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APPLICATIONS OF THE COMPLEX NUMBER IN TRIGONOMETRIC FORM IN SOME PRACTICAL PROBLEMS

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ABSTRACT
In this article, the application of the trigonometric representation of a complex number in some sums, their sum is calculated by some substitutions.
KEYWORDS: ***complex number, radical formula, Moavr formula, sum***

Introduction. As we know, the "Complex number concept" is introduced for students of academic lyceums and specialized schools, and it is appropriate to solve some related issues through the trigonometric representation of a complex number. The trigonometric representation of a complex number and this the formula for raising a number to the nth power

$$z = r(\cos\varphi + i\sin\varphi), \qquad z^n = r^n(\cos n\varphi + i\sin n\varphi) \qquad (1)$$

Example 1.

Calculate the sums below

$$P = \cos\frac{\pi}{2n+1} + \cos\frac{3\pi}{2n+1} + \cdots + \cos\frac{(2n-1)\pi}{2n+1} = \sum_{k=1}^{n}\cos\frac{(2k-1)\pi}{2n+1}$$

$$Q = \sin\frac{\pi}{2n+1} + \sin\frac{3\pi}{2n+1} + \cdots + \sin\frac{2n-1}{2n+1} = \sum_{k=1}^{n}\sin\frac{2k-1}{2n+1}$$

To find the sums above, it is advisable to use the trigonometric form of complex numbers. For this, the second sum is multiplied by i and added to the first:

$$P + iQ = \left(\cos\frac{\pi}{2n+1} + i\sin\frac{\pi}{2n+1}\right) + \left(\cos\frac{3\pi}{2n+1} + i\sin\frac{3\pi}{2n+1}\right) + \cdots + \left(\cos\frac{(2n-1)\pi}{2n+1} + i\sin\frac{2n-1}{2n+1}\right)$$

If

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$$w = \cos\frac{\pi}{2n+1} + i\sin\frac{\pi}{2n+1}$$

according to Muavr's formula

$w^n = \left(\cos\frac{\pi}{2n+1} + i\sin\frac{\pi}{2n+1}\right)^n = \cos\frac{n\pi}{2n+1} + i\sin\frac{n\pi}{2n+1}$ bo'ladi.

$$P + iQ = w + w^3 + w^5 + \cdots + w^{2n-1} = w(1 + w^2 + w^4 + \cdots + w^{2n-2}) = w \cdot \frac{w^{2n}-1}{w^2-1}$$

$$= w \cdot \frac{w^{2n}-1}{w^2-1} \cdot \frac{w^{-1}}{w^{-1}} = \frac{w^{2n}-1}{w-w^{-1}} = \frac{\cos\frac{2n\pi}{2n+1} + i\sin\frac{2n\pi}{2n+1} - 1}{2i\sin\frac{\pi}{2n+1}}$$

$$= \frac{\sin\frac{2n\pi}{2n+1}}{2\sin\frac{\pi}{2n+1}} + i\frac{1-\cos\frac{2n\pi}{2n+1}}{2\sin\frac{\pi}{2n+1}}$$

Then, by equalizing the corresponding parts on both sides, this

$$P = \frac{\sin\frac{2n\pi}{2n+1}}{2\sin\frac{\pi}{2n+1}} \quad \text{va} \quad Q = \frac{1-\cos\frac{2n\pi}{2n+1}}{2\sin\frac{\pi}{2n+1}}$$

we will get the result.

Taking into account the following formulas, the following relations can be written:

$$\sin\frac{2n\pi}{2n+1} = \sin\frac{\pi}{2n+1}$$

$$\cos\frac{2n\pi}{2n+1} = -\cos\frac{\pi}{2n+1}$$

$$1 - \cos\frac{2n\pi}{2n+1} = 2\cos^2\frac{\pi}{2(2n+1)}$$

$$\sin\frac{\pi}{2n+1} = 2\sin\frac{\pi}{2(2n+1)} \cdot \cos\frac{\pi}{2(2n+1)}$$

Based on the above, the following radical formula can be written:

$$P = \frac{1}{2}$$

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$$Q = \frac{1}{2}\cot\frac{\pi}{2(2n+1)}$$

Example 2.

Prove the following equality.

$$\cos\frac{\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{5\pi}{7} = \frac{1}{2}$$

To prove this equality, without using the usual trigonometric properties, we show it by trigonometric substitutions of complex numbers. First of all

$z = \cos\frac{\pi}{7} + i\sin\frac{\pi}{7}$ we enter a complex number whose modulus is equal to 1,

$|z| = 1$. We can find the 7th power of the given complex number using the above Muavr formula and get the following result:

$$z^7 = \left(\cos\frac{\pi}{7} + \cos\frac{\pi}{7}\right)^7 = \cos\pi + i\sin\pi = -1 \text{ va } z^7 + 1 = 0.$$

On the other hand, we have the following equality:

$$\cos\frac{\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{5\pi}{7} = \frac{1}{2}\left(z + \frac{1}{z}\right) + \frac{1}{2}\left(z^3 + \frac{1}{z^3}\right) + \frac{1}{2}\left(z^5 + \frac{1}{z^5}\right)$$
$$= \frac{z^{10} + z^8 + z^6 + z^4 + z^2 + 1}{2z^5}$$

$z^7 + 1 = 0$ orqali quyidagi tengliklarga erishamiz:

$$z^{10} = -z^3 \text{ va } z^8 = -z.$$

From this equation

$$z^{10} + z^8 + z^6 + z^4 + z^2 + 1 = z^6 + z^4 - z^3 + z^2 - z + 1$$
$$= z^6 - z^5 + z^4 - z^3 + z^2 - z + 1 + z^5 = \frac{z^7 + 1}{z + 1} + z^5 = z^5$$

Accordingly, this equality is proved:

$$\cos\frac{\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{5\pi}{7} = \frac{z^5}{2z^5} = \frac{1}{2}$$

Example 3.

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Calculate the following sum.

$$S_n = \sin\alpha + \sin 2\alpha + \cdots + \sin n\alpha$$

To calculate the above sum, we enter the sum C_n

$C_n = \cos\alpha + \cos 2\alpha + \cdots + \cos n\alpha.$

$z = \cos\alpha + i\sin\alpha$ The trigonometric form of the complex number is known. We multiply the sum of $S_n$by i and add it to the sum of C_n to get the following sum:

$$C_n + iS_n = \cos\alpha + i\sin\alpha + \cos 2\alpha + i\sin 2\alpha + \cdots + \sin n\alpha + i\cos n\alpha$$
$$= z + z^2 + \cdots + z^n = z\frac{z^n - 1}{z - 1}$$

hrough trigonometric substitutions known to us $\cos x - 1 = -2\sin^2\frac{x}{2}$ va $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$ accordingly

$$\frac{z^n - 1}{z - 1} = \frac{\cos n\alpha + i\sin n\alpha - 1}{\cos\alpha + i\sin\alpha - 1} = \frac{-2\sin^2\frac{n\alpha}{2} + 2i\sin\frac{n\alpha}{2}\cos\frac{n\alpha}{2}}{-2\sin^2\frac{\alpha}{2} + 2i\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$$
$$= \frac{\sin\frac{n\alpha}{2}}{\sin\frac{\alpha}{2}}\left(\frac{\cos\frac{n\alpha}{2} + i\sin\frac{n\alpha}{2}}{\cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}}\right) = \frac{\sin\frac{n\alpha}{2}}{\sin\alpha}\left(\cos\frac{(n-1)\alpha}{2} + i\sin\frac{(n-1)\alpha}{2}\right).$$

From the above equation, we get the following result:

$$C_n + iS_n = (\cos\alpha + i\sin\alpha)\frac{\sin\frac{n\alpha}{2}}{\sin\alpha}\left(\cos\frac{(n-1)\alpha}{2} + i\sin\frac{(n-1)\alpha}{2}\right)$$
$$= \frac{\sin\frac{n\alpha}{2}}{\sin\alpha}\left(\cos\frac{(n-1)\alpha}{2} + i\sin\frac{(n-1)\alpha}{2}\right).$$

By separating the real and abstract parts of this equation, we find the sums S_n and C_n:

$$S_n = \frac{\sin\frac{n\alpha}{2}\sin\frac{(n+1)\alpha}{2}}{\sin\frac{\alpha}{2}}$$

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$$C_n = \frac{\sin\frac{n\alpha}{2}\cos\frac{(n+1)\alpha}{2}}{\sin\frac{\alpha}{2}}$$

In conclusion, it should be said that when calculating certain sums, it is more convenient to calculate using the trigonometric representation of a complex number, and many sums of this type can be made in practice.

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