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FUZZY HYPERGRAPH AND CONVEX FUZZY SETS

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ABSTRACT

In this paper, we examine the relation between fuzzy sets and fuzzy hypergraphs. How significant are they for the study of fuzzy sets? Fuzzy hypergraphs are used to study convex fuzzy sets.How the projection of hypergraphs to the interval (0,1] at various hyperplanes will be studied.

KEYWORDS: *Fuzzy sets, Convex Fuzzy sets , Fuzzy hypergraphs, hyperplanes.*

I. INTRODUCTION

Earlier Zadeh has used the term fuzzy sets while researching for the project named as pattern recognition.The terms fuzzy set, convex fuzzy set, and α-cut were initially introduced by Prof. Zadeh in 1965[1].Convex and concave fuzzy mappings were added to the notion by Yu-Ru Syau [11]. Sarkar [12] not only introduced concavo-convex fuzzy sets but also illustrated some other intriguing characteristics of this particular kind of fuzzy set. Ban constructed and thoughtfully explored convex temporal intuitionistic fuzzy sets as well as convex intuitionistic fuzzy sets [13, 14]. The generalised features of the aggregation of convex intuitionistic fuzzy sets were thoroughly analysed and characterised by Díiaz et al. [15].

Scholars Syau [5] and Xinmin Yang [2] demonstrated closed and convex fuzzy sets and investigated how they related to one another. In their study, Nadaban and Dzitac[4] discriminated between several forms of fuzzy relations and also gave examples of convex fuzzy relations. Chen-Wei-Xu[6] produced novel fuzzy relations and convexity results for fuzzy relations based on earlier work.We define fuzzy hypergraph; extension of hypergraph of concave function and used it to prove that if a fuzzy set B is convex, if and only if set of points in the fuzzy hypergraph is convex. We used hyperplane projection at different points of the domain to the interval (0,1] of the codomain; we will get length of different intervals which are increasing and decreasing and constant for some points of the domain.Vertical $\alpha - cut$ of the fuzzy set is used to study the length of the vertical line joining the hyperplane.

II. PRELIMINARIES

Throughout this paper, B denotes fuzzy set defined on M denotes fuzzy relation defined on R^2 Here are some definitions that will be useful in this paper.

2.1Definition[6]

A fuzzy set B defined on R is a function; $B: R \to [0,1]$ is called as membership function and $B(x)$ is called membership grade of $B\ at\ x$.

2.2Definition[3]

M be fuzzy relation on $X \times Y$. Then T is convex if and only if $M\big(\mu(x_1, y_1) + (1-\mu)(x_2, y_2)\big) \geq$ $min[M(x_1, y_1) \wedge M(x_2, y_2)]$; $\forall\ (x_1, y_1), (x_2, y_2) \in X \times Y\ and\ \ \mu \in [0,1]$.

2.3Definition[6]

Let M be a fuzzy relation defined on $X \times Y$ and α be such that $0 < \beta \leq 1$.Then $Complementary\ \beta -$ set of M is denoted by $M_\beta -$ is defined by $M_\beta -= \{(x,y) \in X \times Y / T(x,y) \leq\ \beta\}$.

2.4 Definition [1]

B be a fuzzy set defined on R and α be such that $0 < \beta \leq 1$.Then $complementary\ \beta - set$ of B, is denoted by B_β and defined by $C.B_\beta = \{x \in R / B(x) \leq \beta\}$ is a crisp set.

2.5 Definition [1]

B be a fuzzy set defined on R. Then B is concave if and only if
$B(\mu x_1 + (1-\mu)x_2) \geq min[B(x_1), B(x_2)]$; $\forall\ x_1, x_2 \in R\ and\ \mu \in (0,1]$.

2.6 Definition[2]
A fuzzy set B on R is said to be strongly convex fuzzy set if
$B(\mu x_1 + (1-\mu)x_2) > min[B(x_1), B(x_2)]$; $\forall\, x_1, x_2 \in R, x_1 \neq x_2$ and $\mu \in (0,1)$.

2.7 Definition [2]
A fuzzy set B on is said to be strictly concave fuzzy set if
$B(\mu x_1 + (1-\mu)x_2) > min[B(x_1), B(x_2)]$; $B(x_1) \neq B(x_2), \forall x_1, x_2 \in R$ *and* $\mu \in (0,1)$.

2.8 Definition
B be a fuzzy set defined on R. then fuzzy hypergraph of B is denoted by $f.hyperg(B)$ is defined by
$f.hyperg(B) = \{(p,q) \in R \times (0,1]$ *and* $B(p) \geq q\}$.

III. MAIN RESULTS

3.1 Theorem
B be a fuzzy set defined on R then B is Convex fuzzy set if and only if the set of points below the fuzzy hypergraph; f.hyperg(B) is convex.

Proof

Suppose B is a convex fuzzy set defined on R.

to prove that the set of points below the fuzzy hypergraph; f.hyperg(B) is convex.

Let, if possible,the set of points below the fuzzy hypergraph; f.hyperg(B) is not convex.

Then for some $(s,t), (x,y) \in f.hyperg(B)$, we have $\vartheta(s,t) + (1-\vartheta)(x,y) \notin f.hyperg(B)$.

Then, $\vartheta(s,t) + (1-\vartheta)(x,y) = (\vartheta s + (1-\vartheta)x, \vartheta t + (1-\vartheta)y)$;

That is $B(\vartheta s + (1-\vartheta)x) \leq \vartheta t + (1-\vartheta)y, 0 < \vartheta \leq 1$.

Therefore, $B(\vartheta s + (1-\vartheta)t) = \vartheta B(s) + (1-\vartheta)B(t) \leq \vartheta t + (1-\vartheta)y$.

$(\vartheta B(s) \leq \vartheta t\,) + ((1-\vartheta)B(t) \leq (1-\vartheta)y)$.

Implies that, A contradiction.

Therefore, the set of points below the fuzzy hypergraph; f.hyperg(B) is convex.

Conversely suppose that the set of points below the fuzzy hypergraph; f.hyperg(B) is convex.

To prove that B is a convex fuzzy set.

Let $x, y \in B$ *be any arbitrary points and* $\rho \in (0,1]$.

We have $(x, B(x))$ *and* $(y, B(y)) \in f.hyperg(B)$.

Then $\rho(x, B(x)) + (1-\rho)(y, B(y)) = \rho x + (1-\rho)y, \rho B(x) + (1-\rho)B(y) \in f.hyperg(B)$.

$\rho x + (1-\rho)y \in B;\ \rho \in (0,1]$.

Therefore, B is a convex fuzzy set.

3.2 Corollary
B be a strongly (strictly) convex fuzzy set defined on R then $strong\ f.hyperg(B)^{\square}$ is convex; for all $\alpha \in (0, 1)$, where $strong\ f.hyperg(B)^{\square}$ is strong fuzzy hypergraph of B.

Definition
Vertical $\alpha - cut$ of fuzzy set B is defined as $B^{\alpha\uparrow}$, is defined as length of the interval α.

3.3 Theorem
Let B be a convex fuzzy set defined on R if and only if there exists unique Vertical $\alpha - cut$ or of maximum length or interval in R for which vertical $\alpha - cut$ of maximum length.
Proof.
Let B be a convex fuzzy set defined on R

To prove that there exists unique Vertical $\alpha - cut$ or of maximum length or interval in R for which vertical $\alpha - cut$ of maximum length.
Since B is convex if and only if every $\alpha - cut$ of B is convex. Convex subset of R is a singleton set or an interval. If $\alpha - cut$ is singleton then $\alpha\ is\ of\ maximum\ length.$
If $\alpha - cut$ is an interval then that α has maximum length.
Conversely, suppose that there exists unique Vertical $\alpha - cut$ or of maximum length or interval in R for which vertical $\alpha - cut$ of maximum length.
To prove that B is a convex fuzzy set defined on R.
If unique Vertical $\alpha - cut$ is exist then corresponding $\alpha - cut$ is convex.
If Vertical $\alpha - cut$ is an interval in R then the interval in R is convex.
Thus every $\alpha - cut$ in R is convex.therefore, B is convex fuzzy set defined on R.

3.4 Corollary
Let B be a convex fuzzy set defined on R if and only if there exists unique Vertical $\alpha - cut$ of maximum length.

3.5 Theorem
B be a convex fuzzy set defined on R then the projection of hyperplane parallel to (0,1] at point of domain is monotonically increasing in the subset of R following the monotonically decreasing subset of R or constant or vice versa..

Proof
Let B be a convex fuzzy set defined on R.
Let ς be a hyperplane parallel to (0,1].
To prove that the projection of hyperplane parallel to (0,1] at point of domain is monotonically increasing in the subset of R following the monotonically decreasing subset of R
Without loss of generality, assume that first the projection of the hyperplane parallel to (0,1] at point of domain is monotonically increasing or constant.
It is remaining to show that following projection of hyperplane parallel to (0,1] at point of domain is the monotonically decreasing subset of Ror constant
Suppose not.Then there is $x, y \in R\ such\ that\ x < y\ and\ B(x) > B(y).$
Consider, $B(\lambda x + (1-\lambda)y) \geq min[B(x), B(y)] = B(x).$
For $\lambda = 0, we\ have\ B(y) > B(x).$
A contradiction. Therefore, following projection of hyperplane parallel to (0,1] at point of domain is the monotonically decreasing subset of Ror constant

IV. CONCLUSION
Convexity of fuzzy sets has been studied using fuzzy hypergraphs and Vertical α- sets. The relation between the projection of the hyperplane of fuzzy hypergraphs and the convex fuzzy set was shown. Between fuzzy and crisp sets, the Vertical α-set serves as a link. Fuzzy approaches to convexity research are essential on many levels due to the wide applications of convexity in many fields.

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