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THE METHODOLOGY FOR ACCELERATING THE LEARNING PROCESS BASED ON STRUCTURED AND SYSTEMATIZED LEARNING ELEMENTS (ON THE EXAMPLE OF APPROXIMATE ROOT CALCULATION OF A NONLINEAR EQUATION USING NEWTON'S METHOD)

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ANNOTATION
The article is devoted to the consideration of the methodological foundations of the organization and implementation of intensive training in higher educational institutions using the technology of modular training of structured and systematized educational elements. Description of the substantive structure of the subject in the form of systematized educational elements, as well as improvement of the modularly formulated teaching methodology on the topic of approximate calculation of the roots of a nonlinear equation by Newton's method, based on an object-oriented approach to goals. The components of the module are logically structured educational elements, interconnected in content and functionality, and their interconnected blocks are proposed to be presented in the form of a systemically structured logical graph diagram.
KEYWORDS: *Cognitive Development, Systems Thinking, E-Learning, Content Structuring, Modulation, Graph Diagram.*

INTRODUCTION
The principles of innovative teaching technologies can be fully implemented in the educational process by developing and applying open intensive teaching systems. This provides an opportunity to select appropriate teaching technologies tailored to each learner and to develop and activate a personalized educational program. However, the synthesis of open intensive education systems can only be achieved under a number of conditions [1,2]:

• Adhering to the principle of adapting the learning process to the learner's personality. This principle is realized through the logical structure of the subject (constructing its external and internal modules) and the development of a comprehensive program for its study by students;

• Accelerating students' individual acquisition of general scientific and specialized knowledge by designing a "logical structure" of the subject that provides core knowledge in a compact format.

The algorithm for designing the generalized logical structure of a subject includes the following steps [2]:

1. Presenting the content of the subject as a system of individual educational elements;
2. Designing the structure of interrelations between semantic elements to identify the core knowledge;
3. Modeling the core knowledge in verbal, symbolic, graphical, or other forms;
4. Modifying the core knowledge model to determine general concepts and the systemic links between them;
5. Forming the general structures of cognitive activity specific to the subject, and so on.

It is possible to activate individual cognitive processes by deeply shaping the content of the subject and increasing the intensity of information flow. The educational materials of the course should be directed at solving professional training problems of future specialists and should correspond to the system of scientific and practical knowledge being presented to students. The content of key topics should reflect the latest achievements in the field, and practical classes should address professional issues specific to the discipline.

Currently, in non-traditional educational environments, models such as developmental, programmed, algorithmized, project-based, problem-based teaching, artificial intelligence, humanistic pedagogy, neuropsychology, neuropedagogy, and virtual pedagogical agents are rapidly being integrated into modern educational systems that are significantly influenced by practice and are supported by new information and communication technologies [1–13].

LITERATURE REVIEW
The principles of innovative technology define electronic interactive publications as an information-methodological tool for pedagogical management of the educational process. These

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publications provide educational materials necessary for solving specific educational problems, and the method of interaction is defined by a certain algorithm or independently developed by the student [1].

The structural components of the educational material include academic and scientific information, cognitive tasks, exercises, and research assignments. These components are distinguished by their integrity, and their integration creates a synergistic effect that fosters the creative potential of all participants in the educational process and ensures its optimization [11].

Based on existing classifications in psychology and economics, a classification of mathematical modeling methods used in pedagogical research is proposed. In addition, the application areas of mathematical modeling methods in the description of pedagogical objects are highlighted. In the educational process, mathematical models perform descriptive, controlling, investigative, explanatory, and prognostic functions. The algorithm for applying individual methods of mathematical modeling in teaching is generalized as follows: constructing a model of the learning process elements; conducting experiments with the model; pedagogical interpretation of the results. The effectiveness of using mathematical modeling methods in the educational process is substantiated [6].

In the referenced article [3], the use of algorithmic approaches in shaping students' academic and research activities is proposed, with the creation of algorithms for solving mathematical problems playing a key role. Examples of tasks for learners (school and university students) who are mastering the algorithmic approach are provided. According to the authors, a specially organized and systematic approach by the teacher to develop students' algorithmic culture (i.e., preparing modern individuals for an information society) and the targeted selection of educational materials help foster elements of algorithmic culture.

Implementing a cultural approach in education allows the algorithmization process to be considered from the perspective of an individual's algorithmic culture. This culture consists of value-motivational, cognitive, practical, individual-personal, and creative components, along with reflective and structural elements. The features of these components become an integral part of the learner's subjective experience and help form the intellectual potential necessary for successful future learning [9].

In this article [2], the levels of systematization of teaching content are considered in the implementation of a modular approach to organizing the educational process. The issues of identifying the structural elements of educational content and forming a modular curriculum are discussed. Based on the modular-competency approach, the design of the content structure (curriculum) of a four-year bachelor's degree program is considered (using the example of the "Pedagogical Education" program with two educational profiles: "Informatics and Mathematics").

The article analyzes issues related to the development and organization of e-learning content from the perspective of developing systems thinking. The key methodological principles for developing the content of e-learning tools are defined: designing structure based on didactic engineering methods; systematizing educational and methodological materials based on the theory of differential integration of perception and thinking development; linking didactic and methodological resources with pedagogical practice standards and the technical-technological educational environment. Methodological approaches of didactic engineering, as well as the principles of forming cognitive schemes and intellectual development in the learning process, are briefly reviewed [12].

The article is devoted to identifying the principles of selecting and structuring educational content for multimedia learning complexes. A network-based approach is proposed, which allows the establishment of relationships between the selection (compatibility, interdisciplinarity, rationality, comparability) and structuring (nonlinearity, clustering, cascading, modularity) principles of educational materials in multimedia learning complexes, approached from different logical foundations [13].

The analyzed materials briefly consider the structuring and modular breakdown of educational content. However, in higher education institutions, the methodological aspects of teaching the content of programming courses using systematized structures and computer-based technologies, as well as the organization and activation of the learning process, have not been specifically studied. This necessitates special research on the development of a methodology for activating and accelerating students' learning through systematized modular structures of programming foundations using software agents.

METHODOLOGY

The methodology for representing the topic "Approximate determination of the roots of a nonlinear equation" as a structured graph scheme using the method of decomposition into logical parts (concepts) has been considered in the article [7,8].

We examine the teaching methodology of educational elements structured in modular form based on a subject-oriented approach for the topic "Approximate determination of the roots of a nonlinear equation on the interval $x \in [a,b]$ using Newton's method". This involves defining the structure of the educational content and improving its objectives and methods. The components of the module consist of logically organized educational elements (EEs), which are content- and functionally-related structural units dependent on one another. The content of an educational element may be represented by smaller portions of learning information—second- and third-order educational elements.

The modular-structured logical scheme of the topic "Approximate determination of the root of a nonlinear

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equation on a given interval" (Stage 2) can be visualized as follows (Figure 1).

The modular technology primarily consists in developing an effective sequence of content delivery through an innovative approach focused on topics and activities, using model-algorithm-program principles of educational control software tools that integrate educational elements organized based on systemic structure.

We now construct the semantic graph scheme (Figure 2) of the topic to present the educational elements (1, 2, 3, 4, 5, 6, 7, 8) of the modules. These graphical diagrams give students a clear representation of the volume of educational material and the order in which it is developed in the module. The logical structure graph of the module reflects not only the composition and interrelation of the educational elements but also the dynamics of the learning process and the sequential phases of the teacher's instructional activities. In general, the vertices (nodes) of the semantic graph represent the educational elements (EEs), while the edges correspond to the interrelationships between the elements and the serial numbers of the instructional sessions [7,8].

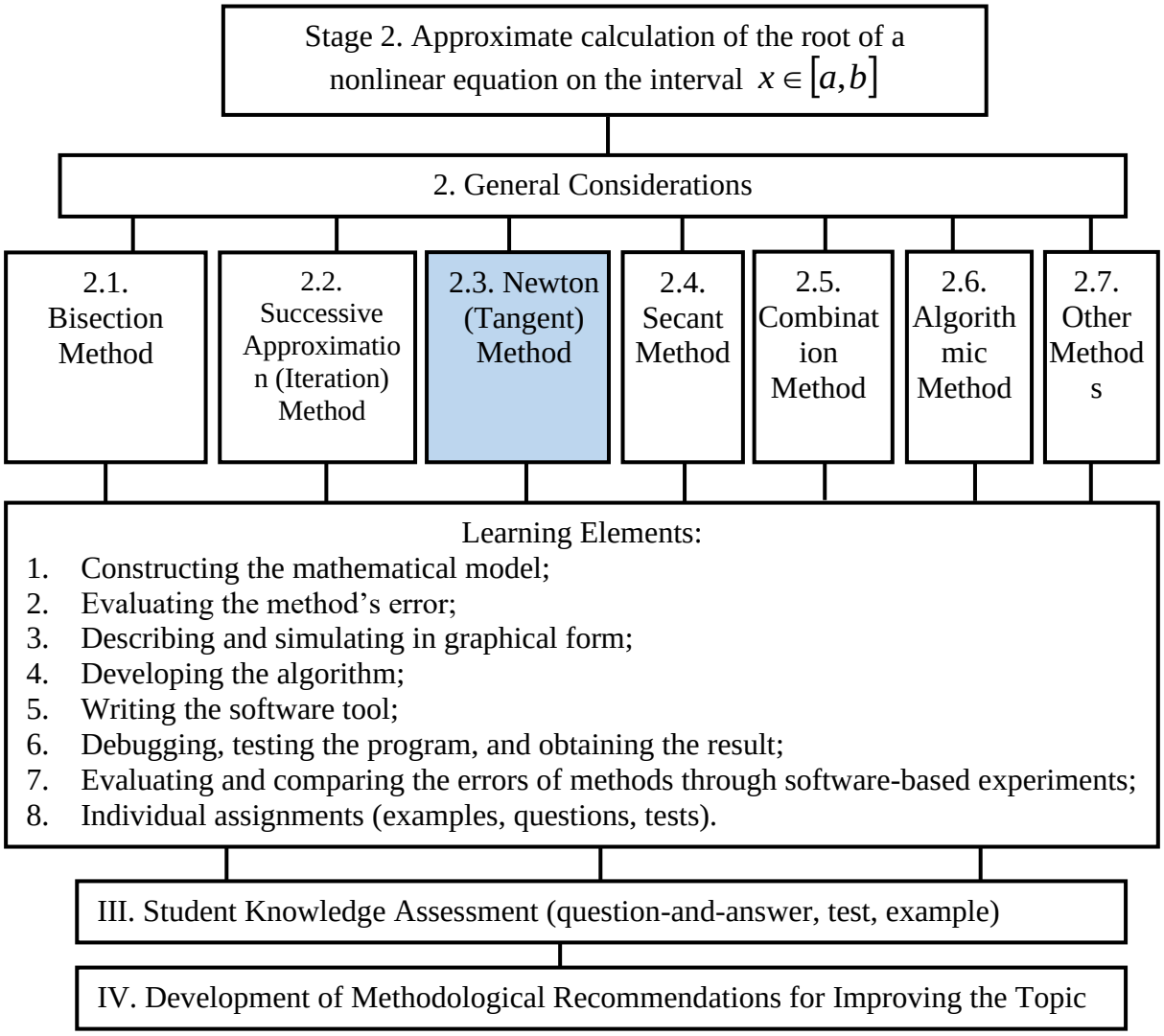


Figure 1. Modular-Structured Logical Scheme of the Topic

Based on the semantic graph, the process of presenting the topic is carried out in the sequence of modules over dynamic time $T_j = \sum_{i=1}^{8} t_i$, $j = \overline{1,5}$, that is, first O.E.1 of Module 2.1 – t_1, then O.E.2 – t_2, and so on, are completed over time t_5.

Based on a system-activity approach to identifying structural elements, the theoretical foundations of the modular teaching system can be developed in the form of educational and methodological manuals, test assignments designed on the basis of a modular structure, and other resources (for students or teachers) for performing laboratory work, practical assignments, coursework, and individual tasks.

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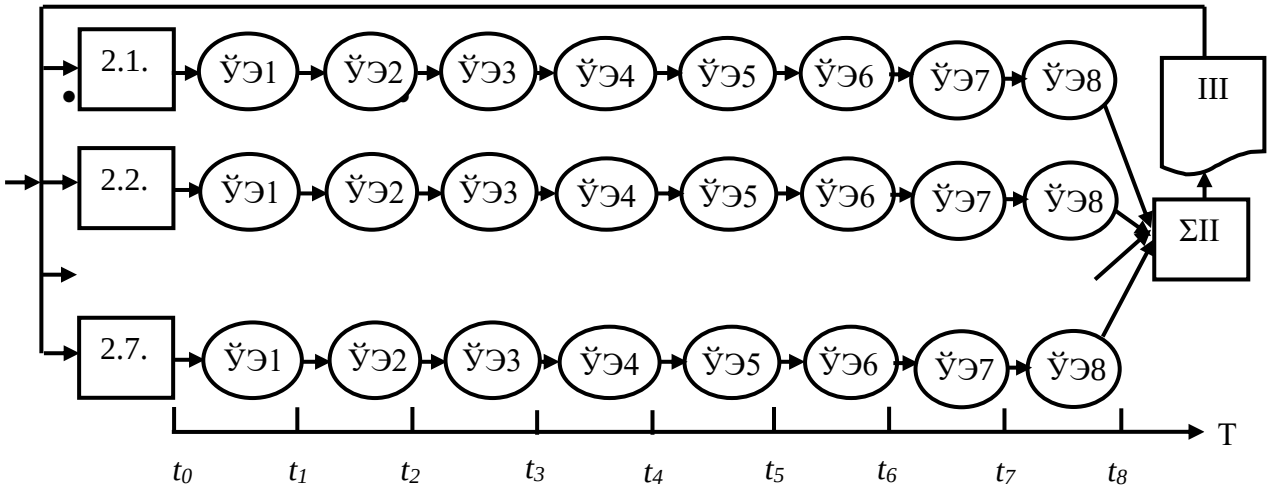


Figure 2. Semantic graph scheme of the process of approximating the root of a nonlinear equation on a given interval using the Newton method, represented in the form of systematized modules.

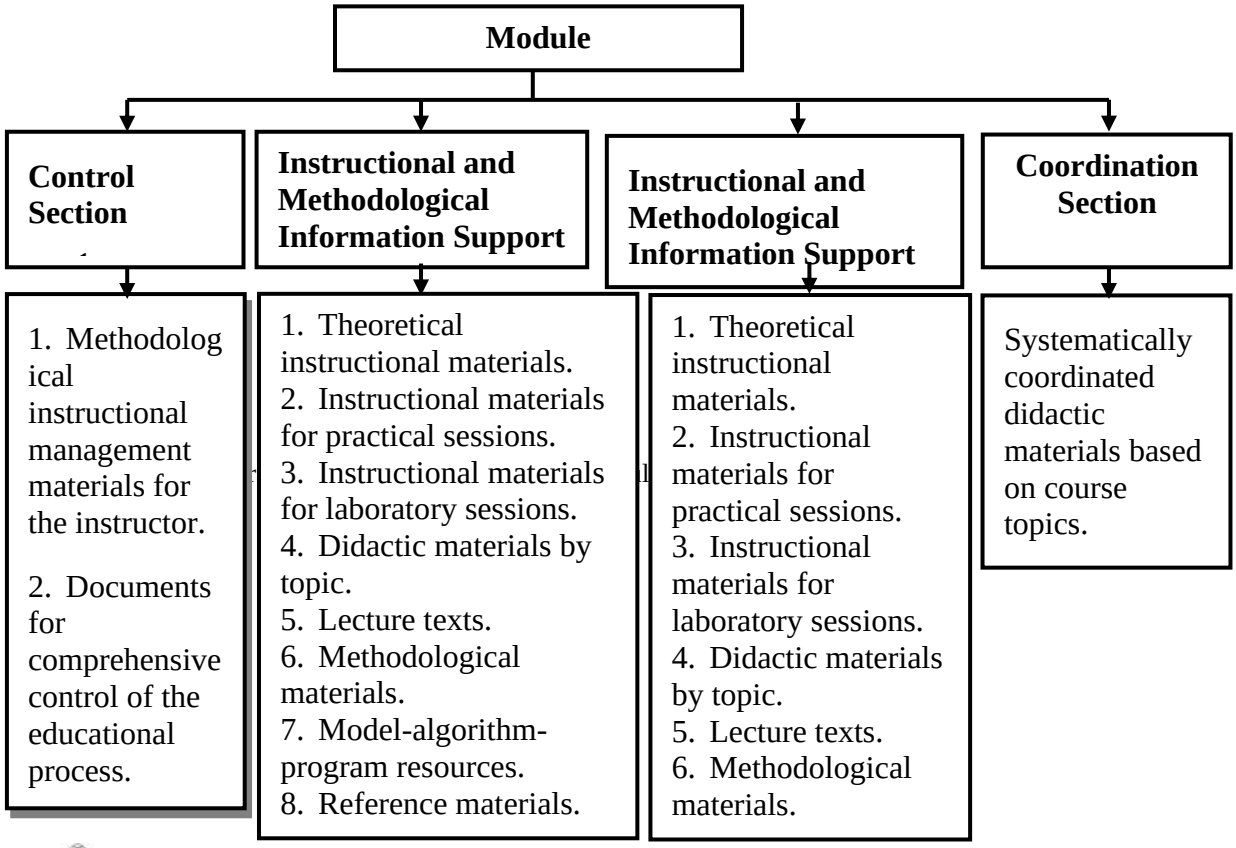
Therefore, each module has a meaningful structure in which we have identified the following components: control, coordination, informational-methodical support, and monitoring (Figure 3).

We will consider the modular teaching methodology based on the semantic graph scheme representing the process of approximately finding the root of a nonlinear equation within a given interval using Newton's method.

1. Sequence of solving algebraic and transcendental equations using Newton's method.

OE1. Given the equation *f(x) = 0*, the interval $x \in [a,b]$ in which the root lies, and the calculation accuracy ξ. There are several options for performing this operation [14, p. 52].

Suppose the equation *f(x) = 0* has a unique solution in the interval $[a,b]$, and $f'(x)$, $f''(x)$ derivatives are defined, continuous, and retain their sign (do not change) within the interval $[a,b]$. To calculate the solution of the equation *f(x) = 0*, the Newton formula $x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$ (1) is denoted and proved.



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$$y = x - \frac{f(x)}{f'(x)} \qquad (2)$$

We derive equation (2). The condition for applying Newton's method is:

$$f(x) \bullet f''(x) > 0,\ x \in [a,b]. \qquad (3)$$

O'E2. Evaluation of Newton's Method Error. The following estimation is considered valid for Newton's method: $\left|x_n - x^*\right| \le \frac{M_2}{2m_1} \bullet \left|x_n - x_{n-1}\right|^2$, бу ерда $M_2 = \max_{a \le x \le b} \left| f''(x) \right|$, $m_1 = \min_{a \le x \le b} \left| f'(x) \right|$.

By applying Lagrange's mean value theorem and Taylor's formula near the point x_{n-1} , we obtain the following inequality to estimate the convergence of Newton's method [14].

$$\left|x_n - x^*\right| \le \frac{M_2}{2m_1} \bullet \left|x_n - x_{n-1}\right|^2. \qquad (4)$$

It can be emphasized that the error estimation of the Newton method according to formula (4) is valid if the condition $\frac{M_2}{2m_1} \bullet \left|x_n - x_{n-1}\right|^2 \le \xi$ is satisfied, that is, $\left|x_n - x_{n-1}\right| < \sqrt{\frac{2m_1\xi}{M_2}}$ holds.

However, in order for this condition to be fulfilled, it is necessary to determine the quantities m_1 and M_2, which in most cases is considered difficult to implement. Therefore, in some cases, this condition is simplified and expressed as follows: $\left|x_n - x_{n-1}\right| < \xi$..

O'E3. Graphical Representation of the Newton Method. The algorithm starts with some initial approximation x_0 that satisfies condition (3), and then constructs new effective solutions iteratively (successively approaching). That is, a tangent is drawn to the function graph at point $x = x_0$, and the point where this tangent intersects the x -axis is taken as the next approximation $x = x_1$, and so on (see Figure 4). As a result, in one of the iterations, we approach the point x^* with the specified accuracy ξ.

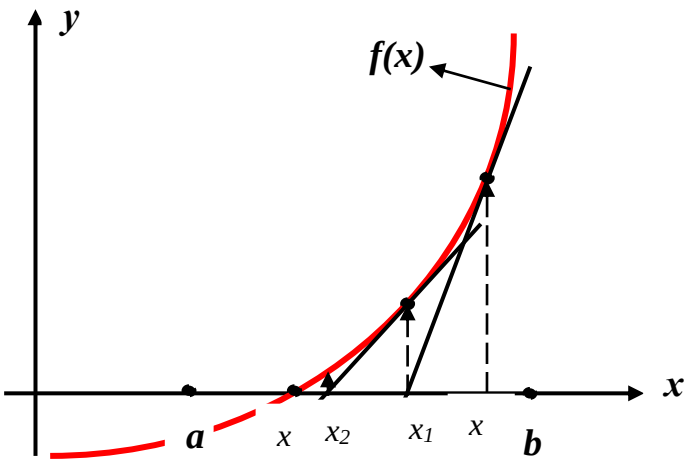


Figure 4. Graphical Description of the Newton Method

The Newton method is considered extremely important in computational mathematics because, in many cases, finding numerical solutions of equations becomes very simple.

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O‘E4. Constructing the Algorithm of the Newton Method.

1. The $f'(x),\ f''(x)$ derivatives of the function f(x) are determined.
2. The Newton method condition (3) is checked for the points “a” and “b” of the interval. The point at which the condition is satisfied is selected as the initial point of the argument (x = a or x = b); otherwise, the Newton method cannot be applied to the given equation.
3. The next approximation is calculated using $y = x - \frac{f(x)}{f'(x)}$.
4. The absolute value of two successive approximations is calculated as $d = |y - x|$, and it is assigned to $x = y$ for further calculations.
5. If the condition $d > \xi$ is satisfied, the calculation process continues from step 3; otherwise, the equation is considered solved to the required accuracy, and the next step is taken.
6. The value of “x” is printed, and the computation process is terminated.

Let us examine the execution of this task in the following example:

$$f(x) = \sin x - x + 0{,}15;\ x \in [0{,}5;1],\ \xi = 0{,}0001.$$

We find the first and second derivatives of the function. $f'(x) = \cos x - 1,\quad f''(x) = -\sin x.$ The formula of the Newton method is given as: $y = x - \frac{\sin x - x + 0{,}15}{\cos x - 1}$. The formula of the Newton method is given as:

$a = 0.5,\ f(a) = \sin 0.5 - 0.5 + 0.15 > 0;\ f''(a) = -\sin 0.5 < 0,\quad f(a) \bullet f''(a) < 0.$ The condition is not satisfied at point x = a.

We now check the Newton method condition at point “b”: $b = 1,\ f(b) = \sin 1 - 0.5 + 0.15 < 0;;\ f''(b) = -\sin 1 < 0,\ f(b) \bullet f''(b) > 0.$

The Newton condition is satisfied at point “b”. The algorithm of the Newton method described above is presented as a block diagram in Figure 5. The variable “n” is used to calculate the number of iterations.

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O‘E4 – Algorithm	O‘E4 – Algorithm

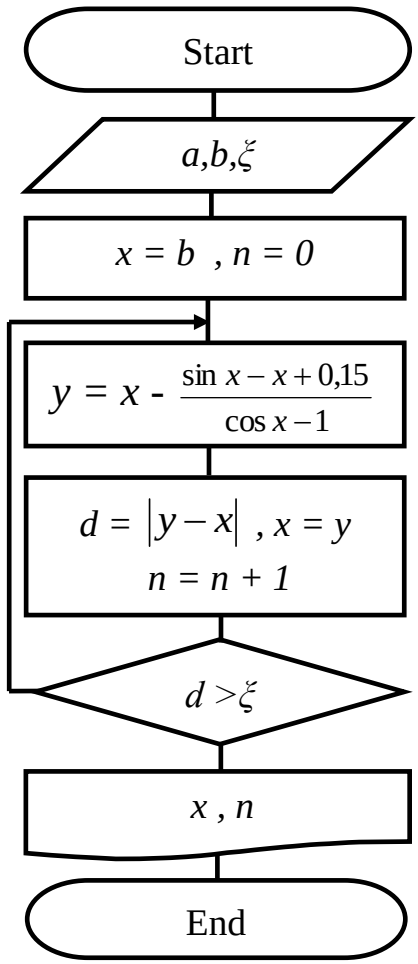
	// metod Ньютона
	#include<iostream.h>
	#include<conio.h>
	#include<math.h>
	int main()
	{
	int n;
	double a,b,d,eps,x,y,fx;cout<<endl;
	cout<<" a, b, eps = ";
	cin>>a>>b>>eps; cout<<endl;
	x=b; n=0;
	
	m4:y=x-(sin(x)-x+0.15)/(cos(x)-1);
	printf(" x = %.8f%. y = %.8f%\n",x,y);
	
	d=fabs(y-x);
	x=y;
	n=n+1;
	
	if (d>eps) goto m4;
	
	printf(" d = %.8f%. n = %2df%\n",d,n);
	fx= sin(x)-x+0.15;
	printf(" x = %.8f%. fx = %.8f%\n",x,fx);
	
	return 0;
	}

Figure 5. Algorithm and Program of the Newton Method.

O‘E5, O‘E6, and O‘E7 are completed during practical sessions: the developed program is debugged, tested, and the result is obtained. The solution to this problem is experimented with using various levels of precision, and the errors are evaluated.

RESULT

The program for solving the example was written in C++, and the obtained result is presented below:

```
a, b, eps = -0.5  1  0.00001

x = 1.00000000    y = 0.98144647
x = 0.98144647    y = 0.98112171
x = 0.98112171    y = 0.98112161
d = 0.00000010    n =  3
x = 0.98112161   fx = -0.00000000
```

O‘E8. Students are given individual assignments (examples, questions, tests) to assess the degree of mastery of the topic.

CONCLUSION

Thus, the methodology of modular design of the structure and content of the course, considered in general within student groups, makes it possible to scientifically justify the development and resolution of the problem of forming the most general principles of educational-methodological activity for students to independently search for knowledge and complement a holistic system of scientific knowledge, as well as to develop abilities.

As can be seen from the above, the development and synthesis of open intensive education systems is one of the most promising directions for the advancement of educational technologies that not only intensify the learning process but also contribute to self-organization, self-formation, and self-awareness of individuals. Therefore, generalizing historical experience and identifying stages of applying mathematical

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modeling methods in pedagogical sciences, determining the main directions for applying mathematical modeling methods in teaching, systematizing and generalizing the classification of mathematical modeling methods used in pedagogy, characterizing the functions of mathematical models in the learning process, and showing the richness of modeling in the content and process of education are of great importance.

The prospects for future research in this area include the use of didactic materials for the development of elements of algorithmic culture in students' independent work, as well as the integration of program materials aimed at developing students' algorithmic culture into the content of training courses for teachers in schools and higher education institutions. Presenting educational material based on systematized structures (lecture, practical, experimental) and organizing it in a computerized, interactive, and mentor-student format provides an opportunity to save time, improve the efficiency of the educational process, and develop optimal methods and strategies for improvement.

RECOMMENDATIONS

Based on the logical graphic-semantic structure of the subject, the methodology presented for integrating modern multimedia technologies into the learning process of university students, computer-based instruction, and professional development serves to implement modern concepts of education in the field of information technology and develop contemporary teaching methods.

The created modern electronic resource base not only increases students' interest in the subjects they study but also enables teachers to preserve invaluable developments that were previously inaccessible to others.

In order to improve the effectiveness of knowledge acquisition and comprehension processes among students, and to create favorable conditions for the development of their intellectual potential and competencies, future research will explore the construction, structuring, decomposition of educational elements in various forms within the informational-methodological section of the subject, as well as the issues of monitoring the learning process.

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