity of human speech underscores the relevance of these theoretical considerations in the design and analysis of speech coding systems [9]. Leonard Kleinrock studied a combination of studies of connected networks and stochastic flows that provided a basis for understanding the general behavior and functioning of communication networks and presented a model of the theory of queuing communication networks with limited channel capacity. Studying Random Processes, the optimal distribution of channel bandwidth, priority effects, the choice of routine procedure, fixed cost constraints, and the design of a topological structure are considered [10]. The researchers studied energy-saving wireless communication networks using the theory of random processes, paying particular attention to channel capacity, transmission schemes, and optimization of energy consumption, and found that energy harvesting is cost-effective for designing and deploying next-generation wireless networks. They also identified open research objectives and future directions in wireless energy collection networks [11].

3.RESULT AND DISCUSSION

A random variable is a variable that, because of the test, depending on the case, takes one of the possible sets of its values (which one is unknown in advance). A random process $X(t)$ is called a process whose value is a random variable for any value of the argument t . In other words, a random process is a function that, because of the test, may take one or another the specific form unknown in advance. For a fixed $t = t_0$, $X(t_0)$ is an ordinary random variable, i.e. the reading of a random process at the time of t_0 .



Examples of random processes in new-generation networks with wired and wireless technologies and intelligent spaces:

- Traffic patterns: Data packets enter the wireless sensor network, and their number can vary over time, often modeling a Poisson process.
- Channel attenuation: The wireless channel's operating conditions can change due to obstacles and interference, and modeling the attenuation process as a stochastic process like Rayleigh or Nakagami helps account for signal level fluctuations' random nature.
- Connecting devices: As devices move or join/leave the network, the network topology can change dynamically. Stochastic processes such as Markov chains can be used to model these changes.
- Interference levels: The interference levels in a wireless network can vary depending on the number of devices and external interference sources. Using a stochastic process to represent the random nature of interference helps understand its effect on communication quality.
- Energy Harvesting: Devices in smart spaces can collect energy from external sources like solar or kinetic energy. Modeling the energy collected as a stochastic process influenced by environmental factors like lighting intensity or movement.
- Packet Loss: During wireless communication, packets may be lost due to channel errors or overload. Modeling packet loss as a random process using a Bernoulli distribution can help understand the probabilistic nature of data loss.
- Quality of Service (QoS) Metrics: QoS metrics like latency and jitter can change randomly in communication networks. Stochastic processes can model the dynamic changes of these indicators over time.
- Device mobility: Modeling the random movement of devices in an intelligent space or mobile peer-to-peer network using processes like random walk or the Gauss-Markov mobility model reflects the unpredictability of device location.
- Network bandwidth: The bandwidth of a wireless network can vary due to changes in channel conditions and traffic load. Stochastic processes like autoregressive models can reflect the time-varying nature of network bandwidth.
- Energy consumption: The energy consumption of devices in smart spaces can fluctuate randomly depending on the tasks performed. Stochastic processes can simulate these fluctuations, helping optimize energy-efficient strategies. Accurately modeling and analyzing these processes can help design and optimize efficient, fault-tolerant network systems.

The examples given earlier cover some key aspects, but there are additional aspects that can be taken into account: Random security events, such as cyber-attacks or attempts to invade the network (modeling the timing and nature of these events can help in developing reliable security strategies), random device failures in the network (device reliability may vary, and modeling failures as a stochastic process can help in predicting the reliability of the system and managing it), dynamic resource allocation (for example, bandwidth or computing resources, in response to changing requirements and stochastic optimization processes can be used for probabilistic resource allocation), human behavior in smart spaces (random patterns of human behavior in smart spaces that affect interaction with devices and network usage and stochastic models can be used for modeling and understanding human behavior to improve system design), spectrum availability in wireless networks (availability of spectrum ranges for wireless communication and spectrum availability can be influenced by factors such as interference or changes in regulation, and modeling this as a stochastic process helps in managing spectrum use), data receipt patterns in cloud computing (patterns of random data requests in cloud computing environments and stochastic models can be applied to account for the variability of data processing load on cloud servers), the signal-to-noise ratio (SNR) in wireless communications (SNR in wireless communication channels can vary depending on environmental conditions and stochastic processes, such as logarithmically regular or rice distribution, can simulate random fluctuations in SNR), dynamic changes in network topology (changes in network topology due to the addition or removal of devices and stochastic models can represent the random nature of network reconfigurations in dynamic environments), etc.

Table 1. Summary of some examples of random processes

The random processes	Description:	Example application
Wiener Process (Brownian Motion)	A continuous-time stochastic process with independent and normally distributed increments.	Modeling the erratic motion of particles suspended in a fluid
Poisson Process	A counting process that represents the number of events occurring in fixed intervals of time or space	Modeling the arrival of customers at a service point or the occurrence of rare events.



Markov Process	A stochastic process where the future state depends only on the current state and not on the sequence of events that preceded it	Modeling systems with memoryless transitions, such as queueing systems.
Gaussian Process	A collection of random variables, any finite number of which have a joint Gaussian distribution	Used in machine learning for regression and classification tasks
Martingale	A stochastic process where the expectation of the future value, given the past values, is equal to the present value	Modeling fair games of chance
Renewal Process	A stochastic process used to model the occurrence of events that renew or restart the process	Analyzing the reliability of systems subject to periodic maintenance
Autoregressive (AR) Process	A linear model in which each value in the time series is a linear combination of past values and a random error term	Time-series analysis in economics and finance
Moving Average (MA) Process	A linear model where the current value is a linear combination of past random errors.	Filtering out noise in time-series data
Ornstein-Uhlenbeck Process	A stationary Gauss–Markov process, often used to model the velocity of a particle undergoing Brownian motion with friction	Modeling the mean-reverting behavior of asset prices
Fractional Brownian Motion	A generalization of Brownian motion with a Hurst parameter, allowing for long-range dependence	Modeling network traffic in telecommunications, where the process captures long-range dependence in data transmission patterns
Kleinrock's Studies	The Ornstein-Uhlenbeck process is a stationary Gauss–Markov process used to model the velocity of a particle undergoing Brownian motion with friction. It exhibits mean-reverting behavior, making it applicable in finance for modeling asset prices that tend to return to a long-term average	Modeling interest rates in finance, where the process represents the tendency for interest rates to revert to a long-term average over time
Gersho-Gray Algorithm	It is related to vector quantization and signal compression. Developed by A. Gersho and R. M. Gray, it involves quantizing vectors into a finite set of representative vectors. This algorithm has applications in image and speech compression	The Gersho-Gray algorithm compresses images while preserving their quality using a limited set of representative vectors, reducing storage and enabling faster transmission.

Random processes are used as models for various phenomena, such as electron emission, noise due to thermal agitation, atmospheric noise, economic changes, population growth, queues, and more[12]. Similar to a random variable represented as a function of an elementary event r_0 resulting from a test, a random process can be expressed as a function of two variables $X(t, r_0)$, where $r_0 \in Q$, $t \in T$, $X(t, r_0) \in R$, and r_0 is an elementary event, Q is the space of elementary events, T is the set of values of the argument t , and R is the set of possible values of a random process $X(t, r_0)$. The realization of a random process $X(t, r_0)$ is termed a non-random function $x(t)$, representing a specific view taken by the random process $X(t)$ as a result of a test (at a fixed r_0), i.e., its trajectory. Therefore, the random process $X(t, r_0)$ combines the characteristics of both a random variable and a function. When fixing the value of the argument t , the random process becomes an ordinary random



variable; when selecting ρ , it transforms into an ordinary non-random function with each test. In subsequent presentations, we will omit the ρ argument, implying it by default (Figure 1).

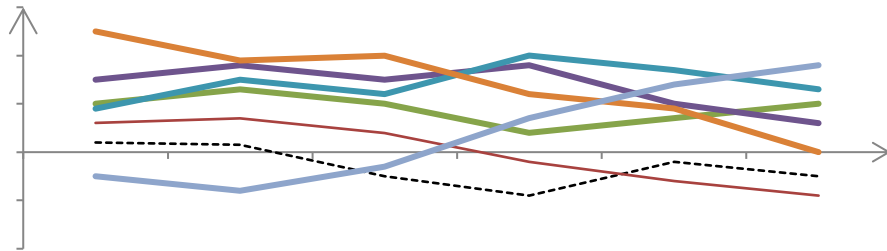


Figure 1. A figure illustrates various implementations of a random process.

If the cross-section of this process at a given t is a continuous random variable, the random process $X(t)$ for a given t is determined by the probability density $\rho(x, t)$. Notably, the density $\rho(x, t)$ does not comprehensively describe the random process $X(t)$ as it fails to express the dependence between sections at different time points. A random process about $X(t)$ encompasses all sections for all possible values of t ; thus, describing it necessitates considering a multidimensional random variable $(X(t_1), X(t_2), \dots, X(t_p))$ consisting of all sections of this process. While there are theoretically infinite such sections, describing a random function often requires considering a relatively small number of sections. A random process has order n if it is entirely determined by the density of the joint distribution $F(x_1, x_2, \dots, x_n; t_1, t_2, \dots, t_p)$ of n arbitrary sections of the process, i.e., the density of the n -dimensional random variable $(X(t_1), X(t_2), \dots, X(t_p))$, where $X(t_i)$ is the cross-section of a random process $X(t)$ at time t_i for $i=1, 2, \dots, n$. Numerical characteristics can also describe a random process. While for a random variable these characteristics are constant numbers, for a random process, they manifest as non-random functions. The mathematical expectation of a random process at time t , denoted as $E[X(t)]$, is a non-random function $ax(t)$. This function is equal to the mathematical expectation of the corresponding section (slice) of the random process at time t , i.e.,

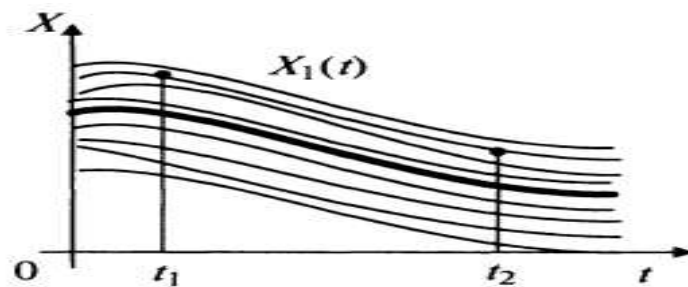


Figure 2. A figure illustrates the random process $X_1(t)$ is characterized by a slow change in the values of implementations with a change in t .

$ax(t)=E[X(t)]$. The variance of a random process at time t , denoted as $D[X(t)]$, is also a non-random function $Dx(t)$. This function is equal to the variance of the corresponding section of the random process at time t , i.e., $Dx(t)=D[X(t)]$. The root means square deviation of a random process at time t , denoted as $\sigma[X(t)]$, is the arithmetic value of the square root of its variance. Thus, $\sigma[X(t)] = \sqrt{D[X(t)]}$. These parameters provide insights into various characteristics of the random process at a specific time t . The mathematical expectation of a random process characterizes the average trajectory of all its possible implementations, and its variance or mean square deviation is the spread of implementations relative to the average trajectory. The characteristics of the random process introduced above are insufficient since they are determined only by the one-dimensional distribution law. Figures 2 and 3 show two random processes $X_1(t)$ and $X_2(t)$ (the with approximately the exact mathematical expectations and variances. If the random process $X_1(t)$ (see Figure 2) is characterized by a slow change in the values of implementations with a change in t , then for the random process $X_2(t)$ (see Figure 3), this change occurs significantly faster. In other words, the random process $X_1(t)$ is characterized by a close probability dependence between its two sections, $X_1(t_1)$ and $X_1(t_2)$, while for the random process $X_2(t)$ this dependence between the sections $X_2(t_1)$ and $X_2(t_2)$ is practically absent. A correlation function characterizes the indicated dependence between the sections. The correlation function of a random process $X(t)$ is a non-random function of two variables t_1 and t_2 , which for each pair of variables t_1 and t_2 is equal to the covariance of the corresponding cross sections $X(t_1)$ and $X(t_2)$ of the random process.

Obviously, for a random process $X_1(t)$, the correlation function $K_{X_1}(t_1, t_2)$ decreases as the difference increases $t_2 - t_1$ is significantly slower than to $K_{X_2}(t_1, t_2)$ for a random process $X_2(t)$. The normalized correlation function of a random process $X(t)$ is a function that expresses the degree of relationship between the values of the process at different time points and is normalized by the variances of the process. Formally, the normalized correlation function $N(t_1, t_2)$ is defined as:

$$N(t_1, t_2) = \frac{Cov(X(t_1), X(t_2))}{\sqrt{Var(X(t_1)) \cdot Var(X(t_2))}}$$

where:

$Cov(X(t_1), X(t_2))$ represents the covariance between the values of the process at times t_1 and t_2 , $Var(X(t_1))$ and $Var(X(t_2))$ denote the variances of the process at times t_1 and t_2 , respectively. The normalized correlation function is useful for measuring the degree of dependence between the values of the process at different time points, independent of the absolute scales of their changes. It is commonly used to analyze the stationarity of a process and identify periodic or random variations in its dynamics.

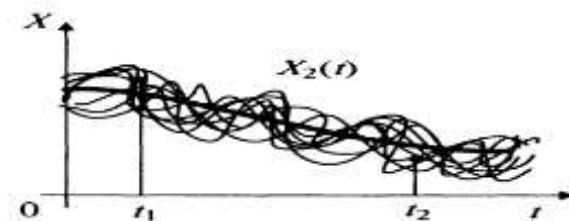


Figure 3. A figure illustrates a random process $X_1(t)$, characterized by a rapid change in the values of implementations when t changes.

To build a mathematical model of a new generation of network variables considering random processes, we will use probabilistic models to describe the relationships. First, let's define the variables:

- B – bandwidth;
- D – latency;
- E - energy consumption;
- S - Service Level.

Now, let's define random processes that affect these variables:

- $FB(t)$ is a random process for throughput;
- $FD(t)$ is a random process for delay;
- $FE(t)$ is a random process for energy consumption;
- $FS(t)$ is a random process for the service level.

The mathematical model can be represented as follows:

$$B = fB(t) + FB(t)$$

$$D = fD(t) + FD(t)$$

$$E = fE(t) + FE(t)$$

$$S = fS(t) + FS(t),$$

where:

$fB(t), fD(t), fE(t), fS(t)$ - deterministic functions representing the main characteristics of the network; $FB(t), FD(t), FE(t), FS(t)$ are random processes describing stochastic fluctuations of variables. This model considers not deterministic and stochastic components, allowing to describe the behavior of a new generation network more accurately. For example, the radio frequency spectrum is a multidimensional object; space, time, polarization, frequency, signal transmission power, and interference are some critical dimensions [13]. Wireless networks typically use Dynamic Spectrum Access (DSA), a stochastic function. DSA has become a vital force for building next-generation wireless networks. Cognitive radio, built on a software-defined radio receiver, is a frequency-dependent and intelligent device that can underlie most forms of these networks. This device provides access to radio frequency spectrum bands and uses them adaptively depending on real-time needs and interference conditions [14]. Unlike traditional fixed spectrum allocation, DSA dynamically allocates spectrum resources, which increases efficiency and reduces interference. An Internet of Things (IoT) device installed to monitor the environment in an intelligent city uses DSA to intelligently select the best spectrum band for data transmission based on the current



RF environment. This dynamic adaptation ensures reliable communication even at different interference levels, contributing to efficient spectrum use in intelligent rooms.

The formula defines a random process $X(t) = X \sin mt$, where X is a random variable, $M(X) = a$, $D(X) = \sigma^2$. Based on the properties of mathematical expectation and variance, it turns out:

$$a_x(t) = M(X \sin mt) = \sin mt \cdot M(X) = a \sin mt,$$

$$D_x(t) = D(X \sin mt) = \sin^2 mt,$$

$$\begin{aligned} \text{Cov}_X(t_1, t_2) &= M[(X \sin mt_1 - a \sin mt_1)(X \sin mt_2 - a \sin mt_2)] = \sin mt_1 \sin mt_2 \cdot M[(X - a)(X - a)] \\ &= \sin mt_1 \sin mt_2 D(X) = \sigma^2 \sin mt_1 \sin mt_2, \end{aligned}$$

$$N_X(t_1, t_2) = \frac{\sigma^2 \sin mt_1 \sin mt_2}{(\sigma \sin mt_1) \cdot (\sigma \sin mt_2)} \equiv 1.$$

The process exhibits complete predictability in terms of linear dependence between its values at different points in time. However, it's worth noting that in real-world scenarios, where randomness is present, achieving complete linear dependence is unlikely.

4. CONCLUSION AND RECOMMENDATIONS

The study focuses on mathematical modeling to understand random processes in modern networks using wired and wireless technologies and intelligent spaces. The wired and wireless networks have undergone significant changes, and integrating smart spaces into our daily lives emphasizes the importance of understanding the stochastic nature of processes in this dynamic environment. The study examined the theoretical foundations of random processes, which act as a vital link between mathematical science and unpredictable phenomena inherent in modern networks' development. Mathematical models and analytical tools were used to unravel the dynamics of random processes, which aimed to provide practical recommendations to operators and stakeholders navigating an ever-evolving communication ecosystem. There is a need for future research to focus on the applied aspects of random processes, especially in emerging technologies like the Internet of Things (IoT) and intelligent spaces. Collaboration between researchers in mathematics, communication systems, and related disciplines can contribute to a holistic understanding of random processes, leading to innovative solutions and applications. With the growing importance of security in wireless networks, further research could explore how random processes enhance the security and privacy aspects of IP-based wireless networks. Developing adaptive strategies using information about random processes to dynamically adjust network parameters in response to changing environmental conditions and user requirements can also be a focus. Integration with machine learning and an understanding of random processes can enhance forecasting capabilities and the adaptive nature of communication systems.

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