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SPLIT DOMINATION INTEGRITY OF SEMI TOTAL POINT GRAPH

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ABSTRACT

The semi-total point graph $T_2(G)$ of a graph G is obtained by taking $V(G) \cup E(G)$ as its vertex set, where two vertices are adjacent if they are adjacent in G, or if one represents a vertex of G and the other an incident edge. In this paper, we investigate the split domination integrity SDI(G), defined as $SDI(G)=min\{|D|+m(G-D):D \subseteq V(G)\}$, where D is a split dominating set and m(G − D) is the order of a maximum component of G − D. We derive sharp bounds and structural constraints for $SDI(T_2(G))$ and compute its exact values for the Cartesian product of semi-total point graphs of several families of graphs. The results generalize and extend previous work on domination parameters of semi-total point graphs by introducing the vulnerability parameter SDI to this class of graphs.

KEYWORDS: *Domination, SplitDomination, Integrity, Split Domination Integrity, Semi Total Point Graph.*

AMS Subject Classification: *05C38, 05C69, 05C76.*

INTRODUCTION

Every graph that is taken into consideration here is finite, non-trivial, undirected, and linked with no loops or multiple edges. Consider a graph G, where the vertex set V(G) with |p| and the edge set E(G) with cardinality |q|.

A communication network can be modeled with a graph where nodes are represented by vertices and links by edges.Therefore, these parameters on graph models are studied in vulnerability analysis of networks.Many parameters have been introduced for the measurement of vulnerability.Integrity is the one of the wellknown parameter of vulnerability. IntegritywasintroducedbyBarefootetal1987anditisdefinedas $I(G)=min\{|S|+m(G-S);S \subseteq V(G)\}$ where m(G−S) denotes the order of a maximum component of $G-S$. R.Sundareswaran and V.Swaminathan introduced the concept of domination integrity of a graph G.$DI(G)=min\{|D|+m(G-D);D \subseteq V(G)\}$where Dis a dominating set and m(G −D) is the order of a maximum componentofG−D. Thedegreeofavertex deg(v) is the number of edges incident to it in G.The symbol $\lceil x \rceil$ denotes the smallest integer that is greater than or equal to x and $\lfloor x \rfloor$ denotes the greatestintegersmallerthanorequaltox.If every vertex in V − D is adjacent to at least one vertex in D, then the set $D \subseteq V(G)$of vertices is referred to as a dominating set of G. The domination number $\gamma(G)$ of G isthe least cardinality of D.

In this paper, we introduce split domination integrity and find split domination integrity of semi-total point graphs for some graphs. And also, we calculate the split domination integrity of the cartesian product of semi total point graphs.

Preliminaries

Semi-total point graphs of graphs were first introduced by Sampathkumar et.al.

Definition 2.1

The graph whose vertex set is V(G) U E(G) is the semitotal point graph $T_2(G)$ of a graph G. Two vertices are adjacent if and only if they are adjacent vertices of G or one is a vertex of G and the other is an edge of G incident with it.

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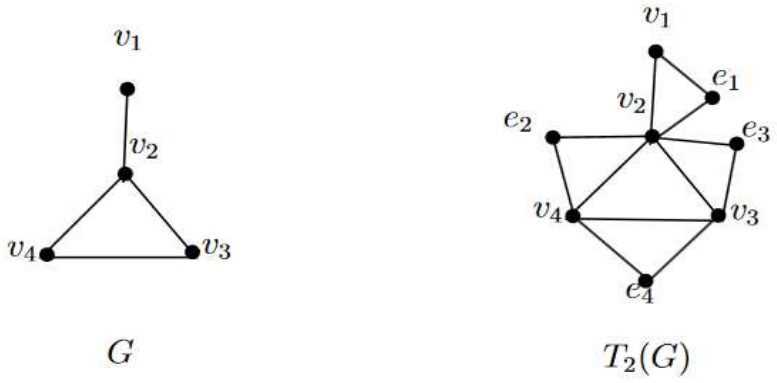


Fig 1. The Graph G and its Semi Total Point Graph $T_2(G)$

Definition 2.2
If N(D)∩D= ϕ for any vertex v ∈ D ⊆ V(G), then a set D ⊆ V(G) is a dominatingset. The lowest cardinality of a dominating set of G is known as the domination number γ(G).If <V-D> is disconnected, then D is a split dominating set (SDI -set).

Definition 2.3
Let G and G' be simple graphs. The cartesian product of the graphs G×G' has the vertex set V(G) × V(G') in which (g, g_1') (g, g_2') is an edge whenever $g_1 = g_2$ and $g_1'g_2' \in E(G')$ or $g_1' = g_2'$and $g_1g_2 \in E(G)$.

Definition 2.4
TheSplit domination integrity of a graph Gis SDI (G)=min{|S|+m(G −S);S⊆V (G)}where S is a split dominating set and m (G −S)is the order of a maximum component of G −S.AsetS⊆V(G)isa SDIsetif SDI(G)=|S|+m(G−S)and S is a dominatingset of G.

Theorem:

For any connected graph G,$SDI(T_2(G)^{++}) \leq 2p$.

Proof : Since G is connected then $\Delta(T_2(G)) \leq p$

−1.Suppose$\Delta(T_2(G))=p-1$,thenGiscompleteand$\gamma(T_2(G))=p-1$.Alsom($T_2(G)$-D)=

p+1.And so$SDI(T_2(G))=2p$.Suppose$\Delta(T_2(G))<p-1$thenclearly$SDI(T_2(G))<2p$.

Theorem1.5. ForanygraphG,$SDI(G) \leq SDI(T_2(G))$.The equality holds for

$G=K_2$ or K_p.

Proof. We consider the following cases:
Case(i):Let G be a graph without isolated vertices.Then $\gamma_s(G) \leq p/2$. Therefore by the definition of $T_2(G)$,$\gamma_s(T_2(G)) \leq (p+q)/2$.Hence $\gamma_s(G) \leq \gamma_s(G^{++})$. that is $|D| \leq |D_1|$, where D_1 is the dominating set of $T_2(G)$. The value of Split domination integrity SDI(G) = |D|+ m(G −D) and $SDI(G^{++}) = |D^{++}|+m(G^{++}-D)$,Since minimal dominating set of G which is also a minimum DI-set of G.Therefore $SDI(G) \leq SDI(T_2(G))$.

Case(ii):Let G be a graph with isolated vertices of the form G=HUtK1

Where H=(p',q')is any connected graph.ThenT2(G) =T2(G1)UtK1.by case (i),

weget$\gamma s(H) \leq \gamma s$ (H1).Therefore $\gamma s(G) \leq p'/2 +$ tand$\gamma(T2(G)) \leq (p'+q')/2 +t$

which implies $\gamma s(G) \leq \gamma s(T2(G))$. Therefore $SDI(G) \leq SDI$ (T2(G)).Further trivially equality holds.

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Theorem 3.1

For every wheel $W_p$with $p \geq 4$, $SDI(T_2(W_p)) = \lceil p-1/2 \rceil + 5$.

Proof.

$T_2(W_p)$ has $3p - 2$ vertices and $6p - 1$ edges. Let $v_1$be the central vertex of $T_2(W_p)$.

Let $\{v_1, v_2,..., v_p, e_1, e_2,..., e_q\}$ be the vertices of $T_2(W_p)$.

Case (i):p is odd

An outer wheel cycle has an order of $p - 1$, which is even. Then $\gamma(C_{p-1}) = \lceil p-1/2 \rceil$. clearly $|D| = \{v\} \cup \lceil p-1/2 \rceil = \lceil p-1/2 \rceil + 1$. When the dominating set D of vertices is removed from $T_2(W_p)$ gives a graph whose components are P′4s and K′1s. Hence $m((T_2(W_p))-D) = 4$. Since D gives the minimum value of $|D| + m((T_2(W_p)) - D)$. Clearly, <V-D> is disconnected. Therefore $SDI((T_2(W_p))) = \lceil p-1/2 \rceil + 5$.

Case (ii):p is even

An outer wheel cycle has an order of $p - 1$, which is odd. Then $\gamma(C_{p-1}) = \lceil p-1/2 \rceil$. Then $|D| = \{v_1\} \cup \{v_2, v_4,...v_{p-2}\} = \lceil p-1/2 \rceil + 1$. By case (i), $SDI(T_2(W_p)) = |D| + m(T_2(W_p) - D) = \lceil p-1/2 \rceil + 5$.

Theorem 3.2

For any complete bipartite graph $K_{p,q}$, $SDI(T_2(K_{p,q}))=2\min\{p, q\} + 1$.

Proof:

Since $T_2(K_{p,q})$ has $2pq$ vertices and $3pq$ edges. Let D be the minimum split dominating set. Clearly $|D| = \min\{p, q\}$ and also $m(T_2(K_{p,q})- D) = \min\{p, q\}+1$. Hence $SDI(T_2(K_{p,q}))= 2\min\{p, q\} + 1$.

Theorem 3.3:

For every connected graph G,$SDI(T_2(G))\leq 2p -1$.

Proof:

Since G is connected then $\Delta(G) \leq p -1$. Suppose$\Delta(G)=p -1$,thenGiscompleteand$\gamma(T_2(G))=p-1$.Alsom $(T_2(G)-D)= p$.AndsoSDI$(T_2(G))=2p -1$.Suppose$\Delta(G)<p-1$thenclearlySDI$(T_2(G))<2p-1$.HenceSDI$(T_2(G))\leq 2p-1$.

Theorem 3.4:

For any connected graph G, $T_2(G)$ is Hamiltonian then $G \cong C_p$, and

$$SDI(T_2(G)) = \begin{cases} 4 & \text{for } p = 3 \\ \lceil \frac{p}{2} \rceil + 3 & \text{for } p \geq 4 \end{cases}$$

Proof.

Let $T_2(G)$ be a Hamiltonian. enough to prove that G has a point vertex u of degree ≥ 3, then $T_2(G)$ is not Hamiltonian. We assume that deg u = 3 in G and $e_1 = uu_1$, $e_2 = uu_2$, and $e_3 = uu_3$ be the lines incident to u. Suppose $T_2(G)$ is hamiltonian and $C = v_1v_2.....v_nv_1$ be a hamiltonian cycle in $T_2(G)$. Let e_1 be the first line incident to u in G, and $v_1 = e_1$. Since no two consecutive elements of C are lines of G, then $v_{i+1} = u$. Then v_{i+2} is one of the elements e_2, e_3, u_1,

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u_2, u_3 of G. Here we consider two cases.

case(i): Suppose v_{i+2} is one of the lines e_2, e_3, say $v_{i+2} = e_2$. Then $v_j = e_3$ for some j. Since no two consecutive elements of C are lines of G, it follows that either $v_{j-1} = u$ or $v_{j+1} = u$. Thus, u occurs again.

case (ii): Suppose v_{j+2} is one of the lines u_1, u_2, u_3. In cycle C, let $v_j = e_2$. Then, in case (i), either $v_{j-1} = u$ or $v_{j+1} = u$. So you occur again, C. Hence, in both cases, u occurs twice in C. This contradicts the fact that C is a Hamiltonian cycle and proves that deg u ≥ 2 for all u in G. Suppose G is a path of length ≥ 2, then obviously $T_2(G)$ is not Hamiltonian. It follows that $G \cong C_p$, and $SDI(T_2(G)) = \begin{cases} 4 & \text{for } p = 3 \\ \lceil \frac{p}{2} \rceil + 3 & \text{for } p \geq 4 \end{cases}$

Theorem 3.5:

Let $S_{p,q}$ be a double star graph, then $SDI(T_2(S_{p,q})) = 4$.

Proof.

The $V(T_2(S_{p,q})) = \{u, v_1, v_2, ..., v_p, v, u_1, u_2, ..., u_q, e_1, e_2, ..., e_n, e_1', e_2', ..., e_q'\}$.

We have two cases:

case (i): **p = q**

Choose a split dominating set $D = \{u, v\}$ that dominates all the point and line vertices of $T_2(S_{p,q})$. Clearly, $|D| = 2$, and so $m(T_2(S_{p,q}) - D) = 2$. Clearly, <V-D> is disconnected. Therefore, $SDI(T_2(S_{p,q})) \leq |D| + m(T_2(S_{p,q}) - D) = 4$. Suppose D_1 is any split dominating set other than D, with $m(T_2(S_{p,q}) - D_1) = 1$, then $|D_1| > 2$, which implies $SDI(T_2(S_{p,q})) \geq |D_1| + m(T_2(S_{p,q}) - D_1) = 4$. Therefore, $SDI(T_2(S_{p,q})) = 4$ is the minimum.

case (ii): **p =q**

From the definition of $T_2(G)$, $\gamma(T_2(S_{p,q})) = 2$. So by case (i), $SDI(T_2(S_{p,q})) = 4$.

Theorem 3.6:

For $p \geq 2$, $SDI(T_2(K_{p,q} \times P_n)) = \begin{cases} \lfloor p/2 \rfloor (p+q) + q + 2 & p > q, n \text{ is even} \\ \lfloor p/2 \rfloor p + \lceil p/2 \rceil q + q + 2 & p > q, n \text{ is odd} \\ \lfloor p/2 \rfloor 3q + 2 & p = q, n \text{ is even} \\ (\frac{p}{2}) 3q + 2, & p = q, n \text{ is odd} \end{cases}$

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Proof:

To Find the SDI($T_2(K_{p,q} \times P_n)$) we consider the following.

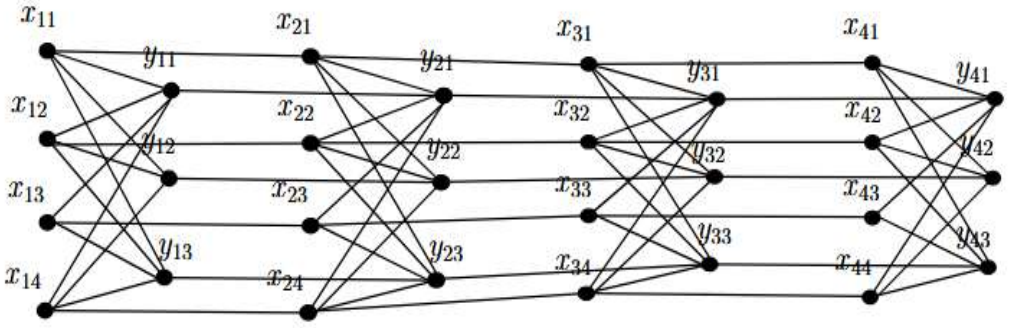


Fig2. The Cartesian Product of($K_{p,q} \times P_n$) for p>q, n=4

Case(i):p> qandniseven

Choose a dominating set of qvertices from the first copy of Kp,q and a set of pvertices from the second copy, qvertices from the third copy, and soon. Sinceniseventhereare $\lfloor p/2 \rfloor$ (p+q)vertices creating a dominating set.Itisevident that m(T2(Kp,q × P_n) – D) = q+2. Therefore SDI($T_2(K_{p,q} \times P_n)$)= $\lfloor p/2 \rfloor$ (p+q)+q+2.

Case(ii):p> qandnisodd

Chooseadominatingsetofqverticesfromthefirstcopyof$K_{p,q}$andasetofpverticesfromthesecondcopy,qverticesfromthethirdcopy,an dsoon.Sincenisoddandp>q, there are $\lfloor p/2 \rfloor$ p+ $\lceil p/2 \rceil$q vertices creating a split dominating set. And m($T_2(K_{p,q} \times P_n)$ – D) = q+2. Therefore SDI ($T_2(K_{p,q} \times P_n)$) = q+2.

Hence SDI ($T_2(K_{p,q} \times P_n)$) = $\lfloor p/2 \rfloor$ p+ $\lceil p/2 \rceil$q+q+2

Case (iii): p=q and nis even

Similar by case (i), SDI (($T_2(K_{p,q} \times P_n)$)= $\lfloor p/2 \rfloor$ 3q +2

Case(iv): p=q and nis odd

Similar by case (ii), SDI (($T_2(K_{p,q} \times P_n)$) = $(\frac{p}{2})$3q +2.

Theorem 3.7:

For a graph $K_2 \times C_p$, ($p \geq 3$)

$$SDI((T_2(K_2 \times C_p)) = \begin{cases} p+4 & \text{if p is even} \\ p+5 & \text{if p is odd} \end{cases}$$

Proof:

The graph ($T_2(K_2 \times C_p)$) has 5p vertices and 9q edges. The vertices of $V(K_2) = \{v_1, v_2\}$ and $V(C_p) = \{u_1, u_2, ..., u_p\}$. We consider two cases.

Case (i): If p is even

Select a split dominating set D containing the independent set of vertices $\{(v_1, u_1), (v_1, u_3), \ldots, (v_1, u_{p-1})\}$ of ($T_2(K_2 \times C_p)$) (that is, $\lceil p/2 \rceil$ vertices). Additionally, each of the two C′ps internal vertices should be arranged so that there is a distance of 2 between the chosen

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vertices (v_2, u_2), (v_2, u_3),…., (v_2, u_p).(that is $\lfloor p/2 \rfloor$ vertices) and so $|D| = p$. After subtracting D vertices from $(T_2(K_2 \times Cp))$, we get a graph with a maximum component of order 4. Hence $m((T_2(K_2 \times C_p)) - D) = 4$. Therefore, $SDI(T_2(K_2 \times C_p)) = p+ 4$.

Case (ii): If p is odd

Select a split dominating set D containing the independent set of vertices (v_1, u_1), (v_1, u_3),..., (v_1, u_p) of $T_2(K_2 \times C_p)$(that is $\lceil p/2 \rceil$ vertices). Additionally, each of the two C'_ps internal vertices should be arranged so that there is a distance of 2 between the chosen vertices (v_2, u_2), (v_2, u_3),..., (v_2, u_{p-1})(that is $\lfloor p/2 \rfloor$ vertices). Ddominates all the vertices of $T_2(K_2 \times C_p)$ except the pendent line vertex of C_p. And so $|D| = \lceil p/2 \rceil + \lfloor p/2 \rfloor + 1 = p + 1$. We remove the vertices of D from $T_2(K_2 \times C_p)$,we get a graph with maximum component of order 4. Hence $m(T_2(K_2 \times C_p) - D) = 4$. Therefore,$SDI(T_2(K_2 \times C_p)) = p+ 5$.

Theorem 3.8:

For a graph $K_2 \times K_{1,p-1}$, then $SDI(T_2(K_2 \times K_{1,p-1})) = p + 6$.

Proof.

Consider the graph $T_2(K_2 \times K_{1,p-1})$, Find a split dominating set D that includes the middle vertices of every two stars of $K_{1,n-1}$ along with the vertices $\{(v_1, u), (v_2, u)\}$. Clearly, $|D| = p+ 2$. By removing the dominating vertices from $T_2(K_2 \times K_{1,p-1})$, we get $m(T_2(K_2 \times K_{1,p-1}) - D) = 4$. Hence $SDI(T_2(K_2 \times K_{1,p-1})) = p + 6$.

Theorem 3.9

For a graph $P_p \times P_q$, then $SDI(T_2(P_p \times P_q)) = \lceil pq/2 \rceil + 3$, $p = q$.

Proof.

Let $p = q = 2$, then $T_2(P_2 \times P_2) \cong T_2(C_4)$, then clearly $SDI(T_2(P_2 \times P_2)) = 5$. Then, for $p = q \geq 2$, In $T_2(P_p \times P_q)$, choose a split dominating set D containing the non-adjacent vertices of each path $P'_p \times P'_q$s. Clearly, $|D| = \lceil pq/2 \rceil$, and so $m((T_2(P_p \times P_q) - D) = 3$. Therefore, $SDI (T_2(P_p \times P_q) = \lceil pq/2 \rceil + 3$.

4. CONCLUSION

A communication system is a graph with nodes representing processors and edges representing communication paths. Reliability is crucial for network designers when stations fail. The number of non-functioning nodes and the maximum order of the surviving subnetwork are key factors in assessing a network's vulnerability. A minimum dominating set of vertices establishes connections with all other vertices. When D is removed from the network, communication decreases, and decision-making becomes paralyzed. Split-domination integrity is a vulnerability parameter discussed in this paper. The study investigated semi-total point graphs and found the SDI of the cartesian product of semi-total point graphs.

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