



PROBLEMS OF ENTERPRISE CAPITAL OPTIMIZATION USING ELEMENTS OF GAME THEORY

Amirzoda HAMRAYEV

Senior Lecturer, Tashkent Kimyo International University,

ABSTRACT

In this article, the influence of the manager's level of optimism on the capital structure, i.e., the probable mathematical expectations of the amount of the company's main operating profit expected in the future, on the capital structure in the optimization of capital of enterprises is studied using the matrix of game theory. As a result, 19 forecast results were developed for several enterprises under 5 different scenarios, and the lowest and highest values of the ratio of debt capital to private capital were determined. This situation ensures the smooth operation of enterprises and helps to increase its fundamental value.

KEYWORDS: *game theory matrix, Bayesian criterion, Laplace criterion, Wald criterion, Maximax criterion, Hurwitz criterion, game nature, strategy.*

INTRODUCTION

In an era marked by dynamic markets, evolving economic landscapes, and increasingly complex financial structures, the optimal allocation and management of capital have become paramount for the sustainable growth and competitiveness of business entities. The intricate decisions surrounding capital optimization, encompassing choices related to debt, equity, and resource allocation, present formidable challenges that demand sophisticated analytical tools. Among these tools, game theory has emerged as a compelling framework that sheds light on the intricate strategic interactions among stakeholders within business entities.

Capital optimization, the process of determining the most advantageous mix of financing sources while aligning with organizational objectives, has a profound impact on a company's financial health, risk profile, and long-term value creation. To navigate this multifaceted terrain effectively, businesses must grapple with issues ranging from the balancing act between debt and equity to the resolution of conflicts of interest between shareholders, creditors, and management. Game theory, with its roots in mathematics, economics, and strategic decision-making, offers a structured and powerful lens through which to explore these complex issues.

This literature review delves into the confluence of capital optimization and game theory, shedding light on the pivotal role played by game theory elements in addressing the challenges inherent in capital allocation within business entities. As we embark on this exploration, we will first delve into the theoretical underpinnings of game theory and its relevance to capital optimization. Subsequently, we will examine how game theory has been applied to model and analyze the strategic decisions related to capital structure, agency relationships, and resource allocation.

In the pages that follow, we will traverse a landscape enriched by empirical insights from studies that have harnessed game-theoretic frameworks to explain real-world capital allocation phenomena. Furthermore, we will highlight the inherent challenges and complexities that persist in this field, suggesting avenues for future research and practical application. As businesses seek to chart a course toward financial resilience and strategic advantage, the fusion of capital optimization problems with game theory elements stands as a compelling frontier for exploration—one that promises to offer both theoretical illumination and practical guidance in the pursuit of optimal capital structures and value creation.

LITERATURE REVIEW

Effective allocation and optimization of capital in business entities is considered one of the decisive factors for achieving long-term stability and competitiveness in today's dynamic economic environment. With its roots in mathematics and economics, game theory has emerged as a valuable framework for addressing the complexities surrounding capital optimization. Game theory, originally developed by Von Neumann and Morgenstern, provided a systematic approach to analyze decision making in competitive settings [1]. In the field

of capital optimization, game theory has provided a solid framework for modeling strategic interactions between various stakeholders, such as shareholders, creditors, and managers. Several scholars have emphasized the versatility of game theory in solving complex problems related to capital allocation and resource management [2, 3].

A firm's capital structure, which includes equity and debt capital, is a classic example of strategic decision-making. Game theory is used to model the interactions between shareholders and creditors in capital structure decisions [4]. Many researchers have studied how the conflict of interests between shareholders and lenders can be resolved through cooperative or non-cooperative game models [5, 6].

The agency problem, characterized by conflicts of interest between shareholders and managers, is a central problem in capital optimization. Game theory was later effectively combined with agency theory to analyze the principal-agent relationship in capital allocation [7, 8]. These studies emphasize the importance of aligning the interests of managers and shareholders through incentive mechanisms derived from game theory models.

Several empirical studies have shown the practical value of game theory in capital optimization. For example, in the research conducted by Smith and Warner, firms use signaling games to explain their financial stability to investors through their dividend policy [9]. Similarly, Song and Thakor investigated the role of static games in influencing firms' capital allocation strategies [10].

RESEARCH METHODOLOGY

During the preparation of this article, methods of scientific data collection, measurement, and analysis were used. Models such as the CAPM model, the discounted cash flow model, the Gordon growth model were used to assess the value of enterprises, and scenarios for improving the formation of optimal capital structures of enterprises were developed using the Bayesian criterion, the Laplace criterion, the Wald criterion, and the Maximax criteria.

ANALYSIS AND RESULTS

It has been practically proven that the formation of the optimal capital structure in enterprises brings the value of enterprises to the highest level. But it should not be forgotten that the factor of uncertainty, which has a probabilistic nature, also answers the problem of how enterprises should implement capital structure in the future.

It is in such a system of uncertainties that several options of efficient capital structure are developed through the matrix of game theory. The main essence of the game theory is that one party moves in the direction of its goals contrary to the decisions of the other party.

Consider the following game theory model. The players, the strategy of the game, the nature of the game, its situation and the winning of the player are selected as the composition of the game. As a player, we take a financial manager, and as his goal, we take the goal of optimizing firm's capital structure. So, as a strategy of this player, we get scenarios of debt capital that the enterprise has attracted in the formation of capital in different ways. Firms can consider ($n=20$) different S_i strategies to increase the share of debt capital by an additional 5%:

$$S_i: q_k^i. \quad i = 1, 2, 3 \dots, 20$$

For example, the strategy S_1 shows the value of the share of debt in the total capital $q_k^i = 0$ when forming the capital of the enterprise, S_2 shows the share of debt of the enterprise in $q_k^i = 5\%$

The nature (T), which includes the uncertainty factor, represents the change in Earnings before interest and taxes ($EBIT$). In short, the state of nature (T_j) indicates that $EBIT$ should be placed in the interval ($a_j\%, a_{j+1}\%$). This state of nature can be chosen in different quantities based on the player's assumptions.

The goal of the player is to maximize the winning game and achieve this T_j by applying (a_{ij}) in the strategy S_i . In this case, the value obtained by changing the capital structure of the enterprise is $EBIT$ taken as a win in the game.

$$a_{ij} = FV_{ij}. \quad i = 1, 2, 3, \dots, n. \quad j = 1, 2, 3, \dots, m \quad (1)$$

here:

a_{ij} - the player wins by applying the strategy S_i in the situation T_j

FV_{ij} - $EBIT$ value of the enterprise in the interval of ($a_j\%, a_{j+1}\%$) and q_k^i is the value of the debt share

Based on the calculation of each strategy of efficiency indicators related to the selected optimality criterion, an optimal strategy is developed to reach the maximum level of this efficiency indicator. Under the criterion of capital optimality there lies the comparison of the selected strategies with each other and the selection of the best among them. It is necessary to remember that the optimal strategy of the player would be optimal according to one criterion and not optimal according to another criterion.

The choice of the criterion of optimality is determined subjectively by the player, and this choice is based on the goal of the player, his exposure to the risk, the conditions for continuing the game. For example, the game

can be proceeded differently when the player has specific information about the probability of the events occurring, or when the player has no information about the probability of the events occurring. In this case, the player can take a conservative path and reduce the possible losses, or choose an aggressive path and take different paths in order to achieve a very large result despite of the risk. What decisions the player makes in any given situation are determined based on the criterion of optimality.

If the probability of occurrence of events is known to the player, or if the player can estimate the probability based on historical data, decisions about the optimal strategy are made based on *Bayesian criterion*.

Based on this criterion, the efficiency indicator of the strategy S_i is based on the sum of the probabilities of occurrence of successful events.

$$B_i^p(e) = \sum_{j=1}^m e_j a_{ij}, \quad i = 1, \dots, n, \quad j = 1, \dots, m \quad (2)$$

e_j - probability of occurrence of the event T_j

Bayesian criterion is determined using the following formula:

$$A^{opt} = \max B_i^p(e) \quad (3)$$

If it is not possible to determine the probability of occurrence of events, then *Laplace criterion*, *Wald criterion*, *Maximax criterion* and *Hurwitz criterion* are used.

According to the *Laplace criterion*, if the player does not have information about the probability of occurrence of events, he assumes that the probability of occurrence of events is similar. As a result, the efficiency indicator L_i^p of the strategy S_i is determined by the arithmetic average of the total winning strategies:

$$L_i^p = \frac{1}{m} \sum_{j=1}^m a_{ij}, \quad i = 1, \dots, n \quad (4)$$

According to the Laplace criterion is completed by choosing the strategy that maximizes the efficiency indicators:

$$A^{opt} = \max L_i^p \quad (5)$$

We can see that Laplace criterion does not take into account the player's goal and risk appetite, but rather the player's goal and risk acceptance. A pessimistic player does not take risks in order to minimize possible losses, while an optimistic player takes large risks in order to maximize expected results.

If the player does not expect any risk, his decision will be such that the expected events will be easy for him, and his profit will be minimal even in the desired strategy. In this case, the optimal strategy is selected according to the *Wald criterion*. In this case, the optimal strategy S_i is the least profitable of the set of W_i strategies:

$$W_i = \min_{1 \leq j \leq m} a_{ij}, \quad i = 1, \dots, n \quad (6)$$

Optimality is completed by choosing the strategy with the maximum efficiency indicators:

$$A^{opt} = \max_{1 \leq j \leq m} W_i \quad (7)$$

If the player thinks that the expected event will give him the best profit, he will make a decision that is focused on the maximum profit in each strategy. In this case, the determination of the optimal strategy is determined based on the *Maximax criterion*. The performance indicator of the strategy S_i is selected from the most successful levels of the total strategies:

$$M_i = \max_{1 \leq j \leq m} a_{ij}, \quad i = 1, \dots, n \quad (8)$$

The optimal strategy according to the *Maximax criterion* is determined as follows:

$$A^{opt} = \max_{1 \leq j \leq m} M_i \quad (9)$$

We can formulate the events that are expected $\Delta EBIT$ to happen in the following ways:

$$\left\{ \begin{array}{l} S_1: \Delta EBIT < -50\% \\ S_2: \Delta EBIT \in [-50\%, -30\%) \\ S_3: \Delta EBIT \in [-30\%, -10\%) \\ S_4: \Delta EBIT \in [-10\%, 10\%) \\ S_5: \Delta EBIT \in [10\%, 30\%) \\ S_6: \Delta EBIT \in [30\%, 50\%) \\ S_7: \Delta EBIT > 50\% \end{array} \right.$$

It is suggested to take the average of the intervals to determine the events expected to occur in order to facilitate calculations. Then we can make the following changes:

- $S_1: \Delta EBIT = -60\%$
- $S_1: \Delta EBIT = -40\%$
- $S_1: \Delta EBIT = -20\%$
- $S_1: \Delta EBIT = 0\%$
- $S_1: \Delta EBIT = 20\%$
- $S_1: \Delta EBIT = 40\%$
- $S_1: \Delta EBIT = 60\%$

Table 1

Game matrix of JSC “Tashkent Fat and Oil” JSC (numbers in strategies are determined in millions of soums)

q_k^i %	S_1	S_2	S_3	S_4	S_5	S_6	S_7	L_i^p	W_i	M_i	Hur_i^p $\tau = 0.5$
0	16076	16076	16076	16076	16076	16076	16076	16076	16076	16076	16076
5	16220	16220	16220	16220	16220	16220	16220	16220	16220	16220	16220
10	16370	16370	16370	16370	16370	16370	16370	16370	16370	16370	16370
15	16521	16521	16521	16521	16521	16521	16521	16521	16521	16521	16521
20	16678	16678	16678	16678	16678	16678	16678	16678	16678	16678	16678
25	16837	16837	16837	16837	16837	16837	16837	16837	16837	16837	16837
30	16760	17002	17002	17002	17002	17002	17002	16967	16760	17002	16881
35	16462	17169	17169	17169	17169	17169	17169	17068	16462	17169	16815
40	16197	17123	17344	17344	17344	17344	17344	17149	16197	17344	16770
45	16060	17019	17522	17522	17522	17522	17522	17241	16060	17522	16791
50	15886	16818	17509	17709	17709	17709	17709	17293	15886	17709	16797
55	15568	16646	17449	17900	17900	17900	17900	17323	15568	17900	16734
60	15271	16557	17405	17791	18101	18101	18101	17332	15271	18101	16686
65	14297	16288	17301	17631	17931	18309	18309	17152	14297	18309	16302
70	12884	15787	17079	17543	17849	17999	18529	16810	12884	18529	15701
75	9893	14180	16166	17002	17561	17863	17772	15777	9893	17863	13878
80	7513	12192	14834	16045	17093	17569	17522	14681	7513	17569	12541
85	5306	9663	12826	14617	16122	16945	17120	13229	5305	17120	11213

(Source: Formed by an author)

What is understood from the matrix of games is that $A_1 - A_5$ strategies are considered dominant strategies and are left open in the analysis. We imagine that the financial manager does not have information about the events that may occur. If the financial manager evaluates all possible events with the same probability, he will achieve the optimal capital structure according to Laplace criterion in A_{16} , when the share of debt capital is 60%.

To put forward a plan to protect the optimal capital structure from the impact of uncertainties, the optimal debt ratio is also achieved in 25% according to the Walda criterion.

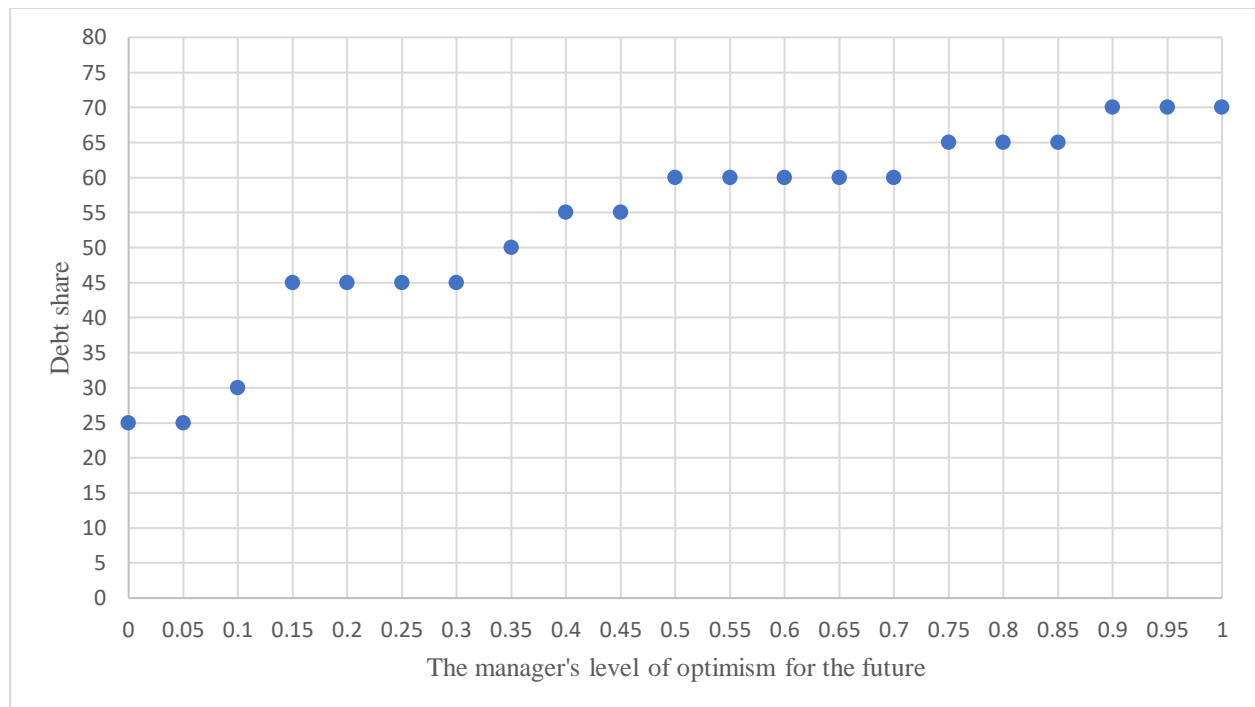
If the financial manager believes that the uncertainty factor will lead the company to the better side and increase its value, then according to the Maximax criterion the optimal capital is reached when debt ratio is 70%.

If the player remains neutral, and also the optimal decision is advanced compared to the above two criteria, the *Hurwitz criterion* is used. Considering the application of the Hurwitz criterion τ coefficient, this coefficient represents a linear combination of two criteria and is evaluated based on the player's level of optimism.

The Hurwitz criterion represents a linear combination between the Wald and Maximax criteria and helps to choose the most optimal strategy:

$$(Hur)_i^p(\tau) = (1 - \tau)W_i + \tau M_i, \quad i = 1, \dots, n$$

This criterion is not considered the right program when we consider the issues of capital structure optimization. Because this criterion only considers the worst and best options that can happen and discards the others. It is worth noting that in the worst-case scenario, the debt burden increases more than in the best-case scenario, and the worst-case scenario increases the cost of capital, and this situation causes the fundamental value of the enterprise to fall. We can see that with a level of optimism, the optimal capital structure is achieved in $\tau < 0.6$ when the debt ratio is 30%, and the company takes the most conservative path. The relationship between the level of optimism of the financial manager and the share of debt capital that the enterprise can receive is shown below.



Graph 1. The relationship between the level of optimism of the financial manager and the optimal capital structure in case of the “Tashkent Fat and Oil” JSC

(Source: prepared by an author)

The graph focuses on the best and worst possible events, but also relies more on the neutral nature of the Bayesian criterion.

If the player evaluates the probability of occurrence of events in the following way, by $e_1 = 0\%$, $e_2 = 0\%$, $e_3 = 10\%$, $e_4 = 60\%$, $e_5 = 30\%$, $e_6 = 0\%$, $e_7 = 0\%$, the highest effective indicator is provided at $q_k^i = 55\%$. This means that if the financial manager most likely does not expect the level of uncertainty factor influence, or evaluates its influence positively, the most optimal capital structure of the enterprise is only provided when the ratio of debt funds to private capital is 55%

Table 2
Game matrix for “Yoggar” JSC (numbers in strategies are determined in millions of soums)

q_k^i	S_1	S_2	S_3	S_4	S_5	S_6	S_7	L_i^p	W_i	M_i	Hur_i^p $\tau = 0.5$
0	332	332	332	332	332	332	332	332	332	332	332
5	337	337	337	337	337	337	337	337	337	337	337
10	341	341	341	341	341	341	341	341	341	341	341
15	345	345	345	345	345	345	345	345	345	345	345
20	349	349	349	349	349	349	349	349	349	349	349
25	354	354	354	354	354	354	354	354	354	354	354
30	359	359	359	359	359	359	359	359	359	359	359
35	363	363	363	363	363	363	363	363	363	363	363
40	377	368	368	368	368	368	368	369	368	377	372.5
45	356	383	373	373	373	373	373	372	356	383	369.5
50	353	367	378	378	378	378	378	373	353	378	365.5
55	349	364	384	384	384	384	384	376	343	384	363.5
60	343	363	381	390	390	390	390	378	343	390	366.5
65	342	361	379	396	396	396	396	381	342	396	369
70	333	357	377	402	402	402	402	382	333	402	367.5
75	319	354	374	388	408	408	408	380	319	408	363.5
80	218	305	361	373	403	403	403	352	218	403	310.5
85	125	225	300	360	382	393	396	312	125	396	260.5

(Source: Prepared by an author)

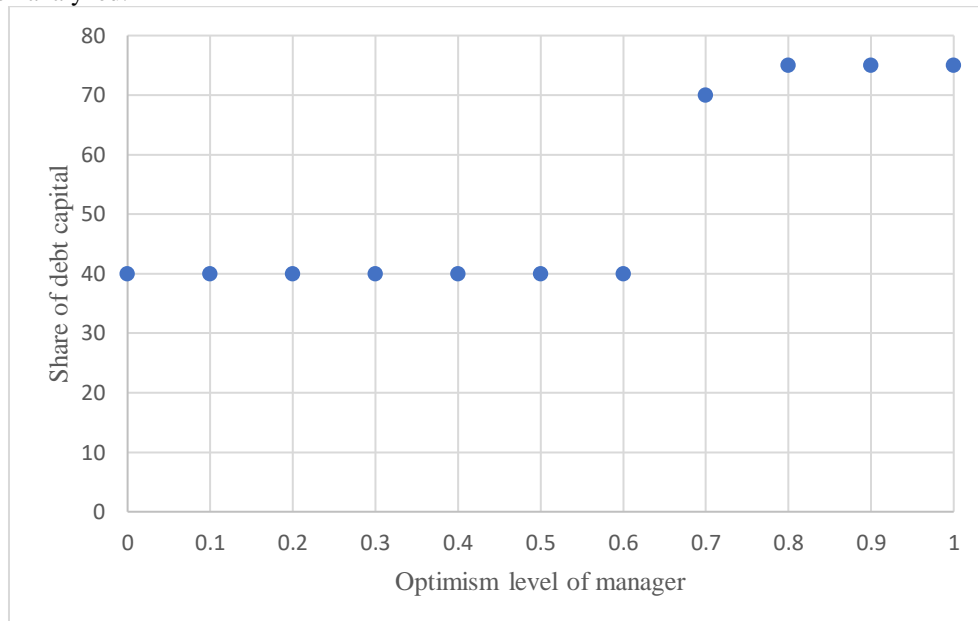
From the matrix of games, it is understood that $A_1 - A_8$ strategies are considered dominant strategies and are not analyzed. Let's imagine that the financial manager has no knowledge of the events that may occur.

If the financial manager evaluates all possible events with the same probability, he will reach the optimal capital structure in A_{15} according to the Laplace criterion, when ratio of debt capital is 70%.

If the financial manager puts forward a plan to protect the optimal capital structure from the effects of uncertainties, according to Wald criterion the optimal debt ratio is achieved in 40%.

If the financial manager believes in this, that is, the factor of uncertainties leads the enterprise in a better direction and increases its value, then according to the Maximax criterion the optimal capital is reached when the debt ratio is equal to 75%.

Below the relationship between the level of optimism of the financial manager and the share of debt capital has been analyzed.



Graph 2. The relationship between the level of optimism of the financial manager and the optimal capital structure in case of “Yoggar” JSC

(Source: Prepared by an author)

Each financial manager can make a different capital structure based on his forecasting situation and his view of the uncertainty factor.

CONCLUSION AND SUGGESTIONS

The table below shows the ways in which a financial manager can subjectively assess the probability of events under several scenarios:

Table 3
Capital structure optimization scenarios of “Tashkent Fat and Oil” JSC

Options	Probability of events							Optimal share
	e_1	e_2	e_3	e_4	e_5	e_6	e_7	
Negative scenario								25*
Forecast 1	100	0	0	0	0	0	0	25
Forecast 2	50	50	0	0	0	0	0	25
Forecast 3	0	100	0	0	0	0	0	30
Slight negative scenario								45*
Forecast 4	25	25	25	25	0	0	0	45
Forecast 5	0	25	50	25	0	0	0	45
Forecast 6	0	25	25	50	0	0	0	45
Forecast 7	0	0	50	50	0	0	0	45
Forecast 8	0	0	100	0	0	0	0	50
Neutral scenario								55*
Forecast 9	0	0	25	50	25	0	0	55
Forecast 10	0	0	33	33	33	0	0	55
Forecast 11	0	0	0	100	0	0	0	60
Slight positive								60*
Forecast 12	0	0	0	0	100	0	0	60
Forecast 13	0	0	0	50	50	0	0	60
Forecast 14	0	0	0	50	25	25	0	60
Forecast 15	0	0	0	25	50	25	0	60
Forecast 16	0	0	0	25	25	25	25	65
Positive								65*
Forecast 17	0	0	0	0	0	100	0	65
Forecast 18	0	0	0	0	0	50	50	65
Forecast 19	0	0	0	0	0	0	100	70

(Source: Prepared by an author)

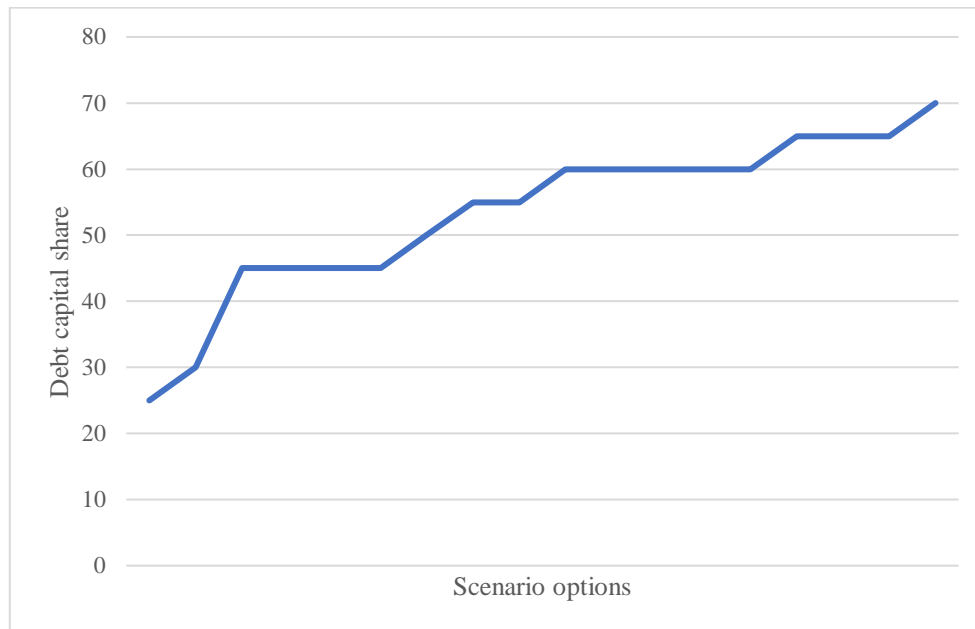
From the data in the table, it is determined that under the most negative scenarios, the company achieves its optimal capital structure when the debt ratio is equal to 25%. And this happens when the company's *EBIT* drops below 60% in the coming years.

If a slightly negative scenario is repeated, the optimal capital structure of the enterprise is ensured by the share of debt capital equal to 45%, and this process occurs when there is a probability that *EBIT* the enterprise will decrease by 30-50%.

If the enterprise is expected to operate at the same level in the future periods and the scenario is evaluated as neutral, the enterprise can increase the share of debt funds by 55%. It is in this case that the optimal capital structure is achieved.

If the future situation is considered to be somewhat positive, the financial manager of the enterprise should consider the possibility of raising the share of debt capital by 60%.

Also, if it is expected that the enterprise will increase *EBIT* to the level of more than 60% in the coming periods, the enterprise will be able to increase the share of debt capital by 65%. Based on the following graph, the scenarios that can be considered by the enterprise manager are presented.



Graph 3. Share of the debt capital of “Tashkent Fat and Oil” JSC under several scenarios

(Source: Prepared by an author)

It should not be forgotten that financial managers in enterprises can make different assessments of the probability of events that may occur through the Bayesian criterion in forming the optimal capital structure, and can make the optimal capital level look different. Therefore, optimal capital formation remains relevant in enterprises.

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