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SEPARATION AXIOMS USING n-OPEN SETS

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ABSTRACT

The aim of this paper is to study some separation axioms (namely nT_0, nT_1 and nT_2) in topological spaces and obtain some basic properties of these separation axioms. Moreover the relationships among nT_0, nT_1 and nT_2 and some other existing separation axioms are investigated and show that the separation axioms nT_0, nT_1 and nT_2 and the separation axioms T_0, T_1 and T_2 are independent of each other, and give some examples in support of these relationships. Further we also introduce nR_0 and nR_1 spaces using n-open sets and obtain some basic properties and characterizations of these spaces.

KEYWORDS: *n-open sets; n-continuous and n-irresolute functions; nT_0, nT_1, nT_2, nR_0 and nR_1 spaces.*
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1. INTRODUCTION

The concept of T_0-axiom was introduced by A. N. Kolmogorov (1803-1887). The notion of T_1-axiom was introduced by Frechet in 1907. In 1914, Hausdorff introduced the T_2-property. In 1925, Urysohn [**12**] introduced and studied a new type of separation axiom, called Urysohn space. In 1943, Shanin [**10**] introduced and studied the notion of R_0-axiom in topological spaces. In 1961, Davis [**2**] introduced the concept of R_1-axiom and investigated R_0-axiom. In 1975, Dunham [**6**] used T_0-identification spaces and obtained some characterizations of R_1-spaces. In 1977, Dorsett [**3**] studied the R_0-axiom and obtained some characterizations of R_0-axiom in the term of T_1. In 1986, Munshi [**9**] introduced and studied some new class of separation axioms (named g_3-axiom, g_4-axiom, T_3-axiom, T_4-axiom etc.) in topological spaces and obtained their characterizations. In 2007, Dorsett [**4**] further investigated T_i-spaces (i = 0, 1, 2) and obtained their characterizations. In 2019, Kumar [**7**] introduced the concepts of ii-T_0, ii-T_1 and ii-T_2 separation axioms utilizing ii-open sets and obtained some characterizations. In 2021, Kumar [**8**] used η-open sets to introduce the concepts of η-T_0, η-T_1 and η-T_2 separation axioms and investigated a relationship among η-separation axioms and other existing separation axioms. In 2021, Baker [**1**] introduced and studied the concept of n-open sets and obtained some properties of n-open sets in topological spaces.

2. PRELIMINARIES

Throughout this paper, spaces (X, ℑ), (Y, τ), and (Z, η) (or simply X, Y and Z) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. For a subset A of X, **cl(A)** and **int(A)** represents the closure of A and Interior of A respectively.

Definition 2.1. A subset A of a topological space (X, ℑ) is said to be **n-open** [**1**] if int(A) ≠ cl(A). A subset A of a space X is called n-closed if its complement is n-open.

We denote the family of all n-open (resp. n-closed) sets of a topological space by n-O(X) (resp. n-C(X)). The **n-closure** of a subset A of X, is the intersection of all n-closed sets containing A and is denoted by **n-cl(A)**. The **n-interior** of a subset A of X is the union of all n-open sets contained in A and is denoted by **n-int(A)**.

Definition 2.2. A space X is said to be:
(i) **T_0-space** if for each pair of distinct points x and y of X, there exists an open set G containing x but not y or an open set H containing y but not x.
(ii) **T_1-space** if for each pair of distinct points x, y of X, there exist an open set G containing x but not y and an open set H containing y but not x.
(iii) **T_2** (or **Hausdorff** or **Separated**) space if for each pair of distinct points x, y of X, there exist two disjoint open sets U and V containing x and y respectively.

For a space, we have following implications:

$T_2 \rightarrow T_1 \rightarrow T_0$

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Where none of the implications are reversible.

Definition 2.3. A space X is said to be:
(i) **ii-T_0-space** [**7**] if for each pair of distinct points x and y of X, there exists an ii-open set G containing x but not y or an ii-open set H containing y but not x.
(ii) **ii-T_1-space** [**7**] if for each pair of distinct points x, y of X, there exist an ii-open set G containing x but not y and an ii-open set H containing y but not x.
(iii) **ii-T_2** [**7**] (or **ii-Hausdorff** or **ii-Separated**) space if for each pair of distinct points x, y of X, there exist two of disjoint ii-open sets U and V containing x and y respectively.

For a space, we have following implications [**7**]:

$$\text{ii-}T_2 \rightarrow \text{ii-}T_1 \rightarrow \text{ii-}T_0$$

Where none of the implications are reversible.

Definition 2.4. Let X and Y be topological spaces. A function $f : X \rightarrow Y$ is said to be **n-continuous** [**1**] if the inverse image of every open set in Y is n-open in X.

3. nT_0-SPACES

In this section, we study nT_0-axiom and obtain some of their basic properties using n-open sets.

Definition 3.1. A space X is said to be **nT_0-space** [**1**] if for each pair of distinct points x and y of X, there exists an n-open set G containing x but not y or an n-open set H containing y but not x.

Example 3.2. Consider the space (X, $\Im$), where X = {a, b, c} and $\Im$ = {ϕ, {a}, {b}, {a, b}, X}. Taking a $\neq$ b two distinct points and U = {a} be an open set such that a $\in$ U = {a} and b $\notin$ U = {a} or V = {b} be any another open set such that b $\in$ V = {b} and a $\notin$ V = {b}. Hence the (X, $\Im$) is T_0-space.
Here n-open sets are {a}, {b}, {c}, {a, b}, {a, c}, {b, c}. Hence the space (X,$\Im$) is also nT_0-space.

Example 3.3. Consider the space (X, $\Im$), where X = {a, b, c} and $\Im$ = {ϕ, {c}, {a, b}, X}. Taking a $\neq$ b two distinct points, there does not exist an open set U such that a $\in$ U and b $\notin$ U or V be any another open set such that b $\in$ V and a $\notin$ V. Hence the (X, $\Im$) is not T_0-space.

Here n-open sets are {a}, {b}, {a, c}, {b, c}. Taking a $\neq$ b two distinct points and U = {a} be an n-open set such that a $\in$ U = {a} and b $\notin$ U = {a} or V = {b} be any another n-open set such that b $\in$ V = {b} and a $\notin$ V = {b}. Hence the (X, $\Im$) is nT_0-space.
Hence the T_0-space and nT_0-space are independent of each other.

Theorem 3.4. [**1**] A subset A of a space X is n-open if A is not clopen.

Lemma 3.5. [**1**] A space X has an n-open subset if and only if it is not discrete.

Remark 3.6. [**1**] Obviously a space is discrete if and only if there are no n-open sets.

Theorem 3.7. [**1**] If X is not discrete, then X is an nT_0-space.

Example 3.8. Every discrete space is a T_0-space but not nT_0-space because in a discrete space, every subset is closed and open both. So every subset is clopen. Hence, there does not exist any n-open set. So, every discrete space is not nT_0-space.

Example 3.9. Any indiscrete space (X, I) is not T_0-space. Let x, y be two distinct points of X. Now the only open nbhd of x is X which also contains y. Thus there exists no open nbhd of x which does not contain y. Hence the space (X, I) is not a T_0-space.

Theorem 3.10. A topological space (X, $\Im$) is nT_0-space if and only if for each pair of distinct points x, y of X, n-cl({x}) $\neq$ n-cl({y}).

Proof. **Necessity**. Let (X, $\Im$) be an nT_0-space and x, y be any two distinct points of X. There exists an n-open set U containing x or y, say x but not y. Then X − U is an n-closed set which does not contain x but contains y. Since n-cl({y}) is the smallest n-closed set containing y, n-cl({y}) $\subset$ X − U and therefore x $\notin$ n-cl({y}). Consequently n-cl({x}) $\neq$ n-cl({y}).

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Sufficiency. Suppose that x, y ∈ X, x ≠ y and n-cl({x}) ≠ n-cl({y}). Let z be a point of X such that z ∈ n-cl({x}) but z ∉ n-cl({y}). We claim that x ∉ n-cl({y}). For, if x ∈ n-cl({y}) then n-cl({x}) ⊂ n-cl({y}). This contradicts the fact that z ∉ n-cl({y}). Consequently x belongs to the n-open set X – n-cl({y}) to which y does not belong.

Theorem 3.11. Every subspace of an nT_0-space is nT_0.
Proof. Let (Y,τ) be a subspace of a topological space (X, ℑ), where τ is the relative topology of ℑ on Y. Let x, y be two distinct points of Y. As Y ⊂ X, x and y are also distinct points of X and there exists an n-open set G such that x ∈ G but y ∉ G, since X is nT_0-space. Then G ∩ Y is an n-open set in (Y, τ) which contains x but does not contain y. Hence (Y, τ) is an nT_0-space.

4. nT_1-SPACES

In this section, we study nT_1-axiom and obtain some basic properties of nT_1-axiom using n-open sets.

Definition 4.1. A topological space (X, ℑ) is said to be **nT_1-space** [**1**] if for each pair of distinct points x, y in X, there exist two n-open sets U and V such that x ∈ U but y ∉ U and y ∈ V but x ∉ V.

Example 4.2. Consider the space (X, ℑ), where X = {a, b, c} and ℑ = {ϕ, {a}, {b}, {a, b}, X}. Taking a ≠ b two distinct points and U = {a} & V = {b} be two open sets such that a ∈ U = {a} but b ∉ U = {a} and b ∈ V = {b} but a ∉ V = {b}. Hence the space (X, ℑ) is T_1-space.
Here n-open sets are {a}, {b}, {c}, {a, b}, {a, c}, {b, c}. Hence the space (X, ℑ) is also nT_1-space.

Example 4.3. Consider the space (X, ℑ), where X = {a, b, c} and ℑ = {ϕ, {a}, {a, b}, {a, c} X}. Then the space (X, ℑ) is not T_1-space for X as a and b are two distinct points of X, there do not exist two open sets one containing a but not b, and the other containing b but not a, that is, there do not exist two open sets U and V such that a ∈ U but b ∉ U and b ∈ V but a ∉ V.
Here n-open sets are {a}, {b}, {c}, {a, b}, {a, c}, {b, c}. Hence the space (X, ℑ) is nT_1-space.

Hence the T_1-space and nT_1-space are independent of each other.

Example 4.4. Consider the space (X, ℑ), where X = {a, b, c} and ℑ = {ϕ, {c}, {a, b}, X}. Taking a ≠ c two distinct points and U = {c} & V = {a, b} be open sets such that c ∈ U = {c} but a ∉ U = {c} and b ∈ V = {a, b} but c ∉ V = {a, b}. Hence the space (X, ℑ) is T_1-space.
Here n-open sets are {a}, {b}, {c}, {a, b}, {a, c}, {b, c}. Hence the space (X, ℑ) is also nT_1-space.

Theorem 4.5. A topological space (X, ℑ) is nT_1-space if and only if the singletons are n-closed sets.
Proof. Let (X, ℑ) be nT_1-space and x be any point of X. Suppose y ∈ X – {x}, then x ≠ y and so there exists an n-open set U such that y ∈ U but x ∉ U. Consequently y ∈ U ⊂ X – {x}, that is X – {x} = ∪ {U : y ∈ X – {x}} which is n-open.

Conversely, suppose {p} is n-closed for every p ∈ X. Let x, y ∈ X with x ≠ y. Now x ≠ y implies y ∈ X – {x}. Hence X – {x} is an n-open set contains y but not x. Similarly X – {y} is an n-open set contains x but not y. Accordingly, X is an nT_1-space.

Definition 4.6. Let X and Y be topological spaces. A function f : X → Y is said to be **n-closed** (resp. **n-open**) if the image of every closed (resp. open) set in X is n-closed (resp. n-open) in Y.

Definition 4.7. Let X and Y be topological spaces. A function f : X → Y is said to be **n-irresolute** if the inverse image of every n-closed set in Y is n-closed in X.

Definition 4.8. Let X be a topological space. A subset N ⊂ X is called an **n-neighbourhood** (briefly **n-nbhd**) of a point x ∈ X if there exists an n-open set G such that x ∈ G ⊂ N.

Theorem 4.9. Let f : X → Y be an n-irresolute, injective map. If Y is nT_1, then X is nT_1.
Proof. Assume that Y is nT_1. Let x, y ∈ X with x ≠ y and f(x), f(y) ∈ Y with f(x) ≠ f(y) because f is injective then there exists a pair of n-open sets U, V of Y such that f(x) ∈ U, f(y) ∈ V and f(x) ∉ V, f(y) ∉ U. Then x ∈ f^{-1}(U), y ∉ f^{-1}(U) and y ∈ f^{-1}(V), x ∉ f^{-1}(V). Since f is n-irresolute, X is nT_1.

Theorem 4.10. Let f : X → Y be bijective.
(i) If f is n-continuous and (Y, $ℑ_2$) is T_1, then (X, $ℑ_1$) is nT_1.
(ii) If f is n-open and (X, $ℑ_1$) is T_1, then (Y, $ℑ_2$) is nT_1.
Proof. Let f : (X, $ℑ_1$) → (Y, $ℑ_2$) be bijective.

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(i) Suppose $f : (X, \Im_1) \to (Y, \Im_2)$ is n-continuous and $(Y, \Im_2)$ is T_1. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Since f is bijective, $y_1 = f(x_1) \neq f(x_2) = y_2$ for some $y_1, y_2 \in Y$. Since $(Y, \Im_2)$ is T_1, there exist open sets G sand H such that $y_1 \in G$ but $y_2 \notin G$ and $y_2 \in H$ but $y_1 \notin H$. Since f is bijective, $x_1 = f^{-1}(y_1) \in f^{-1}(G)$ but $x_2 = f^{-1}(y_2) \notin f^{-1}(G)$ and $x_2 = f^{-1}(y_2) \in f^{-1}(H)$ but $x_1 = f^{-1}(y_1) \notin f^{-1}(H)$. Since f is n-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are n-open sets in $(X, \Im_1)$. It follows that $(X, \Im_1)$ is n-T_1. This proves (i).

(ii) Suppose f is n-open and $(X, \Im_1)$ is T_1. Let $y_1 \neq y_2 \in Y$. Since f is bijective, there exist x_1, x_2 in X, such that $y_1 = f(x_1)$ and $f(x_2) = y_2$ with $x_1 \neq x_2$. Since $(X, \Im_1)$ is T_1, there exist open sets G and H in X such that $x_1 \in G$ but $x_2 \notin G$ and $x_2 \in H$ but $x_1 \notin H$. Since f is n-open, f(G) and f(H) are n-open in Y such that $y_1 = f(x_1) \in f(G)$ and $y_2 = f(x_2) \in f(H)$. Again since f is bijective, $y_2 = f(x_2) \notin f(G)$ and $y_1 = f(x_1) \notin f(H)$. Thus $(Y, \Im_2)$ is nT_1. This proves (ii).

Definition 4.11. A topological space $(X, \Im)$ is said to be **n-symmetric** if for x and y in X, $x \in$ n-cl({y}) implies $y \in$ n-cl({x}).

Theorem 4.12. If $(X, \Im)$ is a topological space, then the following properties are equivalent:
(i) $(X, \Im)$ is an n-symmetric space.
(ii) {x} is n-closed, for each $x \in X$.
Proof. (i) $\Rightarrow$ (ii). Assume that $\{x\} \subset U \in$ n-O(X), but n-cl({x}) $\not\subset$ U. Then n-cl({x}) $\cap (X - U) \neq \phi$. Now, we take $y \in$ n-cl({x}) $\cap (X - U)$, then by hypothesis $x \in$ n-cl({y}) $\subset X - U$ and $x \notin U$, which is a contradiction. Therefore {x} is n-closed, for each $x \in X$.

(ii) $\Rightarrow$ (i). Assume that $x \in$ n-cl({y}), but $y \notin$ n-cl({x}). Then $\{y\} \subset X -$ n-cl({x}) and hence n-cl({y}) $\subset X -$ n-cl({x}). Therefore $x \in X -$ n-cl({x}), which is a contradiction and hence $y \in$ n-cl({x}).

Corollary 4.13. If a topological space $(X, \Im)$ is an nT_1 space, then it is n-symmetric.
Proof. In an nT_1 space, every singleton is n-closed (**Theorem 4.5**) and therefore is by **Theorem 4.12**, $(X, \Im)$ is n-symmetric.

Corollary 4.14. If a topological space $(X, \Im)$ is n-symmetric and nT_0, then $(X, \Im)$ is nT_1.
Proof. Let $x \neq y$ and as $(X, \Im)$ is nT_0, we may assume that $x \in U \subset X - \{y\}$ for some $U \in$ n-O(X). Then $x \notin$ n-cl({y}) and hence $y \notin$ n-cl({x}). There exists an n-open set V such that $y \in V \subset X - \{x\}$ and thus $(X, \Im)$ is an nT_1 space.

5. nT_2-SPACES

In this section, we study nT_2-axiom and investigate some basic properties of nT_2-axiom using n-open sets.

Definition 5.1. A space X is said to be **nT_2** [**1**] (**n-Hausdorff** or **n-Separated**) **space** if for every pair of distinct points x and y in X, there exist disjoint n-open sets U and V of X containing x and y respectively.

Example 5.2. Consider the space $(X, \Im)$, where $X = \{a, b, c\}$ and $\Im = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Taking $a \neq b$ two distinct points and $U = \{a\}$ & $V = \{b\}$ be two disjoint open sets such that $a \in U = \{a\}$ and $b \in V = \{b\}$. Hence the space $(X, \Im)$ is T_2-space.

Here n-open sets are {a}, {b}, {c}, {a, b}, {a, c}, {b, c}. Hence the space $(X, \Im)$ is also nT_2-space.

Example 5.3. Consider the space $(X, \Im)$, where $X = \{a, b, c\}$ and $\Im = \{\phi, \{c\}, \{a, b\}, X\}$. Taking $a \neq c$ two distinct points and $U = \{c\}$ & $V = \{a, b\}$ be two disjoint open sets such that $c \in U = \{c\}$ and $b \in V = \{a, b\}$. Hence the space $(X, \Im)$ is T_2-space.

Here n-open sets are {a}, {b}, {c}, {a, b}, {a, c}, {b, c}. Hence the space $(X,\Im)$ is also nT_2-space.

Example 5.4. Consider the space $(X, \Im)$, where $X = \{a, b, c\}$ and $\Im = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Then the space $(X, \Im)$ is T_2-space, since each singleton set is open so that distinct points have disjoint nbhds.

But the space $(X, \Im)$ is not nT_2-space because every subset of X is open and closed both. So, every subset is clopen. So there does not exist any n-open set in $(X, \Im)$.

Hence the T_2-space and nT_2-space are independent of each other.

Example 5.5. Consider the space $(X, \Im)$, where $X = \{a, b, c\}$ and $\Im = \{\phi, \{a, b\}, X\}$. Then the space $(X, \Im)$ is not T_2-space for X as a and b are distinct points of X which do not having disjoint nbhds.
Here n-open sets are {a}, {b}, {c}, {a, b}, {a, c}, {b, c}. Hence the space $(X, \Im)$ is nT_2-space.

Hence the T_2-space and nT_2-space are independent of each other.

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Theorem 5.6. [**1**] If X is not discrete, then X is an nT_2-space.

Remark 5.7. For a space, we have following implications:

$nT_2 \rightarrow nT_1 \rightarrow nT_0$

Where none of the implications are reversible as can be seen from the above examples.

Theorem 5.8. Every nT_2-space is nT_1.
Proof. Let X be an nT_2-space. Let x and y be two distinct points of X. Since X is nT_2-space, there exist disjoint n-open sets U and V such that $x \in U$ and $y \in V$. Since U and V are disjoint, so $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$. Hence X is nT_1-space.

Theorem 5.9. For a topological space X, the following properties are equivalent:
(i) X is an nT_2-space.
(ii) Let $x \in X$. Then for each $x \neq y$, there exists an n-open set U such that $x \in U$ and $y \notin$ n-cl(U).
(iii) For each $x \in X$, $\cap$ {n-cl(U) : $U \in$ n-O(X) and $x \in U$} = {x}.
Proof. (i) $\Rightarrow$ (ii). Suppose X is an nT_2-space. Then for each $x \neq y$, there exist disjoint n-open sets U and V such that $x \in U$ and $y \in V$. Since V is n-open, V^c is n-closed and $U \subset V^c$. This implies that n-cl(U) $\subset V^c$. Since $y \notin V^c$, $y \notin$ n-cl(U).

(ii) $\Rightarrow$ (iii). If $y \neq x$, then there exists an n-open set U such that $x \in U$ and $y \notin$ n-cl(U). Therefore $y \notin \cap$ {n-cl(U) : $U \in$ n-O(X) and $x \in U$}. Therefore $\cap$ {n-cl(U) : $U \in$ n-O(X) and $x \in U$} = {x}. This proves (iii).

(iii) $\Rightarrow$ (i). Let $y \neq x$ in X. Then $y \notin \{x\} = \cap$ {n-cl(U) : $U \in$ n-O(X) and $x \in U$}. This implies that there exists an n-open set U such that $x \in U$ and $y \notin$ n-cl(U). Let $V = (\text{n-cl}(U))^c$. Then V is n-open and $y \in V$. Now $U \cap V = U \cap (\text{n-cl}(U))^c \subset U \cap (U)^c = \phi$. Therefore, X is nT_2-space.

Theorem 5.10. Let $f : X \rightarrow Y$ be a bijection.
(i) If f is n-open and X is T_2, then Y is nT_2.
(ii) If f is n-continuous and Y is T_2, then X is nT_2.
Proof. Let $f : X \rightarrow Y$ be a bijection.
(i) Suppose F is n-open and X is T_2. Let $y_1 \neq y_2 \in Y$. Since f is a bijection. There exist x_1, x_2 in X such that $f(x_1) = y_1$ and $f(x_2) = y_2$ with $x_1 \neq x_2$. Since X is T_2, there exist disjoint open sets U and V in X such that $x_1 \in U$ and $x_2 \in V$. Since f is n-open, f(U) and f (V) are n-open in Y such that $y_1 = f(x_1) \in f(U)$ and $y_2 = f(x_2) \in f(V)$. Again since f is a bijection, f(U) and f(V) are disjoint in Y. Thus Y is nT_2.

(ii) Suppose $f : X \rightarrow Y$ is n-continuous and Y is T_2. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Let $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since f is one-one, $y_1 \neq y_2$. Since Y is T_2, there exist disjoint open sets U and V containing y_1 and y_2 respectively. Since f is n-continuous bijective, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint n-open sets containing x_1 and x_2 respectively. Thus X is nT_2.

Theorem 5.11. A topological space $(X, \Im)$ is nT_2 if and only if the intersection of all n-closed, n-neighbourhoods of each point of the space is reduced to that point.
Proof. Let $(X, \Im)$ be nT_2 and $x \in X$. Then for each $y \neq x$ in X, there exist disjoint n-open sets U and V such that $x \in U$, $y \in V$. Now $U \cap V = \phi$ implies $x \in U \subset V^c$. Therefore V^c is an n-neighbourhood of x. Since V is n-open, V^c is n-closed and n-neighbourhood of x to which y does not belong. That is there is an n-closed, n-neighbourhood of x which does not contain y. So we get the intersection of all n-closed, n-neighbourhood of x is {x}.

Conversely, let x, $y \in X$ such that $x \neq y$ in X. Then by assumption, there exist an n-closed, n-neighbourgood V of x such that $y \notin V$. Now there exists an n-open set U such that $x \in U \subset V^c$. Thus U and V^c are disjoint n-open sets containing x and y respectively. Thus $(X, \Im)$ is nT_2.

Theorem 5.12. If $f : X \rightarrow Y$ be bijective, n-irresolute map and X is nT_2, then $(X, \Im_2)$ is nT_2.
Proof. Suppose $f : (X, \Im_1) \rightarrow (Y, \Im_2)$ is bijective. f is n-irresolute, and $(Y, \Im_2)$ is n-T_2. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Since f is bijective, $y_1 = f(x_1) \neq f(x_2) = y_2$ for some $y_1, y_2 \in Y$. Since $(Y, \Im_2)$ is nT_2, there exist disjoint n-open sets G and H such that $y_1 \in G$ and $y_2 \in H$. Again since f is bijective, $x_1 = f^{-1}(y_1) \in f^{-1}(G)$ and $x_2 = f^{-1}(y) \in f^{-1}(H)$. Since f is n-irresolute, $f^{-1}(G)$ and $f^{-1}(H)$ are n-open sets in $(X, \Im_1)$. Also f is bijective, $G \cap H = \phi$ implies that $f^{-1}(G) \cap f^{-1}(H) = f^{-1}(G \cap H) = f(\phi) = \phi$. It follows that $(X, \Im_2)$ is nT_2.

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6. nR_k Space (k = 0, 1)

In this section, using n-open sets two new classes of topological spaces called, nR_0 and nR_1 spaces are introduced and obtain their characterizations.

Definition 6.1. A topological space $(X, \Im)$ is said to be **nR_0** if U is an n-open set and $x \in U$ then n-cl({x}) $\subset$ U.

Theorem 6.2. For a topological space $(X, \Im)$ the following properties are equivalent:
(i) $(X, \Im)$ is nR_0.
(ii) For any $F \in$ n-C(X), $x \notin F$ implies $F \subset U$ and $x \notin U$ for some $U \in$ n-O(X).
(iii) For any $F \in$ n-C(X), $x \notin F$ implies $F \cap$ n-cl({x}) = ϕ.
(iv) For any distinct points x and y of X, either n-cl({x}) = n-cl({y}) or n-cl({x}) $\cap$ n-cl({y}) = ϕ.
Proof. (i) $\Rightarrow$ (ii). Let $F \in$ n-C(X) and $x \notin F$. Then by (i), n-cl({x}) $\subset X - F$. Set U = X − n-cl({x}), then U is an n-open set such that $F \subset U$ and $x \notin U$.

(ii) $\Rightarrow$ (iii). Let $F \in$ n-C(X) and $x \notin F$. There exists $U \in$ n-O(X) such that $F \subset U$ and $x \notin U$. Since $U \in$ n-O(X), U $\cap$ n-cl({x}) = ϕ and F $\cap$ n-cl({x}) = ϕ.

(iii) $\Rightarrow$ (iv). Suppose that n-cl({x}) $\neq$ n-cl({y}) for distinct points $x, y \in X$. There exists $z \in$ n-cl({x}) such that $z \notin$ n-cl({y}) (or $z \in$ n-cl({y}) such that $z \notin$ n-cl({x})). There exists $V \in$ n-O(X) such that $y \notin V$ and $z \in V$; hence $x \in V$. Therefore, we have $x \notin$ n-cl({y}). By (iii), we obtain n-cl({x}) $\cap$ n-cl({y}) = ϕ.

(iv) $\Rightarrow$ (i). Let $V \in$ n-O(X) and $x \in V$. For each $y \notin V$, $x \neq y$ and $x \notin$ n-cl({y}). This shows that n-cl({x}) $\neq$ n-cl({y}). By (iv), n-cl({x}) $\cap$ n-cl({y}) = ϕ for each $y \in X - V$ and hence n-cl({x}) $\cap$ ($\cup_{y \in X - V}$ n-cl({y})) = ϕ. On other hand, since $V \in$ n-O(X) and $y \in X - V$, we have n-cl({y}) $\subset X - V$ and hence $X - V = \cup_{y \in X - V}$ n-cl({y}). Therefore, we obtain (X − V) $\cap$ n-cl({x}) = ϕ and n-cl({x}) $\subset$ V. This shows that $(X, \Im)$ is an nR_0 space.

Theorem 6.3. If a topological space $(X, \Im)$ is nT_0 and an nR_0 space then it is nT_1.
Proof. Let x and y be any distinct points of X. Since X is nT_0, there exists an n-open set U such that $x \in U$ and $y \notin U$. As $x \in U$ implies that n-cl({x}) $\subset$ U. Since $y \notin U$, so $y \notin$ n-cl({x}). Hence $y \in V = X -$ n-cl({x}) and it is clear that $x \notin V$. Hence it follows that there exist n-open sets U and V containing x and y respectively, such that $y \notin U$ and $x \notin V$. This implies that X is nT_1.

Theorem 6.4. For a topological space $(X, \Im)$ the following properties are equivalent:
(i) $(X, \Im)$ is nR_0.
(ii) $x \in$ n-cl({y}) if and only if $y \in$ n-cl({x}), for any points x and y in X.
Proof. (i) $\Rightarrow$ (ii). Assume that X is nR_0. Let $x \in$ n-cl({y}) and V be any n-open set such that $y \in V$. Now, by hypothesis, $x \in V$. Therefore, every n-open set which contain y contains x also. Hence $y \in$ n-cl({x}).

(ii) $\Rightarrow$ (i). Let U be an n-open set and $x \in U$. If $y \notin U$, then $x \notin$ n-cl({y}) and hence $y \notin$ n-cl({x}). This implies that n-cl({x}) $\subset$ U. Hence $(X, \Im)$ is nR_0.

From **Definition 4.11** and **Theorem 6.4**, the notions of n-symmetric and nR_0 are equivalent.

Definition 6.5. Let A be a subset of a topological space $(X, \Im)$. The **n-kernel** of A, denoted by n-ker(A) is defined to be the set
n-ker(A) = $\cap$ {U $\in$ n-O(X) : A $\subset$ U}.

Theorem 6.6. Let $(X, \Im)$ be a topological space and $x \in X$. Then $y \in$ n-ker({x}) if and only if $x \in$ n-cl({y}).
Proof. Suppose that $y \notin$ n-ker({x}). Then there exists an n-open set V containing x such that $y \notin V$. Therefore, we have $x \notin$ n-cl({y}). The proof of the converse case can be done similarly.

Theorem 6.7. Let $(X, \Im)$ be a topological space and A be a subset of X. Then, n-ker(A) = {$x \in X$: n-cl({x}) $\cap$ A $\neq \phi$}.
Proof. Let $x \in$ n-ker(A) and suppose n-cl({x}) $\cap$ A = ϕ. Hence $x \notin X -$ n-cl({x}) which is an n-open set containing A. This is impossible, since $x \in$ n-ker(A). Consequently, n-cl({x}) $\cap$ A $\neq \phi$. Next, let $x \in X$ such that n-cl({x}) $\cap$ A $\neq \phi$ and suppose that $x \notin$ n-ker(A). Then, there exists an n-open set V containing A and $x \notin V$. Let $y \in$ n-cl({x}) $\cap$ A. Hence, V is an n-neighbourhood of y which does not contain x. By this contradiction $x \in$ n-ker(A) and the claim.

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Definition 6.8. A subset A of a topological space X is called an **n-difference set** (briefly, **n-D-set**) if there are U, V ∈ n-O(X) such that U ≠ X and A = U – V.

Theorem 6.9. The following properties hold for the subsets A, B of a topological space (X, ℑ):
(i) A ⊂ n-ker(A).
(ii) A ⊂ B implies that n-ker(A) ⊂ n-ker(B).
(iii) If A is n-open in (X, ℑ), then A = n-ker(A).
(iv) n-ker(n-ker(A)) = n-ker(A).
Proof. (i), (ii) and (iii) are immediate consequences of **Definition 6.5**. To prove (iv), first observe that by (i) and (ii), we have n-ker(A) ⊂ n-ker(n-ker(A)). If x ∉ n-ker(A), then there exists U ∈ n-O(X) such that A ⊂ U and x ∉ U. Hence n-ker(A) ⊂ U, and so we have x ∉ n-ker(n-ker(A)). Thus n-ker(n-ker(A)) = n-ker(A).

Proposition 6.10. If a singleton {x} is an n-D-set of (X, ℑ), then n-ker({x}) ≠ X.
Proof. Since {x} is an n-D-set of (X, ℑ), then there exist two subsets U_1, U_2 ∈ n-O(X) such that {x} = $U_1 – U_2$, {x} ⊂ U_1 and U_1 ≠ X. Thus, we have that n-ker({x}) ⊂ U_1 ≠ X and so n-ker({x}) ≠ X.

Theorem 6.11. The following statements are equivalent for any points x and y in a topological space (X, ℑ):
(i) n-ker({x}) ≠ n-ker({y}).
(ii) n-cl({x}) ≠ n-cl({y}).
Proof. (i) ⇒ (ii). Suppose that n-ker({x}) ≠ n-ker({y}), then there exists a point z in X such that z ∈ n-ker({x}) and z ∉ n-ker({y}). From z ∈ n-ker({x}) it follows that {x} ∩ n-cl({z}) ≠ ϕ which implies x ∈ n-cl({z}). By z ∉ n-ker({y}), we have {y} ∩ n-cl({z}) = ϕ. Since x ∈ n-cl({z}), n-cl({x}) ⊂ n-cl({z}) and {y} ∩ n-cl({x}) = ϕ. Therefore, it follows that n-cl({x}) ≠ n-cl({y}). Now n-ker({x}) ≠ n-ker({y}) implies that n-cl({x}) ≠ n-cl({y}).

(ii) ⇒ (i). Suppose that n-cl({x}) ≠ n-cl({y}). Then there exists a point z in X such that z ∈ n-cl({x}) and z ∉ n-cl({y}). Then, there exists an n-open set containing z and therefore x but not y, namely, y ∉ n-ker({x}) and thus n-ker({x}) ≠ n-ker({y}).

Theorem 6.12. Let (X, ℑ) be a topological space. Then ∩ {n-cl({x}) : x ∈ X} = ϕ if and only if n-ker({x}) ≠ X for every x ∈ X.
Proof. **Necessity**. Suppose that ∩ {n-cl({x}) : x ∈ X} = ϕ. Assume that there is a point y in X such that n-ker({y}) = X. Let x be any point of X. Then x ∈ V for every n-open set V containing y and hence y ∈ n-cl({x}) for any x ∈ X. This implies that y ∈ ∩{n-cl({x}) : x ∈ X}. But this is a contradiction.

Sufficiency. Assume that n-ker({x}) ≠ X for every x ∈ X. If there exists a point y in X such that y ∈ ∩ {n-cl({x}) : x ∈ X}, then every n-open set containing y must contain every point of X. This implies that the space X is the unique n-open set containing y. Hence n-ker({y}) = X which is a contradiction. Therefore, ∩ {n-cl({x}) : x ∈ X} = ϕ.

Theorem 6.13. A topological space (X, ℑ) is nR_0 if and only if for every x and y in X, n-cl({x}) ≠ n-cl({y}) implies n-cl({x}) ∩ n-cl({y}) = ϕ.
Proof. **Necessity**. Suppose that (X, ℑ) is nR_0 and x, y ∈ X such that n-cl({x}) ≠ n-cl({y}). Then, there exists z ∈ n-cl({x}) such that z ∉ n-cl({y}) (or z ∈ n-cl({y}) such that z ∉ n-cl({x})). There exists V ∈ n-O(X) such that y ∉ V and z ∈ V, hence x ∈ V. Therefore, we have x ∉ n-cl({y}). Thus x ∈ [X – n-cl({y})] ∈ n-O(X), which implies n-cl({x}) ⊂ [X – n-cl({y})] and n-cl({x}) ∩ n-cl({y}) = ϕ.

Sufficiency. Let V ∈ n-O(X) and let x ∈ V. We still show that n-cl({x}) ⊂ V. Let y ∉ V, that is y ∈ X – V. Then x ≠ y and x ∉ n-cl({y}). This shows that n-cl({x}) ≠ n-cl({y}). By assumption, n-cl({x}) ∩ n-cl({y}) = ϕ. Hence y ∉ n-cl({x}) and therefore n-cl({x}) ⊂ V.

Theorem 6.14. A topological space (X, ℑ) is nR_0 if and only if for any points x and y in X, n-ker({x}) ≠ n-ker({y}) implies n-ker({x}) ∩ n-ker({y}) = ϕ.
Proof. Suppose that (X, ℑ) is an nR_0 space. Thus by **Theorem 6.11**, for any points x and y in X if n-ker({x}) ≠ n-ker({y}) then n-cl({x}) ≠ n-cl({y}). Now we prove that n-ker({x}) ∩ n-ker({y}) = ϕ. Assume that z ∈ n-ker({x}) ∩ n-ker({y}). By z ∈ n-ker({x}) and **Theorem 6.6**, it follows that x ∈ n-cl({z}). Since x ∈ n-cl({x}), by **Theorem 6.2**, n-cl({x}) = n-cl({z}). Similarly, we have n-cl({y}) = n-cl({z}) = n-cl({x}). This is a contradiction. Therefore, we have n-ker({x}) ∩ n-ker({y}) = ϕ.

Conversely, let (X, ℑ) be a topological space such that for any points x and y in X, n-ker({x}) ≠ n-ker({y}) implies n-ker({x}) ∩ n-ker({y}) = ϕ. If n-cl({x}) ≠ n-cl({y}), then by **Theorem 6.11**, n-ker({x}) ≠ n-ker({y}). Hence, n-ker({x}) ∩ n-ker({y}) = ϕ which implies n-cl({x}) ∩ n-cl({y}) = ϕ. Because z ∈ n-cl({x}) implies that x ∈ n-ker({z}) and therefore n-ker({x}) ∩ n-ker({z})

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$\neq \phi$. By hypothesis, we have n-ker({x}) = n-ker({z}). Then $z \in$ n-cl({x}) $\cap$ n-cl({y}) implies that n-ker({x}) = n-ker({z}) = n-ker({y}). This is a contradiction. Therefore, n-cl({x}) $\cap$ n-cl({y}) = ϕ and by **Theorem 6.2**, (X, $\Im$) is an nR_0 space.

Theorem 6.15. For a topological space (X, $\Im$) the following properties are equivalent:
(i) (X, $\Im$) is an nR_0 space.
(ii) For any non-empty set A and G $\in$ n-O(X) such that A $\cap$ G $\neq \phi$, there exists F $\in$ n-C(X) such that A $\cap$ F $\neq \phi$ and F $\subset$ G.
(iii) For any G $\in$ n-O(X), we have G = $\cup$ {F $\in$ n-C(X) : F $\subset$ G}.
(iv) For any F $\in$ n-C(X), we have F = $\cap$ {G $\in$ n-O(X) : F $\subset$ G}.
(v) For every x $\in$ X, n-cl({x}) $\subset$ n-ker({x}).

Proof. (i) $\Rightarrow$ (ii). Let A be a non-empty subset of X and G $\in$ n-O(X) such that A $\cap$ G $\neq \phi$. There exists x $\in$ A $\cap$ G. Since x $\in$ G $\in$ n-O(X), n-cl({x}) $\subset$ G. Set F = n-cl({x}), then F $\in$ n-C(X), F $\subset$ G and A $\cap$ F $\neq \phi$.

(ii) $\Rightarrow$ (iii). Let G $\in$ n-O(X), then G $\supset \cup${F $\in$ n-C(X) : F $\subset$ G}. Let x be any point of G. There exists F $\in$ n-C(X) such that x $\in$ F and F $\subset$ G. Therefore, we have x $\in$ F $\subset \cup$ {F $\in$ n-C(X) : F $\subset$ G} and hence G = $\cup$ {F $\in$ n-C(X) : F $\subset$ G}.

(iii) $\Rightarrow$ (iv). Obvious.

(iv) $\Rightarrow$ (v). Let x be any point of X and y $\notin$ n-ker({x}). There exists V $\in$ n-O(X) such that x $\in$ V and y $\notin$ V, hence n-cl({y}) $\cap$ V = ϕ. By (iv), ($\cap$ {G $\in$ n-O(X) : n-cl({y}) $\subset$ G}) $\cap$ V = ϕ and there exists G $\in$ n-O(X) such that x $\notin$ G and n-cl({y}) $\subset$ G. Therefore n-cl({x}) $\cap$ G = ϕ and y $\notin$ n-cl({x}). Consequently, we obtain n-cl({x}) $\subset$ n-ker({x}).

(v) $\Rightarrow$ (i). Let G $\in$ n-O(X) and x $\in$ G. Let y $\in$ n-ker({x}), then x $\in$ n-cl({y}) and y $\in$ G. This implies that n-ker({x}) $\subset$ G. Therefore, we obtain x $\in$ n-cl({x}) $\subset$ n-ker({x}) $\subset$ G. This shows that (X, $\Im$) is an nR_0 space.

Corollary 6.16. For a topological space (X, $\Im$) the following properties are equivalent:
(i) (X, $\Im$) is an nR_0 space.
(ii) n-cl({x}) = n-ker({x}) for all x $\in$ X.
Proof. (i) $\Rightarrow$ (ii). Suppose that (X, $\Im$) is an nR_0 space. By **Theorem 6.15**, n-cl({x}) $\subset$ n-ker({x}) for each x $\in$ X. Let y $\in$ n-ker({x}), then x $\in$ n-cl({y}) and by **Theorem 6.2**, n-cl({x}) = n-cl({y}). Therefore, y $\in$ n-cl({x}) and hence n-ker({x}) $\subset$ n-cl({x}). This shows that n-cl({x}) = n-ker({x}).

(ii) $\Rightarrow$ (i). Follows from **Theorem 6.15**.

Theorem 6.17. For a topological space (X, $\Im$) the following properties are equivalent:
(i) (X, $\Im$) is an nR_0 space.
(ii) If F is n-closed, then F = n-ker(F).
(iii) If F is n-closed and x $\in$ F, then n-ker({x}) $\subset$ F.
(iv) If x $\in$ X, then n-ker({x}) $\subset$ n-cl({x}).
Proof. (i) $\Rightarrow$ (ii). Let F be an n-closed and x $\notin$ F. Thus (X – F) is an n-open set containing x. Since (X, $\Im$) is n-R_0, n-cl({x}) $\subset$ (X – F). Thus n-cl({x}) $\cap$ F = ϕ and by **Theorem 6.7**, x $\notin$ n-ker(F). Therefore n-ker(F) = F.

(ii) $\Rightarrow$ (iii). In general, A $\subset$ B implies n-ker(A) $\subset$ n-ker(B). Therefore, it follows from (ii), that n-ker({x}) $\subset$ n-ker(F) = F.

(iii) $\Rightarrow$ (iv). Since x $\in$ n-cl({x}) and n-cl({x}) is n-closed, by (iii), n-ker({x}) $\subset$ n-cl({x}).

(iv) $\Rightarrow$ (i). We show the implication by using **Theorem 6.4**. Let x $\in$ n-cl({y}). Then by **Theorem 6.6**, y $\in$ n-ker({x}). Since x $\in$ n-cl({x}) and n-cl({x}) is n-closed, by (iv), we obtain y $\in$ n-ker({x}) $\subset$ n-cl({x}). Therefore x $\in$ n-cl({y}) implies y $\in$ n-cl({x}). The converse is obvious and (X, $\Im$) is nR_0.

Definition 6.18. A topological space (X, $\Im$) is said to be **nR_1** if for x, y in X with n-cl({x}) $\neq$ n-cl({y}), there exist disjoint n-open sets U and V such that n-cl({x}) $\subset$ U and n-cl({y}) $\subset$ V.

Theorem 6.19. A topological space (X, $\Im$) is nR_1 if it is nT_2.
Proof. Let x and y be any points of X such that n-cl({x}) $\neq$ n-cl({y}). By **Theorem 5.8**, every n-T_2 space is n-T_1. Therefore, by **Theorem 4.5**, n-cl({x}) = {x}, n-cl({y}) = {y} and hence {x} $\neq$ {y}. Since (X, $\Im$) is nT_2, there exist disjoint n-open sets U and V such that n-cl({x}) = {x} $\subset$ U and n-cl({y}) = {y} $\subset$ V. This shows that (X, $\Im$) is nR_1.

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Theorem 6.20. If a topological space $(X, \Im)$ is n-symmetric, then the following properties are equivalent:
(i) $(X, \Im)$ is nT_2.
(ii) $(X, \Im)$ is nR_1 and nT_1.
(iii) $(X, \Im)$ is nR_1 and nT_0.
Proof. Straightforward.

Theorem 6.21. For a topological space $(X, \Im)$ the following statements are equivalent:
(i) $(X, \Im)$ is nR_1.
(ii) If $x, y \in X$ such that $n\text{-cl}(\{x\}) \neq n\text{-cl}(\{y\})$, then there exist n-closed sets F_1 and F_2 such that $x \in F_1$, $y \notin F_1$, $y \in F_2$, $x \notin F_2$ and $X = F_1 \cup F_2$.
Proof. Obvious.

Theorem 6.22. If $(X, \Im)$ is nR_1, then $(X, \Im)$ is nR_0.
Proof. Let U be an n-open set such that $x \in U$. If $y \notin U$, since $x \notin n\text{-cl}(\{y\})$, we have $n\text{-cl}(\{x\}) \neq n\text{-cl}(\{y\})$. So, there exists an n-open set V such that $n\text{-cl}(\{y\}) \subset V$ and $x \notin V$, which implies $y \notin n\text{-cl}(\{x\})$. Hence $n\text{-cl}(\{x\}) \subset U$. Therefore, $(X, \Im)$ is nR_0.

Corollary 6.23. A topological space $(X, \Im)$ is nR_1 if and only if for $x, y \in X$, $n\text{-ker}(\{x\}) \neq n\text{-ker}(\{y\})$, there exist disjoint n-open sets U and V such that $n\text{-cl}(\{x\}) \subset U$ and $n\text{-cl}(\{y\}) \subset V$.
Proof. Follows from **Theorem 6.11**.

Theorem 6.24. A topological space $(X, \Im)$ is nR_1 if and only if $x \in X - n\text{-cl}(\{y\})$ implies that x and y have disjoint n-open neighbourhoods.
Proof. **Necessity**. Let $x \in X - n\text{-cl}(\{y\})$. Then $n\text{-cl}(\{x\}) \neq n\text{-cl}(\{y\})$, so, x and y have disjoint n-open neighbourhoods.

Sufficiency. First, we show that $(X, \Im)$ is nR_0. Let U be an n-open set and $x \in U$. Suppose that $y \notin U$. Then, $n\text{-cl}(\{y\}) \cap U = \phi$ and $x \notin n\text{-cl}(\{y\})$. There exist n-open sets U_x and U_y such that $x \in U_x$, $y \in U_y$ and $U_x \cap U_y = \phi$. Hence, $n\text{-cl}(\{x\}) \subset n\text{-cl}(U_x)$ and $n\text{-cl}(\{x\}) \cap U_y \subset n\text{-cl}(U_x) \cap U_y = \phi$. Therefore, $y \notin n\text{-cl}(\{x\})$. Consequently, $n\text{-cl}(\{x\}) \subset U$ and $(X, \Im)$ is nR_0. Next, we show that $(X, \Im)$ is nR_1. Suppose that $n\text{-cl}(\{x\}) \neq n\text{-cl}(\{y\})$. Then, we can assume that there exists $z \in n\text{-cl}(\{x\})$ such that $z \notin n\text{-cl}(\{y\})$. There exist n-open sets V_z and V_y such that $z \in V_z$, $y \in V_y$ and $V_z \cap V_y = \phi$. Since $z \in n\text{-cl}(\{x\})$, $x \in V_z$. Since $(X, \Im)$ is nR_0, we obtain $n\text{-cl}(\{x\}) \subset V_z$, $n\text{-Cl}(\{y\}) \subset V_y$ and $V_z \cap V_y = \phi$. This shows that $(X, \Im)$ is nR_1.

Conclusion. In this paper, we introduce and study some new types of separation axioms (namely nT_0, nT_1, nT_2, nR_0 and nR_1) using n-open sets in topological spaces. The relationships among nT_0, nT_1, nT_2 and some other existing separation axioms are investigated and give some examples in support of these relationships. We also obtained some characterizations and preservation theorems for these separation axioms. This idea can be extended to topological ordered, bitopological, bitopological ordered, fuzzy topological and fuzzy bitopological spaces etc.

Conflicts of Interest. We certify that this work is original, has never been published before, and is not being considered for publication anywhere else at this time. This publication is free from any conflicts of interest. As the Corresponding Author, I certify that the listed author has read the paper and given their approval for submission.

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