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DERIVATION OF CERTIFIED DOMINATING SETS IN BIPARTITE GRAPHS AND ITS APPLICATION

Megha Chandraprakash Gaikwad
Assistant Professor, Shri Sheth Muralidharji Mansingaka Arts, science and commerce College, Pachora ,Dist. Jalgaon

ABSTRACT
A certified dominating set is a refinement of the classical dominating set in graph theory, introduced to ensure that each dominating vertex dominates at least two vertices outside the set. This paper studies the derivation and properties of certified dominating sets in bipartite graphs. We present definitions, theoretical results, illustrative examples, and discuss bounds and applications.
KEYWORDS: *Bipartite Graph, Dominating Set, Certified Domination, Graph Theory.*

INTRODUCTION

Domination in graphs is a well-studied topic with applications in network security, social networks, and resource allocation.
A subset $D \subseteq V$ is called a dominating set if $\forall v \in V \setminus D, N(v) \cap D \neq \emptyset$, where $\boldsymbol{N(v)}$ denotes the open neighborhood of v. The minimum cardinality of such a set is the domination number
$\gamma(G)$. To enhance reliability and structural robustness, the notion of certified domination was introduced.
A dominating set D is said to be certified if every vertex $u \in D$ satisfies:

$$| N(u) \cap (V \setminus D) \neq 1,$$

The minimum size of such a set is called the certified domination number, denoted by

$$\gamma(G) \leq \gamma c(G)$$

A certified dominating set strengthens the concept of domination by imposing additional structural constraints. Bipartite graphs, due to their partitioned structure, provide a natural setting to study certified domination.

Assumptions
If G is bipartite with partition $V = X \cup Y, X \cap Y = \emptyset$ then adjacency occurs only between the two parts.
Let G = (V, E) be a simple graph. A dominating set D ⊆ V is such that every vertex in V − D is adjacent to at least one vertex in D. A dominating vertex v ∈ D is said to be certified if it has at least two neighbors in V − D. A dominating set D is a certified dominating set (CDS) if every vertex in D is certified. A bipartite graph G = (X ∪ Y, E) consists of two disjoint vertex sets X and Y such that every edge joins a vertex in X to a vertex in Y.

Certified Domination in Bipartite Graphs:
In bipartite graphs, domination must respect the partition structure. We analyze how certified dominating sets can be constructed by selecting vertices alternately from partitions X and Y. Necessary and sufficient conditions for existence of CDS are derived.

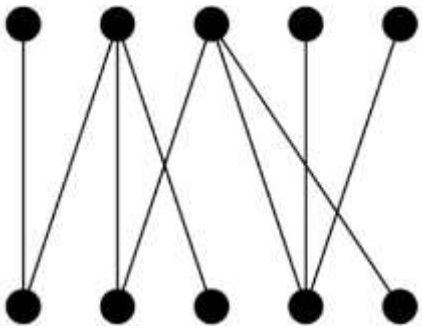


Figure 1 : Bipartite Graph

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Theorem 1:
Every bipartite graph with minimum degree at least 2 admits a certified dominating set.
Proof:
Let G = (X ∪ Y, E) be a bipartite graph with δ(G) ≥ 2. Choose a minimal dominating set D. Since each vertex has degree at least 2, each vertex in D dominates at least two vertices outside D, hence D is certified.

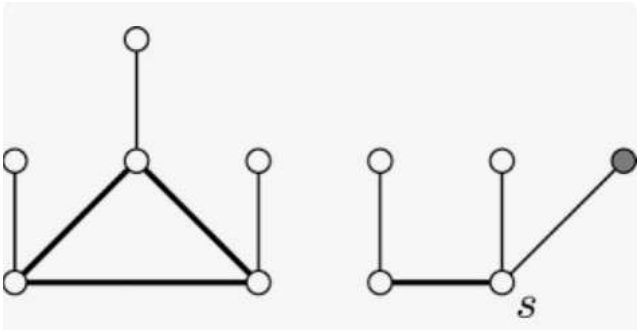


Figure 2: Graph with Certified Dominating set

Bounds on Certified Domination Number
The certified domination number γ_c(G) is the minimum cardinality of a CDS. For a bipartite graph G with n vertices,

$$\gamma(G) \leq \gamma_c(G) \leq n/2.$$

We discuss tightness of these bounds with examples.

Algorithmic Construction
We propose a greedy algorithm to construct a CDS in bipartite graphs by iteratively selecting high-degree vertices while maintaining certification conditions.

For a bipartite graph: $G = (U \cup V, E)$. To form a certified dominating set $D \subseteq U \cup V$

Consider a set $D \neq \emptyset$ involving the other vertices that are not in the dominated set.

Let $R \supset V(G)$ the set of other vertices that are not in the dominating set. Involving the greedy selection: For $R \neq \emptyset \vee vertex\ x \notin D$.

Evaluating: for each vertex : $deg_R(x) = |\{y \epsilon R: (x, y) \epsilon E\}|$.

Selecting a unique vertex such that $v = \arg\ arg^{max}_{x \notin D} \deg R^{(x)}$

Add $v + D = v \in v + D$, Remove v and its neighbors from R: $R(p, v) \vee p \in R$

For Evaluation of Certification on domination
Each vertex $v \in D$: Declaring D to be a dominating set. Evaluate: $ext(v) = |\{u \notin D: (u, u)|$
After considering the dominating condition i.e. every vertex in V\D is adjacent to at least one vertex in D. later consider: Every vertex $v \in D$ has either no neighbor in V\DV or at least two neighbors in V\D later giving this statement:

$$\forall v \in D, |\ N(v) \cap (V \setminus D)\ | \neq 1$$

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So each vertex in D has 0 or ≥2 external neighbors**,** satisfying certification. Thus, the algorithm correctly determines certified domination**.**

Applications

Certified domination in bipartite graphs has applications in fault-tolerant network design, monitoring systems, and matching-based resource allocation.

Let $D_x = D \cap X$, $D_y = D \cap Y$. Edges exist only between X and Y. $For\ x \in Dx$:

$N(x) \subseteq Y$ Certification becomes: $| N(x) \cap (Y \setminus DY) | \in \{0\} \cup [2, \infty)$ For $y \in Dy$: $| N(y) \cap (X \setminus DX) | \in \{0\} \cup [2, \infty)$

Later using certified domination condition in bipartite graphs as:

$$\forall x \in X \setminus Dx, N(x) \cap Dy \neq \emptyset$$

Certification conditions: $\forall x \in DX, | N(x) \cap (Y \setminus DY) | \neq 1$

Later supporting the lower bounds in bipartite graphs as $\gamma(G) \leq \gamma c(G)$. Where $\gamma c(G)$ is the certified domination number. To see the implementation on the Fault-Tolerance Requirement, collaborating with the above results we get; $| N(u) \cap (V \setminus D) | \geq 2\ or\ 0$

This further guarantees the redundancy property as:

Every active controller either:

- Monitors at least two independent devices
- Also is fully internal to the control backbone.

CONCLUSION

This paper presents a foundational study of certified dominating sets in bipartite graphs. Future work includes complexity analysis and extension to weighted and dynamic graphs.

Later we see a brief conclusion of the domination parameters in bipartite graphs supporting $\forall v \in V \setminus D, N(v) \cap D \neq \emptyset$ when every non-controller vertex is supervised by at least one controller.

Therefore, in bipartite graphs:

1. Coverage is guaranteed by domination.
2. Fault tolerance is guaranteed by certification.
3. The certified domination number satisfies

$$\gamma(G) \leq \gamma c(G)$$

Thereby giving a lower bound on the number of controllers required for reliable operation.

Consequently, the parameter $\gamma c(G)$ represents a minimum redundancy-preserving control configuration in bipartite network structures.

This establishes certified domination as a rigorous combinatorial model for fault-tolerant monitoring, resource allocation stability, and resilient distributed control in bipartite systems.

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