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PREDICTION OF PRESSURE DROP IN HORIZONTAL PIPES WITH GAS-LIQUID FLOW

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ABSTRACT

Predicting the flow pattern when two gas-liquid phases circulate through a horizontal pipe is a priority. In a previous article[16], the authors reviewed some of the maps used to identify those flow patterns. This article presents a review of some of the correlations used to calculate pressure drops, once the existing flow pattern type has been identified. Some examples are also presented to clarify the use of these correlations.

KEYWORDS: ***Flow to two gas-liquid phases, flow patterns, pressure drops, correlations.***

1. - INTRODUCTION

In the design of pipes with two-phase flow, the engineer is mainly concerned with the calculation of pressure drop, which can be estimated quite accurately. It has been recognized for years that in order to improve the prediction of the different constituent parameters of this phenomenon, which are the volumetric fraction of each phase (holdup), pressure drop, heat and mass transfer, as well as other hydraulic parameters, it was necessary to consider the detailed structure of the flow configuration. These configurations, which are related to the distribution of phases within the pipe, are called flow patterns or regions[16].

Many experimental and theoretical work has been carried out to predict the pressure drop and the type of flow pattern produced in the pipes, but so far no general correlation has been found, This is due to the existence of a certain number of complications that hinder the use of a single correlation. The largest of these in the two-phase flow is the variety of flow patterns that can occur. Although no general correlation has been found applicable to all flow types, correlations have been developed for specific flow patterns. One of the first to do a visual classification of flow patterns was Alves [1].

Flow patterns are empirically correlated based on the flow and properties of fluids. The prediction of pressure drop is made by correlations of different kinds. The first to propose one of them was Lockhart and Martinelli [2], which depends on the type of flow. Currently there are semi-empirical correlations independent of the flow pattern present in the pipe, which brings us closer to the development of a general model of gas-liquid biphasic flow in the perhaps not distant future.

Pressure drop generally receives contributions of three different effects: friction, acceleration, and elevation. In the case of horizontal flow, the latter effect does not intervene in pressure drop. The simplest calculation of the pressure drop in flow to two liquid-gas phases is based on the work of Lockhart and Martinelli [2]. They found that the general equation for the calculation of frictional pressure losses in the two-phase flow was given by:

$$\Delta P_{2F} = \phi^2 \, \Delta P_{1F} \quad (1)$$

Where: ΔP_{2F} = pressure drop in flow to two phases. ; ΔP_{1F} = pressure drop in one of the phases.
ϕ = Function that depends on the Lockhart-Martinelli module (X):

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$$X = \left(\frac{\Delta P_L}{\Delta P_G} \right)^{1/2} \qquad (2)$$

Where: X =Lockhart-Martinelli module for flow to two liquid-gas phases.
ΔP_L - pressure drop in the liquid phase.
Subsequently, other researchers were tasked with developing more detailed and complex theoretical models, which give the engineer greater precision in calculating pressure drops in two phases.

2.- CORRELATION OF LOCKHART-MARTINELLI

The basic idea of this correlation is, that the pressure drop in the concurrent flow to two phases can be calculated using the equations and graphs commonly used for the calculation of pressure drop in the flow to a single fluid phase, once the individual speeds of each phase are known. By assuming that the two phases are circulating along the line completely separated from each other, it is possible to define their respective velocity in terms of a diameter called hydraulics and a form factor. In their article, they publish in detail the analysis of the phenomenon and the development of this correlation. Lockhart and Martinelli launched two basic postulates for their analysis:

1. The static pressure drop for the liquid phase is always equivalent to that of the gas phase regardless of the flow pattern adopted by the moving mixture, there is also no noticeable radial static pressure difference.
2. The volume occupied by the liquid phase plus the volume occupied by the gas phase at any time and position, must be equal to the total volume of the pipe. These postulates suggest the non-existence of a change in the flow model along the pipe. In this way, the plug and battering ram flows are eliminated in this consideration.

Based on their experimental observations, Lockhart and Martinelli graphed ϕ vs. X similarly to the one shown in Figure 1. These researchers actually obtained four curves of ϕ for each phase, as they defined the following flow regimes:

1. Both liquid and gas phases under turbulent (tt) regime.
2. Turbulent flow in the liquid phase and viscous flow in the gas phase (tv)
3. Viscous flow in the liquid and turbulent in the gas phase (vt).
4. Both phases in viscous (vv) regimen.

The graphs were made in logarithmic scale in order to narrow the data scattered around a single correlation curve.

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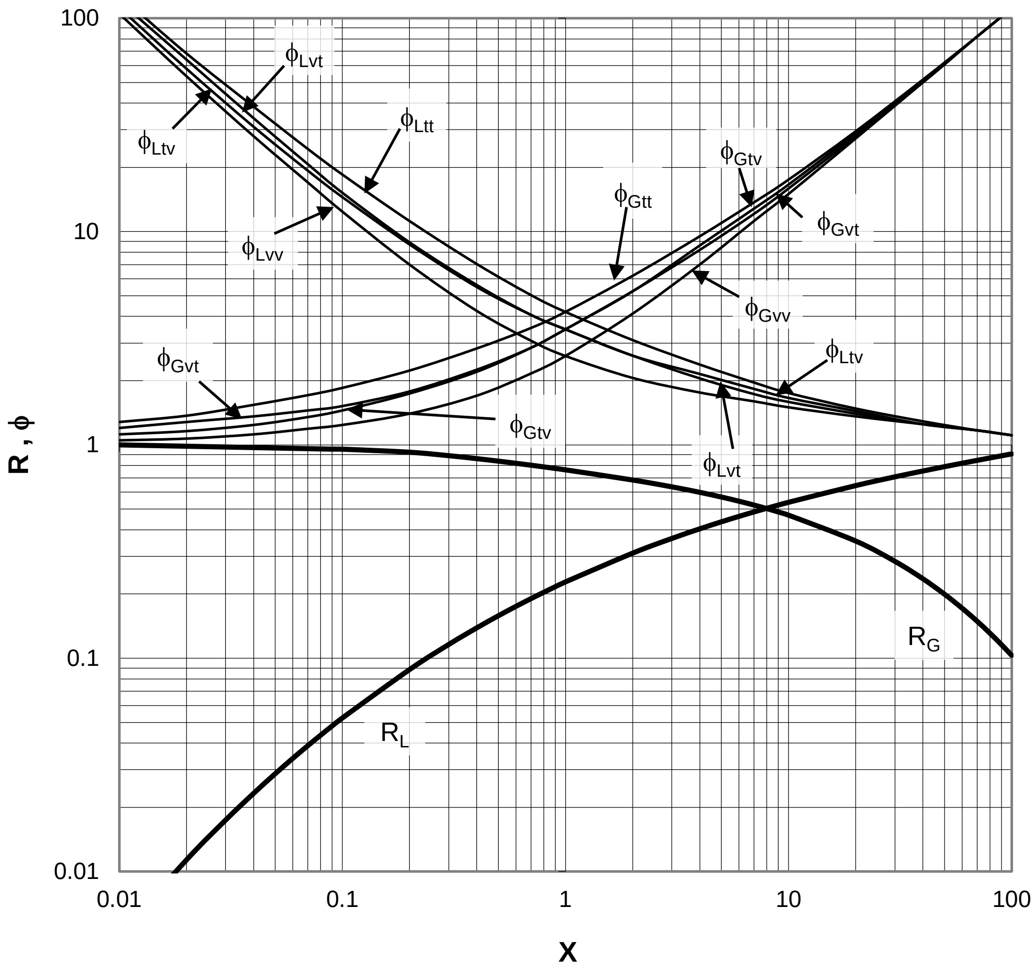

Figure 1.- Lockhart-Martinelli graph for s and R based on X. (1949)

In this graph, R is the fraction of the total volume of the pipe occupied by one of the phases, known as holdup. By calculating the value of X, you can get the R value for each phase from the graph in Figure 1.

Although Lockhart-Martinelli's method only provides approximate solutions to pressure drops, the way both researchers attack the problem based on an idealized physical model is perhaps the most satisfactory solution available and from which numerous work and correlations have been developed.

Lockhart-Martinelli method:

1.- Determine the flow rate of each phase according to Lockhart-Martinelli's criterion, obtaining the superficial Reynolds:

$$Re = \frac{D v_S \rho}{\mu} \qquad (3)$$

Where: D - internal diameter of the pipe in m. v_S - surface velocity of the phase in m/s.
ρ = phase density to operating conditions in kg/m^3. ; μ= The viscosity of the phase in kg/(m s).

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If Re <1000, the phase regimen is viscous (v). If Re > 2000, the phase regimen is turbulent (t).
2.- Calculate the pressure drop for each phase, using the Darcy equation: For the liquid phase:

$$\Delta P_L = \frac{f_D\, V_{SL}^2\, L\, \rho_L}{2 g_C D} \quad \left[\frac{kgf}{m^2}\right] \qquad (4)$$

Where:
f_D = Darcy friction factor. V_{SL} - surface velocity of the liquid phase in m/s.
L - length of the pipe in m. ; ρ_L = - density of the liquid phase in kg/m3.
gc = 9.81 m kg/(s^2 kgf) = 32.2 ft lb/(s^2 lbf) ; D - internal diameter of the pipe in m.
To convert kgf/m^2 to kgf/cm^2, divide between 10000, and to convert from kgf/m^2 to lbf/in^2 or psi, multiply by 0.0014.
For the gas phase:

$$\Delta P_G = 6.379 \times 10^{-7}\, L\, W_G^2 \left(\frac{f_D}{D^5\, \rho_G}\right) \quad \left[\frac{kgf}{m^2}\right] \qquad (5)$$

Where:
L - length of the pipe in m. ; W_G - mass flow of the gas in kg/h. ; f_D - Darcy's friction factor.
D - internal diameter of the pipe in m. ; ρ_G - density of the gas phase in kg/m^3.

If Re <2100, the phase is at laminate regimen. Darcy's friction factor is only a function of Reynolds' number and is obtained using the Moody chart (Figure 2 or 3) or is calculated using the Hagen-Poiseuille equation:

$$f_D = \frac{64}{Re} \qquad (6)$$

If Re > 2100, the phase in flow is in transition. If Re > 10000, the phase flows into turbulent regime. For these last two regimes, Darcy's friction factor is then a function of the relative roughness of the pipe (ε/D) and the Reynolds number, and can be obtained using the Moody chart or by using the Chen[3] equation:

$$\frac{1}{\sqrt{f_D}} = -2\log\left[\frac{\varepsilon}{3.7065D} - \frac{5.0452}{Re}\log\left(\frac{1}{2.8257}\left(\frac{\varepsilon}{D}\right)^{1.1098} + \frac{5.8506}{Re^{0.8981}}\right)\right] \qquad (7)$$

It is worth noting the difference between this transitional criterion from laminar to turbulent regime (Re = 2100), and that of Lockhart-Martinelli defined in step 1 of this method. If Figure 2 is used, the friction factor obtained is Fanning (f_f), whose value is one-fourth of the Darcy friction factor (f_D).

3.- Calculate the X parameter with equation 2, or with the following equation, which applies only to turbulent-turbulent flow (tt):

$$X^2 = 0.0084\left(\frac{W_L}{W_G}\right)^{1.8}\left(\frac{\rho_G}{\rho_L}\right)\left(\frac{\mu_L}{\mu_G}\right)^{0.2}$$

4.- In the Lockhart-Martinelli graph (Figure 1), X reads the parameter ϕ_G corresponding to the flow type (tt, tv, vt or vv).
5.- Calculate the pressure drop to two phases with equation 1, using the pressure drop for the gas phase.

$$\Delta P_{2F} = \phi_G^2\, \Delta P_G \qquad (1)$$

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Figure 2.- Moody graph of friction factor for any type of pipe.

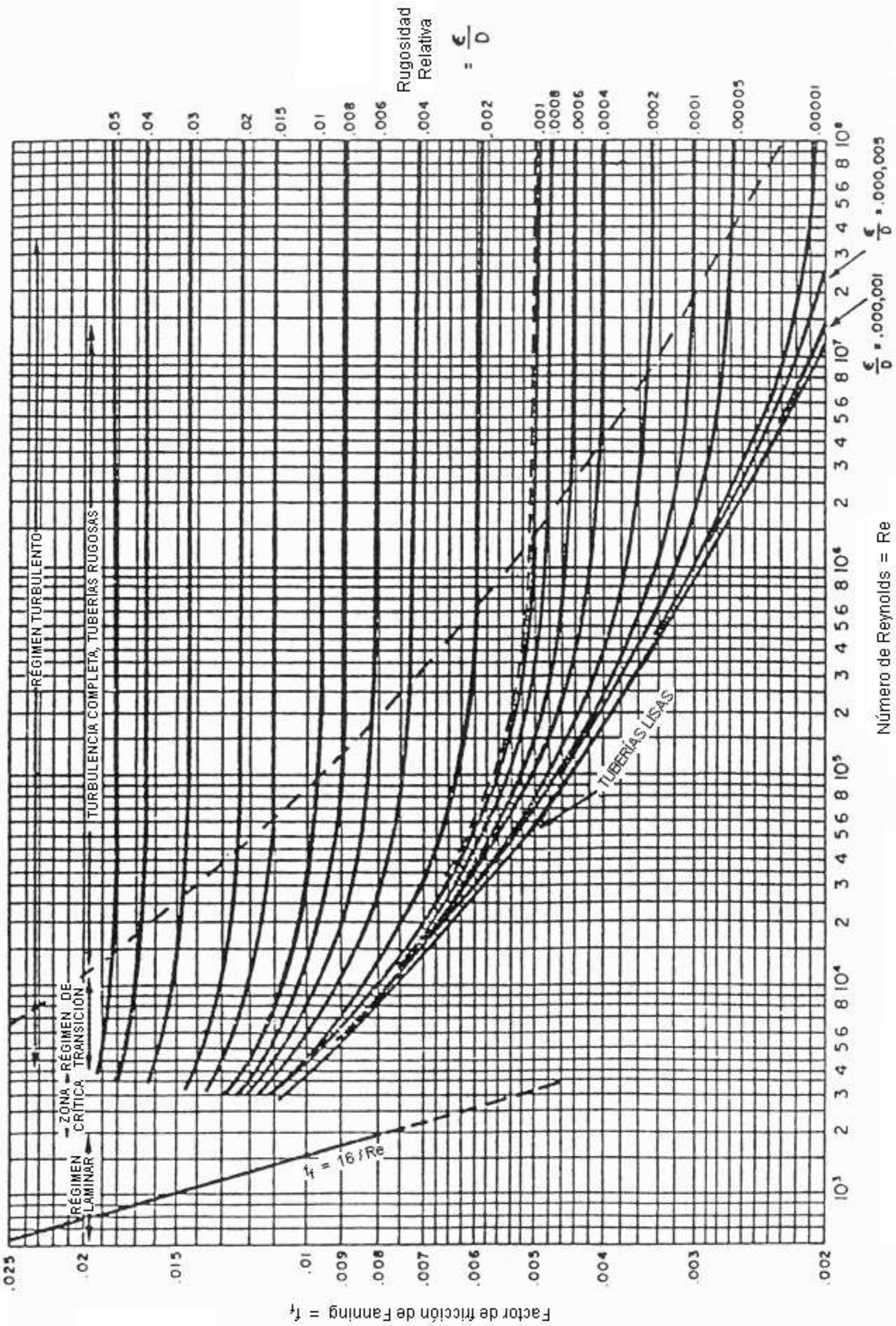

Figure 3.- Moody graph of friction factor for commercial steel and wrought iron pipes

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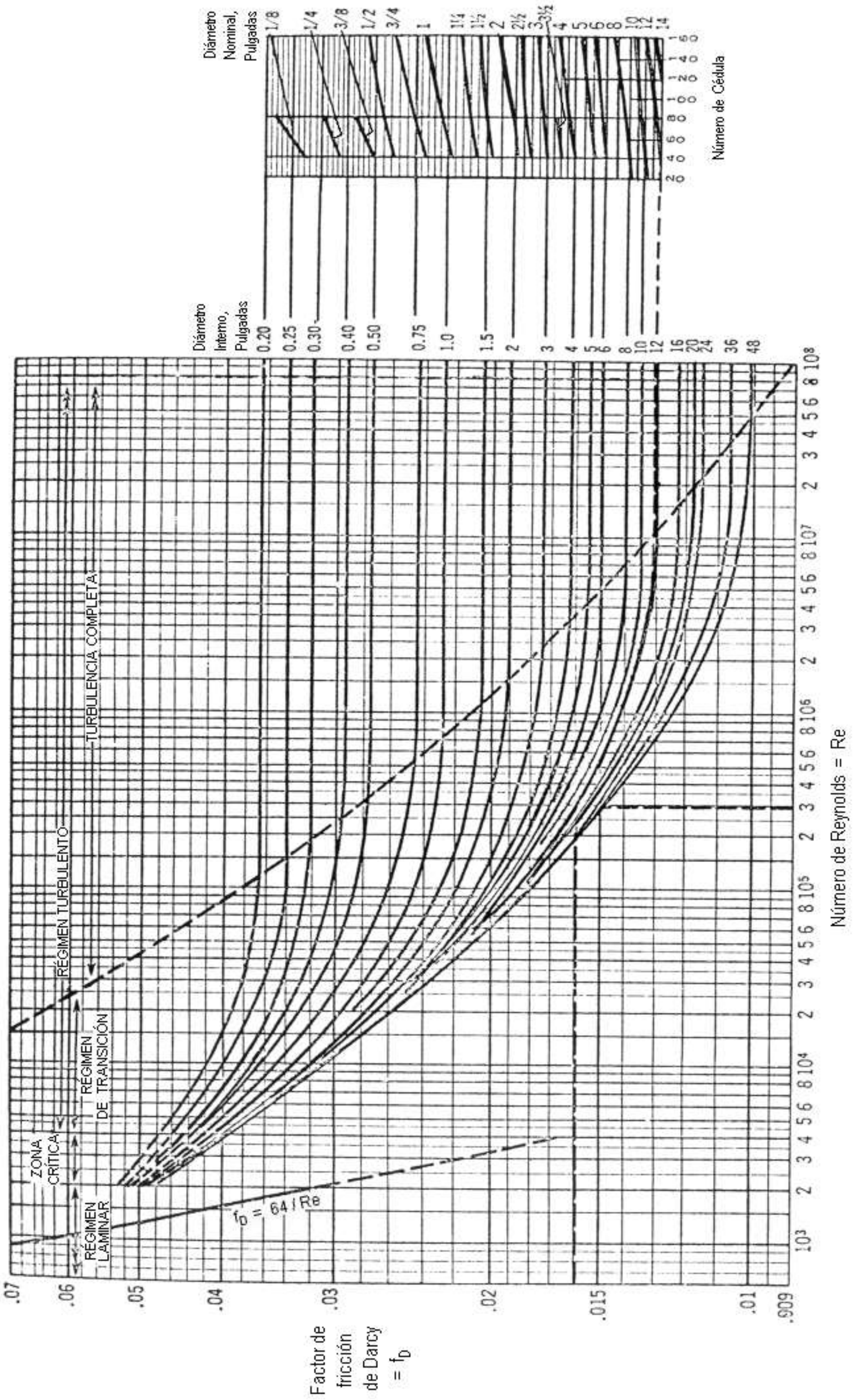

A simplified way to obtain pressure losses with two-phase flow is :

a) Calculate pressure drops first as if each phase were alone in the pipe.
b) Calculate Martinelli's module

c) $X = \left[\frac{\Delta P_L}{\Delta P_G}\right]^{0.5}$
d) Calculate the total pressure drop:
e) $\Delta P_{total} = Y_L \Delta P_L = Y_G \Delta P_G$
Where:

$$Y_L = 4.6X^{-1.78} + 12.5X^{-0.68} + 0.65$$
$$Y_G = X^2 Y_L$$

3.- BAKER EQUATIONS AND COLLABORATORS

Using lockhart and Martinelli's correlation, Baker[4] developed a series of equations for each flow pattern, in which it relates differently the X parameter to the parameter ϕ, so that its application is a more accurate method for estimating the value of the pressure drop to two phases. It should be noted that the application of Baker's method is limited to a single particular case of two-phase flow: the one in which both phases of the mixture have a turbulent flow according to the Lockhart-Martinelli classification. Therefore, Baker's modification is to replace the ϕ_{Gtt} curve with the equations proposed by this researcher. The parameter ϕ_{Gtt} for the different flow patterns according to Baker is given by the following equation [5].

bubble flow:
$$\phi_{Gtt} = \frac{16.64\,X^{0.75}}{\left(\frac{W_L}{A}\right)^{0.1}} \quad (9)$$

plug flow:
$$\phi_{Gtt} = \frac{35.766\,X^{0.855}}{\left(\frac{W_L}{A}\right)^{0.17}} \quad (10)$$

estratified flow:
$$\phi_{Gtt} = \frac{54756\,X}{\left(\frac{W_L}{A}\right)^{0.8}} \quad (11)$$

slug flow:
$$\phi_{Gtt} = \frac{2629\,X^{0.815}}{\left(\frac{W_L}{A}\right)^{0.5}} \quad (12)$$

anular flow:
$$\phi_{Gtt} = (4.8 - 12.303\,D)\,X^{(0.343-0.827D)} \quad (13)$$

D = 0.254 m para D >10 in

Wave flow:
$$\Delta P_{2F} = 9.074 \times 10^{-12} \frac{f_H W_G^2}{D^5 \rho_G} \quad [psi] \quad (14)$$

$$f_H = 0.0043\left(\frac{W_L\,\mu_L}{W_G\,\mu_G}\right)^{0.214} \quad (15)$$

Dispersed flow:
$$\phi_{Gtt} = \exp\left\{1.4659 + 0.49138(\ln X) + 0.04887(\ln X)^2 - 0.000349(\ln X)^3\right\} \quad (16)$$

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Where: W_L - mass flow of the liquid in kg/h.; A - transverse area of the pipe in m2.
D - internal diameter of the pipe in m.; f_H - Huntington's friction factor for wave flow.
W_G - mass flow of the gas in kg/h. ; ρ_G - gas density in kg/m^3 ;.
μ_L - viscosity of the liquid in cp. μ_G - viscosity of the gas in cp.
To convert psi to kgf/m^2, multiply by 703.07.

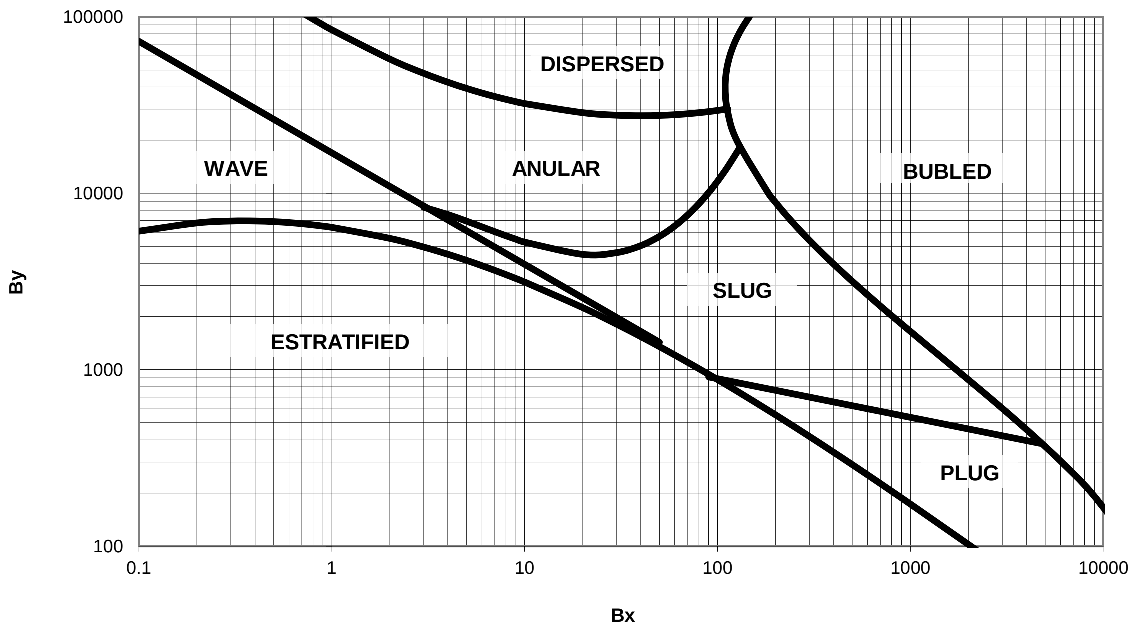

Figure A - Map of Baker patterns for horizontal flow in gas-liquid systems. (1954).

The authors presented this map in the article "*Prediction of the flow pattern to two- phases, vapor-liquid in horizontal pipes* " [16]
To determine the flow pattern using the Baker map, the Baker parameters (Bx and By) must first be calculated, which determines the expected flow type on the graph:

$$Bx = 0.0341\frac{W_L}{W_G}\frac{\rho_G^{1/2}\,\mu_L^{1/3}}{\sigma_L\,\rho_L^{1/6}}\quad\left[cp^{1/3}\right] \qquad (A)$$

$$By = 7.092\frac{W_G}{A\sqrt{\rho_G\rho_L}}\quad\left[\frac{lb}{h\,ft^2}\right] \qquad (B)$$

Lockhart-Martinelli method modified by Baker:
1.- Determine the flow pattern with Baker's parameters and map A.
2.- Determine the flow rate for each phase, calculating the corresponding surface Reynolds with equation 3.
3.- Calculate the pressure drop for the gas phase with equation 5.
4.- Calculate the X parameter with equation 2. If the flow rate is turbulent-turbulent, equation 8 can be used.
5.- Get the parameter ϕ_G corresponding to the flow rate, on the Lockhart-Martinelli graph (Figure 1). If the regime is turbulent-turbulent, Baker equations (equations 9 to 16) can be used.

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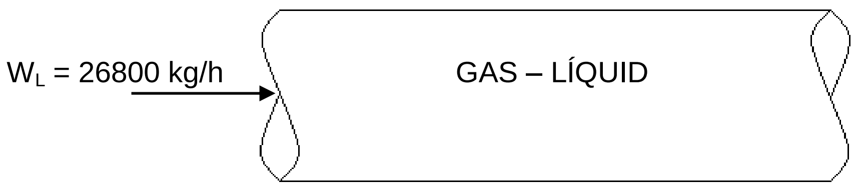
6.- Calculate the pressure drop to two phases with equation 1.

Example 1

What are the friction losses in a horizontal pipe 100 m in length and 4 inches in diameter of 40, through which pass 26800 kg/h of a liquid with a density of 500 kg/m^3, viscosity of 0.11 cp and 5.07 dina/cm of surface tension? And there also travel through the same pipe. 4250 kg/h of vapours with a density of 27 kg/m^3 and a viscosity of 0.0105 cp.

1.-TRANLATION



2.-Palnning.

2.1.-Discussion

To obtain friction losses it is necessary , first to determine the type of flow pattern present in the pipe. This can be accomplished using Baker's map (Fig A). Friction losses are calculated using the Lockhart-Martinelli method and the complementary Baker equations corresponding to the parameter . ϕ.

3.-CALCULATIONS

3.1.-Flow pattern.

For a 4" nominal diameter pipe card 40, its internal diameter is:

D = 4.026 in = 0.10226 m

$A = 8.2x10^{-3}\ m^2$

By = 31585

σ_L = 5.07 dina/cm = $5.17x10^{-4}$ kgf/m

Bx = 367

With these coordinates, the Baker map (Fig. A) [3][16] reads bubble flow.

3.2.-Pressure drop in the gas phase.

$$v_{SG} = \frac{4250\,\frac{kg}{h}}{3600\,\frac{s}{h}\left(27\,\frac{kg}{m^3}\right)\left(8.2\times10^{-3}\,m^2\right)} = 5.3322\,\frac{m}{s}$$

$$Re_{SG} = \frac{0.10226\,m\left(5.3322\,\frac{m}{s}\right)\left(27\,\frac{kg}{m^3}\right)}{\left(0.001\,\frac{kg/ms}{cp}\right)(0.0105\,cp)} = 1.4\times10^6$$ turbulent flow

From fig. 2 y 3:

e/D = 0.00045

$f_D = 0.0165$

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$$\Delta P_G = 6.379\times 10^{-7}\left(4250\,\frac{kg}{h}\right)^2 (100\,m)\left(\frac{0.0165}{(0.10226\,m)^5\left(27\,\frac{kg}{m^3}\right)}\right) = 62968\,\frac{kgf}{m^2}$$

3.3.-Caída de presión en la fase líquida

$$v_{SL} = \frac{26800\,\frac{kg}{h}}{3600\,\frac{s}{h}\left(500\,\frac{kg}{m^3}\right)\left(8.2\times 10^{-3}\,m^2\right)} = 1.8157\,\frac{m}{s}$$

$$Re_{SL} = \frac{0.10226\,m\left(1.8157\,\frac{m}{s}\right)\left(500\,\frac{kg}{m^3}\right)}{\left(0.001\frac{kg/ms}{cp}\right)(0.11cp)} = 8.44\times 10^5$$

turbulent flow

From fig. 2 y 3:

$\varepsilon/D = 0.00045$

$f_D = 0.017$

$$\Delta P_L = \frac{0.017\left(1.8157\,\frac{m}{s}\right)^2\left(500\,\frac{kg}{m^3}\right)(100\,m)}{2\left(9.81\frac{m\,kg}{s^2\,kgf}\right)(0.10226\,m)} = 1397\,\frac{kgf}{m^2}$$

3.4.- Two - phase pressure drop.

$$X = \left(\frac{1397}{62968}\right)^{0.5} = 0.149$$

For turbulent flow and bubble flow:

$$\phi_{Gtt} = \frac{16.64(0.149)^{0.75}}{\left(\frac{26800\,\frac{kg}{h}}{8.2\times 10^{-3}\,m^2}\right)^{0.1}} = 0.89$$

$$\Delta P_{2F} = (0.89)^2 \left(62968 \frac{kgf}{m^2} \right) = 49877 \frac{kgf}{m^2}$$

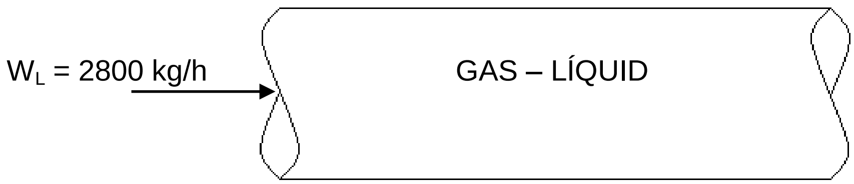
4.-RESULT

Friction pressure losses shall be 49877 kgf/m ^{2} per 100 m of tube length.

Example 2

What will be the expected pressure drop in a horizontal pipe of 6 inches of 40 and 10 m in diameter, through which flow 2800 kg/h of liquid with a density of 834 kg/m^3, viscosity of 0.1 cp and surface tension of 6.25 dina/cm? There is also a flow of 9800 kg/h of steam with 30.75 kg/m ^{3} density and a viscosity of 0.01 cp.

1.-TRANSLATION



2.-Planning

2.1.-Discussion .

To solve the problem you must find the type of flow present in order to be able to select the appropriate Baker correlation for the calculation of the pressure drop.

3.-CALCULATIONS

3.1.-Flow Path

D = 6.065 in = 0.154 m

A = 0.018639 m^2

By = 23285

Bx = 12.82

These values get annular flow on Baker's map (Fig A] [4] [16].

3.2.-Two-phase pressure drop

$Re_{SG} = 2.25\text{x}10^6$ Turbulent flow

From fig. 9 y 10:

e/D = 0.0003

$f_D = 0.015$

$\Delta P_G = 3450.22$ kgf/m^2

To calculate the X parameter, equation 8 is used:

$$X^2 = 0.0084 \left(\frac{2800}{9800} \right)^{1.8} \left(\frac{30.75}{834} \right) \left(\frac{0.1}{0.01} \right)^{0.2} = 5.15 \times 10^{-5}$$

$$X = \sqrt{5.15 \times 10^{-5}} = 0.0072$$

For anular flow:

$$\phi_{Gtt} = (4.8 - 12.303(0.154\,m))0.0072^{(0.343 - 0.827(0.154\,m))} = 1.0026$$

$$\Delta P_{2F} = (1.0026)^2 \left(3450.22 \frac{kgf}{m^2} \right) = 3468.50 \frac{kgf}{m^2}$$

4.-RESULT

The pressure drop is de 3468.50 kgf/m^2.

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Lockhart-Martinelli correlation should preferably be used for pipes smaller than 4 inches, and Baker equations apply for pipes up to 10 inches. Baker [4] [5] found that friction pressure losses in large diameter pipes are smaller , than predicted by Lockhart-Martinelli's original method. In addition, these methods only calculate the friction pressure drop, and therefore the contribution per acceleration should be estimated independently. Experience shows that the latter is generally not significant, and therefore the friction pressure drop is roughly equal to the total, as proposed by Lockhart and Martinelli.

4.- HOMOGENEOUS MODELS [17]

As well as the above correlations, which can be called semi-empirical, are based on Lockhart and Martinelli's work on pressure drops using a physical mixing model, it is also possible to consider fluid flow phenomena in two phases using the homogeneous flow concept[6]. The visualization of the flow in two phases as a homogeneous mixture assumes that the gas and the liquid can be considered as a single uniform phase. In this case, therefore there is no difference between the speeds of both phases. At the above difference in speeds, it is commonly referred to as the slip velocity. The homogeneous model is based on the essential postulate that the average speed of the liquid phase is continuously equal to the average speed of the gas phase. In this way, there is no difference between the speeds of both phases, as mentioned in the previous paragraph, and therefore . one phase does not slide over the other by traveling faster, but rather flow together at the same speed. Based on this postulate, Dukler and collaborators [7] studied this phenomenon using a similarity analysis, the basic premise of which is: If two single-phase flow systems are dynamically similar, then The Reynolds and Euler numbers for each system are the same. Euler's number is twice Fanning's friction factor, which in turn is a quarter of Darcy's friction factor:

$$Eu = 2f_f \qquad (17)$$

$$f_D = 4 f_f \qquad (18)$$

In simplified form, the pressure drop of a homogeneous fluid in a pipe can be expressed with the following equation:

$$\left[\frac{\Delta P}{\Delta L}\right]_{Total} = \left[\frac{\Delta P}{\Delta L}\right]_{aceleración} + \left[\frac{\Delta P}{\Delta L}\right]_{fricción} + \left[\frac{\Delta P}{\Delta L}\right]_{elevación} \qquad (19)$$

The first term gives the pressure drop by acceleration, the second term corresponds to that of frictional pressure losses and the third term is the elevation pressure drop, which takes the value of zero for horizontal pipes. For the study of pressure drop, Dukler proposes four cases, two being the most used:

Case I: No slippage and homogeneous flow

There is no relative slippage between phases and two-phase flow is considered as homogeneous. In this case, the properties of the mixture are obtained by simple relationships between the properties of both phases, as noted below:

$$\rho_{NS} = \rho_L \lambda + \rho_G (1-\lambda) \qquad (20)$$

$$\mu_{NS} = \mu_L \lambda + \mu_G (1-\lambda) \qquad (21)$$

Where: ρ_{NS} = density of the homogeneous mixture without slippage. ; μ_{NS} = viscosity of the homogeneous mixture without slippage. ; λ = fraction of the volume of the pipe occupied by the liquid phase without slipping:

$$\lambda = \frac{Q_L}{Q_L + Q_G} = \frac{v_{SL}}{v_{SL} + v_{SG}} \qquad (22)$$

Q - volumetric flow of the liquid and gas phases.

The Reynolds number can be expressed in terms of these properties of the mixture:

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$$Re_{NS} = \frac{D\, v_{NS}\, \rho_{NS}}{\mu_{NS}} \qquad (23)$$

Where: V_{NS} = surface speed of homogeneous mixture without slippage.

$$v_{NS} = v_{SL} + v_{SG} \qquad (24)$$

For the calculation of the friction factor without phase slippage, Dukler used the Koo [7] equation for the friction factor in a single phase:

$$f_{NS} = 0.0014 + \frac{0.125}{Re_{NS}^{0.32}} \qquad (25)$$

Friction pressure drop without slippage between phases is given by the Fanning equation:

$$\left[\frac{\Delta P_{NS}}{L}\right]_{fricción} = \frac{2\, f_{NS}\, v_{NS}^2\, \rho_{NS}}{g_C\, D} \qquad (26)$$

To obtain the total pressure drop assuming homogeneous flow, Dukler proposed the following equation:

$$\left[\frac{\Delta P_{NS}}{L}\right]_{Total} = \frac{\left[\frac{\Delta P_{NS}}{L}\right]_{fricción}}{1 - AC} \qquad (27)$$

Where: AC = acceleration pressure drop:

$$AC = \frac{16\, W_T\, W_G\, P_{av}}{\pi^2\, g_C\, D^4\, P_1\, P_2\, \rho_{Gav}} \qquad (28)$$

W_T = Total mass flow kg/s:

$$W_T = W_L + W_G \qquad (29)$$

W_L & W_G = mass flows of the liquid and gas, respectively, in kg/s.
Pav - average pressure in the L-length tube section, in kgf/m^2.
P_1 - pressure at the inlet of the pipe section in kgf/m^2.
P_2 - pressure at the outlet of the pipe section in kgf/m^2.
ρ_{Gav} = average gas density in the same pipe section in kg/m^3.
To transform atmospheres into kgf/m^2, multiply the pressure by 10332.7

The pressure drop in this case is always less than the actual pressure drop present in a pipe, therefore, the assumption of a homogeneous flow without slip inter phase (non slip) allows to obtain the lowest possible pressure drop, providing the engineer with a limiting case in the design of a pipe with flow to two phases gas-liquid.

Case II: Constant slipping .

The ratio of the speed of the phases at the average velocity is constant through the section. When phases flow simultaneously, gas generally flows faster than liquid, causing an increase in the volume of the pipe occupied by the liquid. This phenomenon is known as slip, and the fraction of the volume of the pipe occupied by the liquid under these conditions is known as holdup (R_L). It is clear that R_L cannot be determined from the entry flows, so special correlations resulting from experimental measurements are used. In his article, Dukler [8] recommends using the Hughmark [9] correlation to calculate that volumetric fraction or holdup. For this case, Dukler defined the following parameter:

$$\beta = \frac{\rho_L}{\rho_{NS}}\left(\frac{\lambda^2}{R_L}\right) + \frac{\rho_G}{\rho_{NS}}\left[\frac{(1-\lambda)^2}{R_G}\right] \qquad (30)$$

Where, λ = fraction of liquid without slippage between phases. R = holdup of the liquid and gas phases. The number of Reynolds for two-phase flow can be expressed as:

$$Re_{2F} = \frac{4\, W_T}{\pi\, D\, \mu_{NS}}\beta \qquad (31)$$

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In order to calculate the friction pressure drop, he defined a assumed friction factor, based on Koo's equation for the single-phase friction factor:

$$f_O = 0.0014 + \frac{0.125}{Re_{2F}^{0.32}} \qquad (32)$$

Using a data bank carefully selected by him, whose sources are the works of researchers such as Lockhart and Martinelli [2], Baker [4], Bankoff [10], among others, Dukler found a relationship between the assumed friction factor (fo) and the actual two-phase (f), which he called $\alpha(\lambda)$,which are related to the graph of Figure 4.
The behavior in Figure 4 can be represented by the following equation:

$$\alpha(\lambda) = \frac{f}{f_O} = 1 - \frac{\ln\lambda}{1.281 + 0.478(\ln\lambda) + 0.444(\ln\lambda)^2 + 0.094(\ln\lambda)^3 + 0.00843(\ln\lambda)^4} \qquad (33)$$

With all well-defined parameters and variables, the pressure drop due to friction in the two-phase flow can be calculated:

$$\left[\frac{\Delta P_{2F}}{L}\right]_{fricción} = \frac{2 G_T^2 f_O}{g_C D \rho_{NS}} \alpha(\lambda)\beta \qquad (34)$$

Where: G_T = total mass velocity in kg/(m^2 s):

$$G_T = \frac{W_L + W_G}{3600\,A} \qquad (35)$$

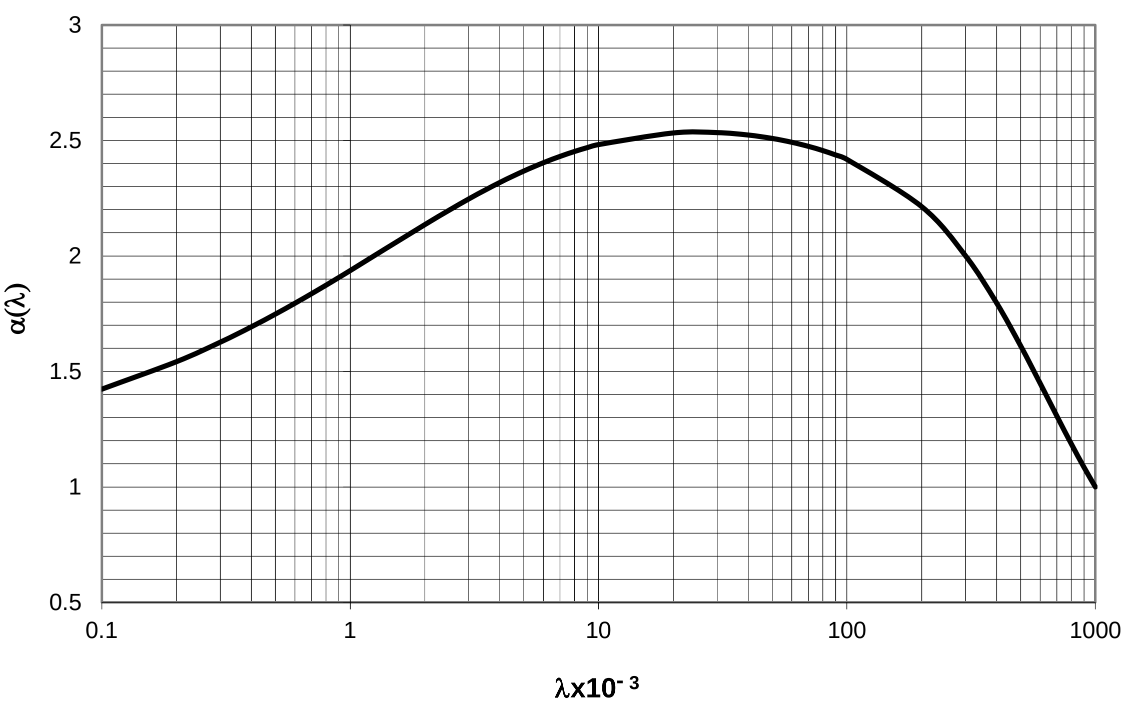

Figure 4.- Dukler's graphic for α λ. (1964)

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In addition to the pressure drop due solely to the effects of friction, it is important to consider acceleration losses due to the expansion of the gas phase in the biphasic mixture, as it moves through the horizontal pipe. The acceleration pressure drop given by Dukler is :

$$\left[\frac{\Delta P_{2F}}{L}\right]_{aceleración} = \frac{1}{g_C A^2 L}\left[\frac{W_G^2}{R_G}\left(\frac{1}{\rho_{G2}} - \frac{1}{\rho_{G1}}\right) + \frac{W_L^2}{\rho_L R_L}\right] \quad (36)$$

Where:

R_G y R_L = holdup gas and liquid phases , ρ_{G1} = gas density and the entrance of the pipe of L lengt; ρ_{G2} = gas density at the end of the pipe.

The total drop pressure in an horizontal tube is the sum of friction and acceleration effects.

$$\left[\frac{\Delta P_{2F}}{L}\right]_{Total} = \left[\frac{\Delta P_{2F}}{L}\right]_{fricción} + \left[\frac{\Delta P_{2F}}{L}\right]_{aceleración} \quad (37)$$

In this way, the acceleration effect is only important when handling very large flows or very low pressures.

5.- DUKLER METHOD:

1.- Calculate the fraction of the volume of the pipe occupied by the liquid without slippage between the phases (λ) with equation 22:

$$\lambda = \frac{Q_L}{Q_L + Q_G} = \frac{v_{SL}}{v_{SL} + v_{SG}} \quad (22)$$

2.- Calculate density without slippage between phases with equation 20:

$$\rho_{NS} = \rho_L \lambda + \rho_G (1-\lambda) \quad (20)$$

3.- Calculate the viscosity without slippage between phases with equation 21:

$$\mu_{NS} = \mu_L \lambda + \mu_G (1-\lambda) \quad (21)$$

4.- Calculate the liquid holdup (R_L) using the Hughmark method:

4.1.- Assume the R_L value. An initial value can be: $R_L = \lambda$

4.2.- Get the Reynolds based on R_L:

$$Re = \frac{D G_T}{R_L \mu_L + (1-R_L)\mu_G} \quad (38)$$

4.3.- Calculate Froude number: $Fr = \frac{v_{NS}^2}{gD}$ (39)

4.4.- Calculate the Hughmark Z parameter:

$$Z = \frac{Re^{1/6} Fr^{1/8}}{\lambda^{1/4}} \quad (40)$$

4.5.- Get the K parameter of Hughmark [9]:

If Z < 10:

$$K = -0.163673 + 0.310372\, Z - 0.0352491\, Z^2 + 0.001366\, Z^3 \quad (41)$$

If Z > 10:

$$K = 0.755454 + 0.00358499\, Z - 1.43604\times10^{-5}\, Z^2 \quad (42)$$

4.6.- Calculate R_L:

$$R_L = 1 - (1 - \lambda) K \quad (43)$$

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4.7.- Compare the R_L value assumed in step 4.1 with the one calculated in step 4.6. If they are the same, continue to step 4.8; and if they are different, return to step 4.2 using the R_L calculated in step 4.6.

4.8.- Calculate R_G:

$$R_G = 1 - R_L \qquad (44)$$

5.- Calculate β with equation 30:

$$\beta = \frac{\rho_L}{\rho_{NS}}\left(\frac{\lambda^2}{R_L}\right) + \frac{\rho_G}{\rho_{NS}}\left[\frac{(1-\lambda)^2}{R_G}\right] \qquad (30)$$

6.- Calculate the Reynolds with slippage between phases using equation 31:

$$Re_{2F} = \frac{4W_T}{\pi D \mu_{NS}}\beta \qquad (31)$$

7.- Calculate the assumed friction factor with equation 32:

$$f_O = 0.0014 + \frac{0.125}{Re_{2F}^{0.32}} \qquad (32)$$

8.- Get $\alpha(\lambda)$ from the figure 4, or from equation 33:

$$\alpha(\lambda) = \frac{f}{f_O} = 1 - \frac{\ln\lambda}{1.281 + 0.478(\ln\lambda) + 0.444(\ln\lambda)^2 + 0.094(\ln\lambda)^3 + 0.00843(\ln\lambda)^4} \qquad (33)$$

9.- Calculate friction pressure drop with equation 34:

$$\left[\frac{\Delta P_{2F}}{L}\right]_{fricción} = \frac{2G_T^2 f_O}{g_C D \rho_{NS}}\alpha(\lambda)\beta \qquad (34)$$

10.- Get the total pressure drop:

10.1.- Assume a pressure drop in the L-length pipe run. An initial value can be that of the frictional pressure drop:

$$\left[\frac{\Delta P_{2F}}{L}\right]_{supuesto} = \left[\frac{\Delta P_{2F}}{L}\right]_{fricción}$$

10.2.- Calculate the output pressure of the pipe section in question:

$$P_2 = P_1 - L\left[\frac{\Delta P_{2F}}{L}\right]_{supuesto} \qquad (45)$$

10.3.- Obtain the densities of the gas at the inlet and outlet of the pipe section.

10.4.- Get the pressure drop by acceleration with equation 36:

$$\left[\frac{\Delta P_{2F}}{L}\right]_{aceleración} = \frac{1}{g_C A^2 L}\left[\frac{W_G^2}{R_G}\left(\frac{1}{\rho_{G2}} - \frac{1}{\rho_{G1}}\right) + \frac{W_L^2}{\rho_L R_L}\right]$$

10.5.- Calculate the total pressure drop with equation 37:

$$\left[\frac{\Delta P_{2F}}{L}\right]_{Total} = \left[\frac{\Delta P_{2F}}{L}\right]_{fricción} + \left[\frac{\Delta P_{2F}}{L}\right]_{aceleración} \qquad (37)$$

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If the total pressure drop is not roughly equal to the one assumed in step 10.1, return to that step and assume another pressure. The value of the total pressure drop calculated in step 10.5 can be used to make the new assumption.

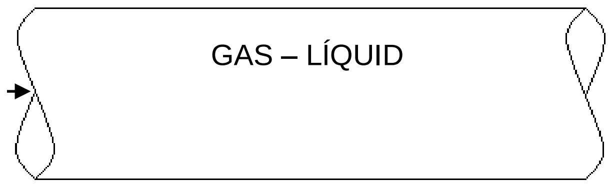
Example 3

Estimate the pressure drop in 100 m of four-inch steel tube 40, through which 2400 kg/h of water and 950 kg/h of air pass through. The initial pressure in the pipe is 7 kgf/cm^2. The viscosity of the liquid is 1 cp and that of the gas is 0.018 cp. The liquid density is 1000 kg/m ^{3}. Use Lockhart- Martinelli's method and Dukler's case I.

1.-TRANSLATION

$P_1 = 7$ kgf/cm^2

$W_L = 2400$ kg/h



2.-Planning

2.1.-Discussion

According to the statement of the problem, the methodologies of Lockhart-Martinelli and Dukler Case I will be used to estimate pressure drops.

3.-CALCULATIONS

3.1.- Lockhart-Martinelli method.

$D = 4.026$ in $= 0.1023$ m

$A = 0.0082$ m^2

Pressure drop in liquid phase:

$$v_{SL} = \frac{2400\,\frac{kg}{h}}{3600\,\frac{s}{h}\left(1000\,\frac{kg}{m^3}\right)\left(0.0082\,m^2\right)} = 0.0813\,\frac{m}{s}$$

$$Re_{SL} = \frac{0.1023\,m\left(0.0813\,\frac{m}{s}\right)\left(1000\,\frac{kg}{m^3}\right)}{\left(0.001\,\frac{kg/ms}{cp}\right)(1\,cp)} = 8317$$ turbulent flow

$e/D = 0.00045$

$f_D = 0.032$

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$$\Delta P_L = \frac{0.032\left(0.0813\,\frac{m}{s}\right)^2\left(1000\,\frac{kg}{m^3}\right)(100\,m)}{2\left(9.81\,\frac{m\,kg}{s^2\,kgf}\right)(0.1023\,m)} = 10.54\,\frac{kgf}{m^2}$$

Pressure in gas phase :

$$\rho_G = \frac{PM}{RT} = \frac{7\,\frac{kgf}{cm^2}\left(0.9678\,\frac{atm}{kgf/cm^2}\right)\left(29\,\frac{kg}{kgmol}\right)}{0.082\,\frac{m^3atm}{kgmol\,K}(20+273.15)K} = 8.173\,\frac{kg}{m^3}$$

$$v_{SG} = \frac{950\,\frac{kg}{h}}{3600\,\frac{s}{h}\left(8.173\,\frac{kg}{m^3}\right)\left(0.0082\,m^2\right)} = 3.938\,\frac{m}{s}$$

$$Re_{SG} = \frac{0.1023\,m\left(3.938\,\frac{m}{s}\right)\left(8.173\,\frac{kg}{m^3}\right)}{\left(0.001\,\frac{kg/ms}{cp}\right)(0.018\,cp)} = 1.83\times10^5$$ turbulent flow.

e/D = 0.00045

$f_D = 0.0185$

$$\Delta P_G = 6.379\times10^{-7}\left(950\,\frac{kg}{h}\right)^2(100\,m)\left(\frac{0.0185}{(0.1023m)^5\left(8.173\,\frac{kg}{m^3}\right)}\right) = 11630.86\,\frac{kgf}{m^2}$$

Parameter X:

$$X = \left(\frac{10.54}{11630.86}\right)^{0.5} = 0.0301$$

For turbulent flow :

$$\log\phi_{Gtt} = 0.00176(\log 0.0301)^3 + 0.1148(\log 0.0301)^2 + 0.4821(\log 0.0301) + 0.6358$$

$$\log\phi_{Gtt} = 0.1619$$

$$\phi_{Gtt} = 1.4516$$

Two phase pressure drop:

$$\Delta P_{2F} = (1.4516)^2 \left(11630.86 \frac{kgf}{m^2}\right) = 24507.88 \frac{kgf}{m^2}$$

3.2.- Case I Dukler

$$\lambda = \frac{0.0813 \frac{m}{s}}{0.0813 \frac{m}{s} + 3.938 \frac{m}{s}} = 0.0202$$

$$\rho_{NS} = 1000 \frac{kg}{m^3}(0.0202) + 8.173 \frac{kg}{m^3}(1 - 0.0202) = 28.208 \frac{kg}{m^3}$$

$$\mu_{NS} = 1cp(0.0202) + 0.018\,cp(1 - 0.0202) = 0.038\,cp$$

$$v_{NS} = 0.0813 \frac{m}{s} + 3.938 \frac{m}{s} = 4.019 \frac{m}{s}$$

$$Re_{NS} = \frac{0.1023\,m\left(4.019 \frac{m}{s}\right)\left(28.208 \frac{kg}{m^3}\right)}{0.038\,cp\left(0.001 \frac{kg/ms}{cp}\right)} = 305198$$

$$f_{NS} = 0.0014 + \frac{0.125}{(305198)^{0.32}} = 0.003597$$

$$\Delta P_{NS} = \frac{2(0.003597)\left(4.019 \frac{m}{s}\right)^2 \left(28.208 \frac{kg}{m^3}\right)(100\ m)}{9.81 \frac{m\,kg}{s^2\,kgf}(0.1023\,m)} = 326.61 \frac{kgf}{m^2}$$

4.-RESULT

With the Lockhart- Martinelli method a pressure drop of 24507.88 kgf/m^2 is obtained per 100 m of tube, and by Dukler's I case a pressure drop of 326.61 kgf/m^2 is obtained per 100 m of tube. Recalling that Dukler's case I provides the minimum possible actual pressure drop, the pressure drop calculated using the Lockhart-Martinelli method is valid, as it gives a higher value than Dukler's case I.

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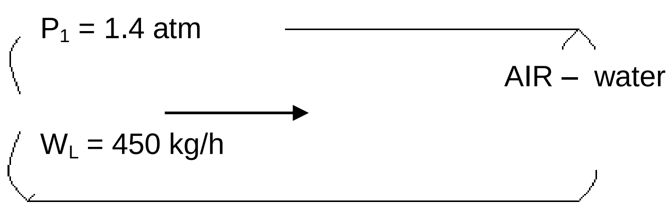
Example 4

By a smooth pipe 100 m long and 1 inch in diameter 40, 450 kg/h of water and 7 kg/h of air flow. The pipe inlet is at 1.4 atm and the system is isothermal at 20 °C. Find the pressure drop by Dukler's method. The physical properties of the fluids are: ρ_L = 1000 kg/m^3; ρ_G = 1.4 kg/m^3; μ_L = 1 cp; μ_G =0.018cp.

1.- Translation.



2.-Planning.

2.1.-Discussion

Dukler's method will be used to obtain the total pressure drop in the pipe .

3.-CALCULATIONS

3.1.-Calculation of mixing properties without slip between phases (non slip)

D = 1.049 in = 0.0266 m

A = 5.576x10^{-4} m^2

$$v_{SL} = \frac{450\,\frac{kg}{h}}{3600\,\frac{s}{h}\left(1000\,\frac{kg}{m^3}\right)\left(5.576\times10^{-4}\,m^2\right)} = 0.2242\,\frac{m}{s}$$

$$v_{SG} = \frac{7\,\frac{kg}{h}}{3600\,\frac{s}{h}\left(1.4\,\frac{kg}{m^3}\right)\left(5.576\times10^{-4}\,m^2\right)} = 2.4909\,\frac{m}{s}$$

$$\lambda = \frac{0.2242\,\frac{m}{s}}{0.2242\,\frac{m}{s} + 2.4909\,\frac{m}{s}} = 0.0826$$

$$\rho_{NS} = 1000\,\frac{kg}{m^3}(0.0826) + 1.4\,\frac{kg}{m^3}(1-0.0826) = 83.853\,\frac{kg}{m^3}$$

$$\mu_{NS} = 1cp(0.0826) + 0.018\,cp(1-0.0826) = 0.0991cp$$

3.2.-Obtaining holdup using the Hughmark method.

First iteration:

R_L = 0.75

$$G_T = \frac{450\,\frac{kg}{h} + 7\,\frac{kg}{h}}{5.576\times10^{-4}\,m^2} = 819584\,\frac{kg}{m^2\,h}$$

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$$Re = \frac{0.0266\,m\left(819584\,\frac{kg}{m^2\,h}\right)\left(\frac{1h}{3600\,s}\right)}{\left[0.75(1cp)+(1-0.75)(0.018\,cp)\right]\left(0.001\frac{kg/ms}{cp}\right)} = 8026$$

$$v_{NS} = 0.2242\frac{m}{s} + 2.4909\frac{m}{s} = 2.7151\frac{m}{s}$$

$$Fr = \frac{\left(2.7151\frac{m}{s}\right)^2}{9.81\frac{m}{s^2}(0.0266\,m)} = 28.25$$

$$Z = \frac{(8026)^{1/6}(28.25)^{1/8}}{(0.0826)^{1/4}} = 12.67 > 10$$

$K = 0.755454 + 0.00358499\ (12.67) - 1.43604x10^{-5}\ (12.67)^2 = 0.799$

$R_L = 1 - (1 - 0.0826)\ (0.799) = 0.267 \neq 0.75$

Second iteration:

$R_L = 0.267$

Re = 21583

Z = 14.94 > 10

$K = 0.755454 + 0.00358499\ (14.94) - 1.43604x10^{-5}\ (14.94)^2 = 0.806$

$R_L = 1 - (1 - 0.0826)\ (0.806) = 0.26 \approx 0.267$

therefore: $R_L = 0.26$

$R_G = 1 - 0.26 = 0.74$

3.3.-Calculation of frictional pressure drop

$$\beta = \frac{1000\,\frac{kg}{m^3}}{83.853\,\frac{kg}{m^3}}\left[\frac{(0.0826)^2}{0.26}\right] + \frac{1.4\,\frac{kg}{m^3}}{83.853\,\frac{kg}{m^3}}\left[\frac{(1-0.0826)^2}{0.74}\right] = 0.332$$

$$W_T = 450\frac{kg}{h} + 7\frac{kg}{h} = 457\frac{kg}{h}$$

$$Re_{2F} = \frac{4\left(457\,\frac{kg}{h}\right)\left(\frac{1h}{3600\,s}\right)}{\pi(0.0266\,m)(0.0991cp)\left(0.001\,\frac{kg/m\,s}{cp}\right)}(0.332) = 20353$$

$$f_O = 0.0014 + \frac{0.125}{(20353)^{0.32}} = 0.0066$$

$$\alpha(\lambda) = \frac{f}{f_O}$$

$$= 1 - \frac{\ln 0.0826}{1.281 + 0.478(\ln 0.0826) + 0.444(\ln 0.0826)^2 + 0.094(\ln 0.0826)^3 + 0.00843(\ln 0.0826)^4}$$

$$= 2.451$$

$$[\Delta P_{2F}]_{fricción} = \frac{2\left[\left(819584\,\frac{kg}{m^2\,h}\right)\left(\frac{1h}{3600\,s}\right)\right]^2(100\,m)(0.0066)}{9.81\,\frac{m\,kg}{s^2\,kgf}(0.0266\,m)\left(83.853\,\frac{kg}{m^3}\right)}(2.451)(0.332) = 2554.24\,\frac{kgf}{m^2}$$

3.4.-Calculation of acceleration pressure drop

$[\Delta P_{2F}]_{supuesto}$ = 2555 kgf/m^2

P_1 = 1.4 atm = 14466 kgf/m^2

P_2 = 14466 kgf/m^2 – 5110 kgf/m^2 = 9356 kgf/m^2 = 0.91 atm

$$\rho_1 = \frac{1.4\,atm\left(29\,\frac{kg}{kgmol}\right)}{0.082\,\frac{m^3\,atm}{kgmol\,K}(20 + 273.15)K} = 1.69\,\frac{kg}{m^3}$$

$$\rho_2 = \frac{0.91atm\left(29\,\frac{kg}{kgmol}\right)}{0.082\,\frac{m^3\,atm}{kgmol\,K}(20 + 273.15)K} = 1.10\,\frac{kg}{m^3}$$

$$[\Delta P_{2F}]_{aceleración} = \frac{1}{9.81\,\frac{m\,kg}{s^2\,kgf}\left(5.576 \times 10^{-4}\,m^2\right)^2}$$

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$$*\left[\frac{\left(7\frac{kg}{h}\right)^2}{\left(3600\frac{s}{h}\right)^2(0.74)}\left(\frac{1}{1.10\frac{kg}{m^3}}-\frac{1}{1.69\frac{kg}{m^3}}\right)+\frac{\left(450\frac{kg}{h}\right)^2}{\left(3600\frac{s}{h}\right)^2\left(1000\frac{kg}{m^3}\right)(0.26)}\right]$$

$$\left[\Delta P_{2F}\right]_{aceleración}=20.23\frac{kgf}{m^2}$$

3.5.-Calculation of total pressure drop.

$$\left[\Delta P_{2F}\right]_{Total}=2554.24\frac{kgf}{m^2}+20.23\frac{kgf}{m^2}=2574.47\frac{kgf}{m^2}$$

Since this value is close to the assumption to get the pressure drop by acceleration, then the calculations are correct.

4.-RESULT

The pressure drop estimated by Dukler case II is 2574.47 kgf/m^2 by 100 m tube length. By Dukler's I case, a total pressure drop of 2404.22 kgf/m^2 per 100m of tube is obtained, which is less than that obtained by case II and is therefore valid for this example

6.- CONCLUSIONS

The Dukler method is the best one to date, as it predicts with greater precision (from 15% to 20% error) pressure drops to two phases: gas-liquid in horizontal pipes, compared to other methodologies. However, Lockhart-Martinelli's method is still the most used due to its simplicity of calculation, despite its lower accuracy (up to 50% error). Over the years many other correlations have appeared to find pressure drops and liquid holdup for horizontal flow to two gas-liquid phases. The interested reader can consult among them those of Hoogendoorn [11], Bertuzzi [12], Baxendell [13], Eaton [14], Beggs [15], Bankoff [10], among other methodologies.

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