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APPLICATIONS OF THE COMPLEX NUMBER IN TRIGONOMETRIC FORM IN SOME PRACTICAL PROBLEMS

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ABSTRACT
In this article, the application of the trigonometric representation of a complex number in some sums, their sum is calculated by some substitutions.
KEYWORDS: *complex number, radical formula, Moavr formula, sum*

INTRODUCTION

As we know, the "Complex number concept" is introduced for students of academic lyceums and specialized schools, and it is appropriate to solve some related issues through the trigonometric representation of a complex number. The trigonometric representation of a complex number and this the formula for raising a number to the nth power

$$z = r(\cos\varphi + i\sin\varphi), \qquad z^n = r^n(\cos n\varphi + i\sin n\varphi) \qquad (1)$$

Example 1.

Calculate the sums below

$$P = \cos\frac{\pi}{2n+1} + \cos\frac{3\pi}{2n+1} + \cdots + \cos\frac{(2n-1)\pi}{2n+1} = \sum_{k=1}^{n}\cos\frac{(2k-1)\pi}{2n+1}$$

$$Q = \sin\frac{\pi}{2n+1} + \sin\frac{3\pi}{2n+1} + \cdots + \sin\frac{2n-1}{2n+1} = \sum_{k=1}^{n}\sin\frac{2k-1}{2n+1}$$

To find the sums above, it is advisable to use the trigonometric form of complex numbers. For this, the second sum is multiplied by i and added to the first:

$$P + iQ = \left(\cos\frac{\pi}{2n+1} + i\sin\frac{\pi}{2n+1}\right) + \left(\cos\frac{3\pi}{2n+1} + i\sin\frac{3\pi}{2n+1}\right) + \cdots + \left(\cos\frac{(2n-1)\pi}{2n+1} + i\sin\frac{2n-1}{2n+1}\right)$$

If

$$w = \cos\frac{\pi}{2n+1} + i\sin\frac{\pi}{2n+1}$$

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according to Muavr's formula

$w^n = \left(\cos\frac{\pi}{2n+1} + i\sin\frac{\pi}{2n+1}\right)^n = \cos\frac{n\pi}{2n+1} + i\sin\frac{n\pi}{2n+1}$ bo'ladi.

$$P + iQ = w + w^3 + w^5 + \cdots + w^{2n-1} = w(1 + w^2 + w^4 + \cdots + w^{2n-2}) = w \cdot \frac{w^{2n}-1}{w^2-1}$$

$$= w \cdot \frac{w^{2n}-1}{w^2-1} \cdot \frac{w^{-1}}{w^{-1}} = \frac{w^{2n}-1}{w-w^{-1}} = \frac{\cos\frac{2n\pi}{2n+1} + i\sin\frac{2n\pi}{2n+1} - 1}{2i\sin\frac{\pi}{2n+1}}$$

$$= \frac{\sin\frac{2n\pi}{2n+1}}{2\sin\frac{\pi}{2n+1}} + i\frac{1-\cos\frac{2n\pi}{2n+1}}{2\sin\frac{\pi}{2n+1}}$$

Then, by equalizing the corresponding parts on both sides, this

$P = \frac{\sin\frac{2n\pi}{2n+1}}{2\sin\frac{\pi}{2n+1}}$ va $Q = \frac{1-\cos\frac{2n\pi}{2n+1}}{2\sin\frac{\pi}{2n+1}}$

we will get the result.

Taking into account the following formulas, the following relations can be written:

$$\sin\frac{2n\pi}{2n+1} = \sin\frac{\pi}{2n+1}$$

$$\cos\frac{2n\pi}{2n+1} = -\cos\frac{\pi}{2n+1}$$

$$1 - \cos\frac{2n\pi}{2n+1} = 2\cos^2\frac{\pi}{2(2n+1)}$$

$$\sin\frac{\pi}{2n+1} = 2\sin\frac{\pi}{2(2n+1)} \cdot \cos\frac{\pi}{2(2n+1)}$$

Based on the above, the following radical formula can be written:

$$P = \frac{1}{2}$$

$$Q = \frac{1}{2}\cot\frac{\pi}{2(2n+1)}$$

Example 2.

Prove the following equality.

$$\cos\frac{\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{5\pi}{7} = \frac{1}{2}$$

To prove this equality, without using the usual trigonometric properties, we show it by trigonometric substitutions of complex numbers. First of all

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$z=\cos\frac{\pi}{7}+i\sin\frac{\pi}{7}$ we enter a complex number whose modulus is equal to 1,

$|z|=1$. We can find the 7th power of the given complex number using the above Muavr formula and get the following result:

$$z^7=\left(\cos\frac{\pi}{7}+\cos\frac{\pi}{7}\right)^7=\cos\pi+i\sin\pi=-1 \text{ va } z^7+1=0.$$

On the other hand, we have the following equality:

$$\cos\frac{\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{5\pi}{7}=\frac{1}{2}\left(z+\frac{1}{z}\right)+\frac{1}{2}\left(z^3+\frac{1}{z^3}\right)+\frac{1}{2}\left(z^5+\frac{1}{z^5}\right)=\frac{z^{10}+z^8+z^6+z^4+z^2+1}{2z^5}$$

$z^7+1=0$ orqali quyidagi tengliklarga erishamiz:

$z^{10}=-z^3$ va $z^8=-z$.

From this equation

$$z^{10}+z^8+z^6+z^4+z^2+1=z^6+z^4-z^3+z^2-z+1=z^6-z^5+z^4-z^3+z^2-z+1+z^5=\frac{z^7+1}{z+1}+z^5=z^5$$

Accordingly, this equality is proved:

$$\cos\frac{\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{5\pi}{7}=\frac{z^5}{2z^5}=\frac{1}{2}$$

Example 3.

Calculate the following sum.

$$S_n=\sin\alpha+\sin 2\alpha+\cdots+\sin n\alpha$$

To calculate the above sum, we enter the sum C_n

$C_n=\cos\alpha+\cos 2\alpha+\cdots+\cos n\alpha.$

$z=\cos\alpha+i\sin\alpha$ The trigonometric form of the complex number is known. We multiply the sum of $S_n$by i and add it to the sum of C_n to get the following sum:

$$C_n+iS_n=\cos\alpha+i\sin\alpha+\cos 2\alpha+i\sin 2\alpha+\cdots+\sin n\alpha+i\cos n\alpha=z+z^2+\cdots+z^n=z\frac{z^n-1}{z-1}$$

hrough trigonometric substitutions known to us $\cos x-1=-2\sin^2\frac{x}{2}$ va $\sin x=2\sin\frac{x}{2}\cos\frac{x}{2}$ accordingly

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$$\frac{z^n-1}{z-1}=\frac{\cos n\alpha+i\sin n\alpha-1}{\cos\alpha+i\sin\alpha-1}=\frac{-2\sin^2\frac{n\alpha}{2}+2i\sin\frac{n\alpha}{2}\cos\frac{n\alpha}{2}}{-2\sin^2\frac{\alpha}{2}+2i\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}=\frac{\sin\frac{n\alpha}{2}}{\sin\frac{\alpha}{2}}\left(\frac{\cos\frac{n\alpha}{2}+i\sin\frac{n\alpha}{2}}{\cos\frac{\alpha}{2}+i\sin\frac{\alpha}{2}}\right)$$
$$=\frac{\sin\frac{n\alpha}{2}}{\sin\alpha}\left(\cos\frac{(n-1)\alpha}{2}+i\sin\frac{(n-1)\alpha}{2}\right).$$

From the above equation, we get the following result:

$$C_n+iS_n=(\cos\alpha+i\sin\alpha)\frac{\sin\frac{n\alpha}{2}}{\sin\alpha}\left(\cos\frac{(n-1)\alpha}{2}+i\sin\frac{(n-1)\alpha}{2}\right)$$
$$=\frac{\sin\frac{n\alpha}{2}}{\sin\alpha}\left(\cos\frac{(n-1)\alpha}{2}+i\sin\frac{(n-1)\alpha}{2}\right).$$

By separating the real and abstract parts of this equation, we find the sums S_n and C_n:

$$S_n=\frac{\sin\frac{n\alpha}{2}\sin\frac{(n+1)\alpha}{2}}{\sin\frac{\alpha}{2}}$$

$$C_n=\frac{\sin\frac{n\alpha}{2}\cos\frac{(n+1)\alpha}{2}}{\sin\frac{\alpha}{2}}$$

In conclusion, it should be said that when calculating certain sums, it is more convenient to calculate using the trigonometric representation of a complex number, and many sums of this type can be made in practice.

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