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CONSTRUCTION OF DIOPHANTINE 3-TUPLES THROUGH 3D NUMBERS

S. Vidhyalakshmi[1]
[1]Professor,
Department of Mathematics,
Shrimati Indira Gandhi College,
Trichy-620 002,
Tamil Nadu,
India.

T. Mahalakshmi[2]
[2]Assistant Professor,
Department of Mathematics,
Shrimati Indira Gandhi College,
Trichy-620 002,
Tamil Nadu,
India.

M.A. Gopalan[3]
[3]Professor, Department of Mathematics,
Shrimati Indira Gandhi College,
Trichy-620 002,
Tamil Nadu,
India.

ABSTRACT

This paper deals with the construction of diophantine 3-tuples based on two given 3D numbers such that the product of any two elements of the set added by a polynomial with integer coefficient is a perfect square.

KEYWORDS: *Diophantine 3-tuple, Pyramidal numbers.*
2010 Mathematics Subject Classification: 11D99

Notations:

- $SO_n = n(2n^2 - 1)$ = Stella Octangula number of rank n
- $P_n^5 = \frac{n^2(n+1)}{2}$ = Pentagonal Pyramidal number of rank n
- $CP_n^3 = \frac{n(n^2+1)}{2}$ = Centered triangular Pyramidal number of rank n

- $CS_n^4 = \frac{2n^3+n}{2}$ = Centered square Pyramidal number of rank n

1. INTRODUCTION

The problem of constructing the sets with property that product of any two of its distinct elements is one less than a square has a very long history and such sets have been studied by Diophantus. A set of m positive integers $\{a_1, a_2, a_3, ..., a_m\}$ is said to have the property $D(n), n \in Z - \{0\}$ if $a_i a_j + n$ is a perfect square for all $1 \le i \le j \le m$ and such a set is called a Diophantine m-tuple with property $D(n)$.

Many Mathematicians considered the construction of different formulations of diophantine triples with the property $D(n)$ for any arbitrary integer n [1] and also, for any linear polynomials in n. In this context, one may refer [2-14] for an extensive review of various problems on diophantine triples.

This paper aims at constructing sequences of diophantine triples based on two given 3D numbers where the product of any two members of the triple with the polynomial with integer coefficients satisfies the required property.

2. METHOD OF ANALYSIS

Sequence: 1

Let $a = SO_n = n(2n^2-1) = 2n^3 - n, \; b = 4P_n^5 = 2n^2(n+1) = 2n^3 + 2n^2$

It is observed that

$$ab - n^4 + n^2 = (2n^3 + n^2 - n)^2$$

Therefore, the pair (a, b) represents diophantine 2-tuple with the property $D(-n^4 + n^2)$.

Let c_1 be any non-zero polynomial in x such that

$$ac_1 - n^4 + n^2 = p^2 \quad (1)$$

$$bc_1 - n^4 + n^2 = q^2 \quad (2)$$

Eliminating c_1 between (1) and (2), we have

$$bp^2 - aq^2 = (b-a)(-n^4 + n^2) \quad (3)$$

Introducing the linear transformations

$$p = X + aT \; , \; q = X + bT \quad (4)$$

in (3) and simplifying we get

$$X^2 = abT^2 - n^4 + n^2$$

which is satisfied by $T = 1$, $X = 2n^3 + n^2 - n$

In view of (4) and (1), it is seen that

$$c_1 = 8n^3 + 4n^2 - 3n$$

Note that (a,b,c_1) represents diophantine 3-tuple with property $D(-n^4+n^2)$

Taking (a,c_1) and employing the above procedure, it is seen that the triple (a,c_1,c_2) where

$$c_2 = 18n^3 + 6n^2 - 8n$$

exhibits diophantine 3-tuple with property $D(-n^4+n^2)$

Taking (a,c_2) and employing the above procedure, it is seen that the triple (a,c_2,c_3) where

$$c_3 = 32n^3 + 8n^2 - 15n$$

exhibits diophantine 3-tuple with property $D(-n^4+n^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a,c_s,c_{s+1})$where

$$c_s = (2s^2+4s+2)n^3 + (2s+2)n^2 - (s^2+2s)n\ ,\ s=1,2,3,...$$

Now, consider (b,c_1) and employing the above procedure, it is seen that the triple (b,c_1,c_2) where

$$c_2 = 18n^3 + 12n^2 - 5n$$

exhibits diophantine 3-tuple with property $D(-n^4+n^2)$

Taking (b,c_2) and employing the above procedure, it is seen that the triple (b,c_2,c_3) where

$$c_3 = 32n^3 + 24n^2 - 7n$$

exhibits diophantine 3-tuple with property $D(-n^4+n^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(b,c_s,c_{s+1})$where

$$c_n = (2s^2+4s+2)n^3 + (2s^2+2s)n^2 - (2s+1)n\ ,\ s=1,2,3,...$$

Sequence: 2

Let $a = SO_n = 2n^3 - n,\ b = 4P_n^5 = 2n^3 + 2n^2$

It is observed that

$$ab + 3n^4 + 2n^3 = (2n^3 + n^2)^2$$

Therefore, the pair (a,b) represents diophantine 2-tuple with the property $D(3n^4+2n^3)$.

Let c_1 be any non-zero polynomial in x such that

$$ac_1+3n^4+2n^3=p^2 \quad (5)$$

$$bc_1+3n^4+2n^3=q^2 \quad (6)$$

Eliminating c_1 between (5) and (6), we have

$$bp^2-aq^2=(b-a)(3n^4+2n^3) \quad (7)$$

Introducing the linear transformations

$$p=X+aT \ , \ q=X+bT \quad (8)$$

in (7) and simplifying we get

$$X^2=abT^2+3n^4+2n^3$$

which is satisfied by $T=1$, $X=2n^3+n^2$

In view of (8) and (5), it is seen that

$$c_1=8n^3+4n^2-n$$

Note that (a,b,c_1) represents diophantine 3-tuple with property $D(3n^4+2n^3)$

Taking (a,c_1) and employing the above procedure, it is seen that the triple (a,c_1,c_2) where

$$c_2=18n^3+6n^2+4n$$

exhibits diophantine 3-tuple with property $D(3n^4+2n^3)$

Taking (a,c_2) and employing the above procedure, it is seen that the triple (a,c_2,c_3) where

$$c_3=32n^3+8n^2-9n$$

exhibits diophantine 3-tuple with property $D(3n^4+2n^3)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a,c_s,c_{s+1})$where

$$c_s=(2s^2+4s+2)n^3+(2s+2)n^2-s^2n, \ s=1,2,3,...$$

Now, consider (b,c_1) and employing the above procedure, it is seen that the triple (b,c_1,c_2) where

$$c_2=18n^3+12n^2-n$$

exhibits diophantine 3-tuple with property $D(3n^4 + 2n^3)$

Taking (b, c_2) and employing the above procedure, it is seen that the triple (b, c_2, c_3) where

$$c_3 = 32n^3 + 24n^2 - n$$

exhibits diophantine 3-tuple with property $D(3n^4 + 2n^3)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (b, c_s, c_{s+1}) where

$$c_s = (2s^2 + 4s + 2)n^3 + (2s^2 + 2s)n^2 - n\ ,\ s = 1,2,3,...$$

Sequence: 3

Let $a = 2CP_n^3 = n^3 + n,\ \ b = 3CS_n^4 = 2n^3 + n$

It is observed that

$$ab - n^6 - n^4 = (n^3 + n)^2$$

Therefore, the pair (a, b) represents diophantine 2-tuple with the property $D(-n^6 - n^4)$.

Let c_1 be any non-zero polynomial in x such that

$$ac_1 - n^6 - n^4 = p^2 \quad (9)$$

$$bc_1 - n^6 - n^4 = q^2 \quad (10)$$

Eliminating c_1 between (9) and (10), we have

$$bp^2 - aq^2 = (b - a)(-n^6 - n^4) \quad (11)$$

Introducing the linear transformations

$$p = X + aT\ ,\ q = X + bT \quad (12)$$

in (11) and simplifying we get

$$X^2 = abT^2 - n^6 - n^4$$

which is satisfied by $T = 1\ , X = n^3 + n$

In view of (12) and (9), it is seen that

$$c_1 = 5n^3 + 4n$$

Note that (a, b, c_1) represents diophantine 3-tuple with property $D(-n^6 - n^4)$

Taking (a,c_1) and employing the above procedure, it is seen that the triple (a,c_1,c_2) where

$$c_2 = 10n^3 + 9n$$

exhibits diophantine 3-tuple with property $D(-n^6 - n^4)$

Taking (a,c_2) and employing the above procedure, it is seen that the triple (a,c_2,c_3) where

$$c_3 = 17n^3 + 16n$$

exhibits diophantine 3-tuple with property $D(-n^6 - n^4)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (a,c_s,c_{s+1}) where

$$c_s = (s^2 + 2s + 2)n^3 + (s+1)^2 n\ ,\ s = 1,2,3,...$$

Now, consider (b,c_1) and employing the above procedure, it is seen that the triple (b,c_1,c_2) where

$$c_2 = 13n^3 + 9n$$

exhibits diophantine 3-tuple with property $D(-n^6 - n^4)$

Taking (b,c_2) and employing the above procedure, it is seen that the triple (b,c_2,c_3) where

$$c_3 = 25n^3 + 16n$$

exhibits diophantine 3-tuple with property $D(-n^6 - n^4)$

Taking (b,c_3) and employing the above procedure, it is seen that the triple (b,c_3,c_4) where

$$c_4 = 41n^3 + 25n$$

exhibits diophantine 3-tuple with property $D(-n^6 - n^4)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (b,c_s,c_{s+1}) where

$$c_s = (2s^2 + 2s + 1)n^3 + (s+1)^2 n\ ,\ s = 1,2,3,...$$

Sequence: 4

Let $a = SO_n = 2n^3 - n\ ,\ b = 2n^3 + n$

It is observed that

$$ab + n^2 = (2n^3)^2$$

Therefore, the pair (a,b) represents diophantine 2-tuple with the property $D(n^2)$.

Let c_1 be any non-zero polynomial in x such that

$$ac_1 + n^2 = p^2 \tag{13}$$

$$bc_1 + n^2 = q^2 \tag{14}$$

Eliminating c_1 between (13) and (14), we have

$$bp^2 - aq^2 = (b-a)(n^2) \tag{15}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + bT \tag{16}$$

in (15) and simplifying we get

$$X^2 = abT^2 + n^2$$

which is satisfied by $T = 1$, $X = 2n^3$

In view of (16) and (13), it is seen that

$$c_1 = 8n^3$$

Note that (a,b,c_1) represents diophantine 3-tuple with property $D(n^2)$

Taking (a,c_1) and employing the above procedure, it is seen that the triple (a,c_1,c_2) where

$$c_2 = 18n^3 - 3n$$

exhibits diophantine 3-tuple with property $D(n^2)$

Taking (a,c_2) and employing the above procedure, it is seen that the triple (a,c_2,c_3) where

$$c_3 = 32n^3 - 8n$$

exhibits diophantine 3-tuple with property $D(n^2)$

Taking (a,c_3) and employing the above procedure, it is seen that the triple (a,c_3,c_4) where

$$c_4 = 50n^3 - 15n$$

exhibits diophantine 3-tuple with property $D(n^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a,c_s,c_{s+1})$where

$$c_s = (2s^2 + 4s + 2)n^3 - (s^2 - 1)n, \; s = 1,2,3,...$$

Now, consider (b, c_1) and employing the above procedure, it is seen that the triple (b, c_1, c_2) where

$$c_2 = 18n^3 + 3n$$

exhibits diophantine 3-tuple with property $D(n^2)$

Taking (b, c_2) and employing the above procedure, it is seen that the triple (b, c_2, c_3) where

$$c_3 = 32n^3 + 8n$$

exhibits diophantine 3-tuple with property $D(n^2)$

Taking (b, c_3) and employing the above procedure, it is seen that the triple (b, c_3, c_4) where

$$c_4 = 50n^3 + 15n$$

exhibits diophantine 3-tuple with property $D(n^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (b, c_s, c_{s+1}) where

$$c_s = (2s^2 + 4s + 2)n^3 + (s^2 - 1)n, \; s = 1,2,3,...$$

Sequence: 5

Let $a = SO_n = 2n^3 - n$, $b = 2n^3 + n$

It is observed that

$$ab + n^2 + 4sn^3 + s^2 = (2n^3 + s)^2$$

Therefore, the pair (a, b) represents diophantine 2-tuple with the property $D(n^2 + 4sn^3 + s^2)$.

Let c_1 be any non-zero polynomial in x such that

$$ac_1 + n^2 + 4sn^3 + s^2 = p^2 \tag{17}$$

$$bc_1 + n^2 + 4sn^3 + s^2 = q^2 \tag{18}$$

Eliminating c_1 between (17) and (18), we have

$$bp^2 - aq^2 = (b - a)(n^2 + 4sn^3 + s^2) \tag{19}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + bT \tag{20}$$

in (19) and simplifying we get

$$X^2 = abT^2 + n^2 + 4sn^3 + s^2$$

which is satisfied by $T = 1 \ , X = 2n^3 + s$

In view of (20) and (17), it is seen that

$$c_1 = 8n^3 + 2s$$

Note that (a, b, c_1) represents diophantine 3-tuple with property $D(n^2 + 4sn^3 + s^2)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 18n^3 + 4s - 3n$$

exhibits diophantine 3-tuple with property $D(n^2 + 4sn^3 + s^2)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 32n^3 + 6s - 8n$$

exhibits diophantine 3-tuple with property $D(n^2 + 4sn^3 + s^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$where

$$c_s = (2s^2 + 4s + 2)n^3 + 2sn - (s^2 - 1)n, \ s = 1, 2, 3, ...$$

Now, consider (b, c_1) and employing the above procedure, it is seen that the triple (b, c_1, c_2) where

$$c_2 = 18n^3 + 4s + 3n$$

exhibits diophantine 3-tuple with property $D(n^2 + 4sn^3 + s^2)$

Taking (b, c_2) and employing the above procedure, it is seen that the triple (b, c_2, c_3) where

$$c_3 = 32n^3 + 6s + 8n$$

exhibits diophantine 3-tuple with property $D(n^2 + 4sn^3 + s^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(b, c_s, c_{s+1})$where

$$c_s = (2s^2 + 4s + 2)n^3 + 2sn + (s^2 - 1)n \ , \ s = 1, 2, 3, ...$$

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