



PREDICTION OF PRESSURE DROP BY FLOW TO TWO PHASES, GAS-LIQUID IN VERTICAL PIPES

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ABSTRACT

Prediction of pressure drops in vertical pipes when the flow is two phases gas-liquid, requires knowledge of the flow pattern that is being presented. In an earlier article ^[17] the authors presented these flow patterns and how to evaluate them. This article introduces some methods for evaluating the pressure drops in the two-phase, gas-liquid, vertical flow, and presents examples for clarifying these correlations.

KEYWORDS : *Flow to two gas-liquid phases, vertical pipes, pressure drops.*

1.-INTRODUCTION

In the design of pipes with two-phase flow, the engineer is primarily concerned with the calculation of pressure drop, which should be estimated quite accurately. It has been recognized for years that in order to improve the prediction of the different constituent parameters of this phenomenon, which are the volumetric fraction of each phase (holdup), pressure drop, heat and mass transfer, as well as other hydraulic parameters, it was necessary to consider the detailed structure of the flow configuration. These configurations, which are related to the distribution of phases within the pipe, are called flow patterns or regions. Many experimental and theoretical work has been carried out to predict the pressure drop and the type of flow pattern produced in the pipes, but so far, no general correlation has been found. This is due to the existence of a certain number of complications that hinder the use of a single correlation. The largest of these, in the two-phase flow is the variety of flow patterns that can occur. The type of flow pattern found depends on fluid properties, flows, and equipment geometry.

Although no general correlation has been found applicable to all flow types, correlations have been developed for specific flow patterns. One of the first to do a visual classification of flow patterns was Alves ^[1]. Flow patterns are empirically correlated based on the flows and properties of fluids. The mechanism of momentum transfer varies with the flow pattern.

2.- PREDICTION OF PRESSURE FALL IN VERTICAL PIPES

Similar to flow in horizontal pipes, in the vertical flow there are correlations or semi-empirical models to calculate pressure drops, and there are also theoretical models for the same purpose. Both classes of models are based on the work of the researchers who developed the correlations for horizontal flow. In the case of semi-empirical models, Lockhart and Martinelli's work is the basis, and in theoretical models, the assumption of homogeneous flow is fundamental in the development of new models.

Semi-empirical correlations

Following the approaches of Lockhart, Martinelli and Baker, Kern^[2] proposes Davis ^[3] correlation to find friction losses in flow to two vertical gas-liquid phases. In his correlation, Davis modified the Lockhart-Martinelli module as follows:



$$X_D = 0.19 X(Fr)^{0.185} \quad (1)$$

Where: X_D = Lockhart-Martinelli module modified by Davis. ;

X = Lockhart-Martinelli module.

Fr = Froude number:

$$Fr = \frac{V_M^2}{gD} \quad (2)$$

V_M = mixing velocity

$$V_M = \frac{\frac{W_L}{\rho_L} + \frac{W_G}{\rho_G}}{3600 A} \quad (3)$$

Davis Method:

1.- Determine the flow rate by calculating the Reynolds number for each phase and using the Lockhart-Martinelli criteria: If $Re > 2000$: Turbulent Regime ; If $Re < 1000$: Viscous Regime.

2- Get Lockhart-Martinelli parameter X See reference [3].

3.- Calculate the velocity of the mixture consisting of the liquid and gas phases, with equation 3.

4.- Calculate Froude number with equation 2.

5.- Get the X_D parameter of Davis with equation 1.

6.- Calculate the Lockhart-Martinelli ϕ_G parameter using the Davis equation:

$$\phi_G = \exp \{ 1.4659 + 0.49138 (\ln X_D) + 0.04887 (\ln X_D)^2 - 0.000349 (\ln X_D)^3 \} \quad (4)$$

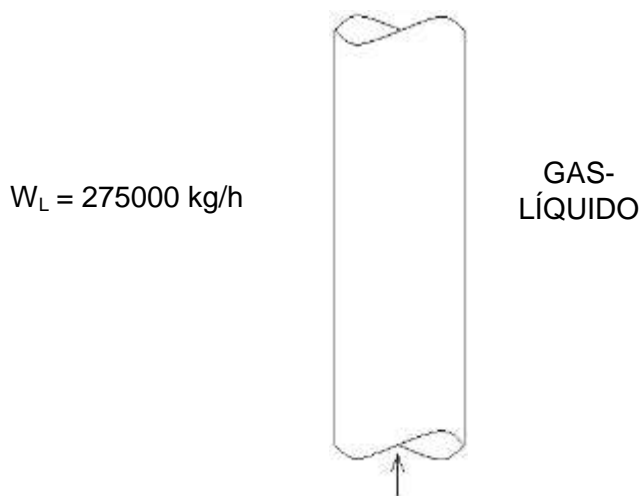
7.- Calculate friction pressure drop with equation A.

$$\Delta P_{2F} = \phi^2 \Delta P_{1F} \quad (A)$$

Example 1

What is the pressure drop that can be obtained in a 18-inch 40-inch pipe, if 275000 kg/h of liquid pass up with a density of 537 kg/m³, viscosity of 0.1 cp and 5.7 dinas/cm of surface tension? In addition, 325000 kg/h of vapours with density of 32 kg/m³ and viscosity of 0.01 cp pass through the pipe.

1.- Translation



2.-Planning

2.1.-Discusion

Flow pattern can be obtained by map of González Ortiz ^[4] or by using Oshinowo-Charles ^[5] for upstream flow.



2.2.-Pressure drop.

Davis correlation will be used to obtain frictional pressure losses.

3.-CALCULS

3.1.-Flow pattern.

D = 16.876 in=.04287 m ; A . 0.1443 m²

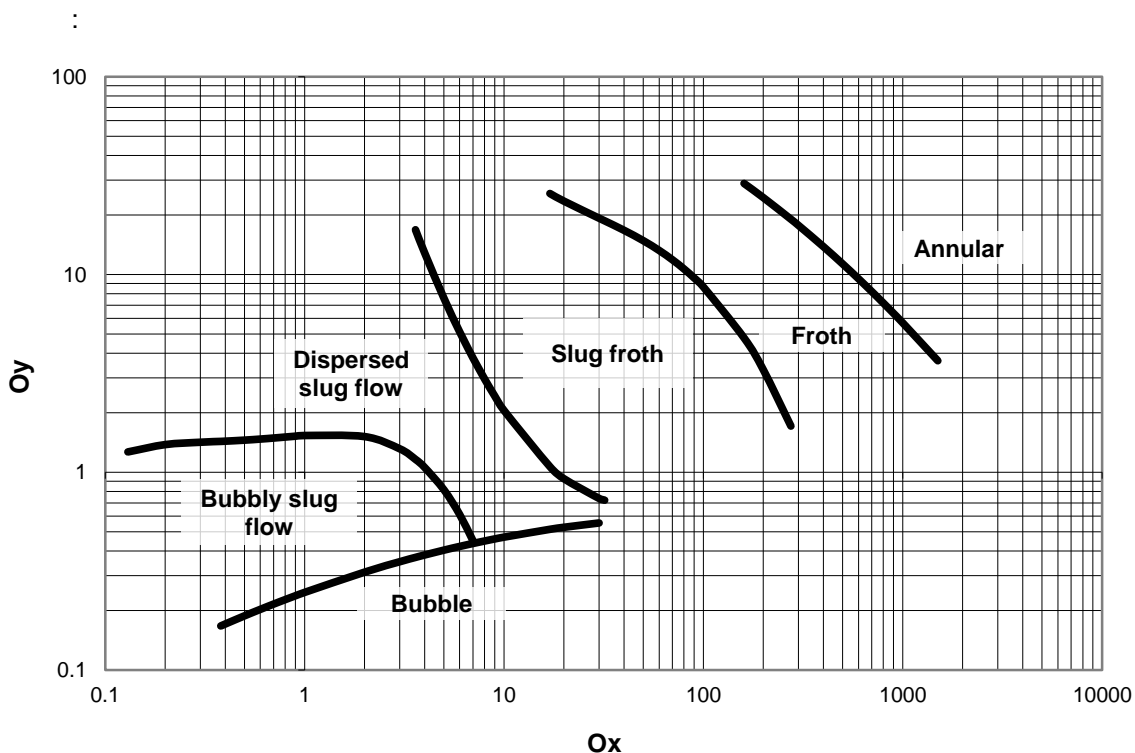
From the Map of González Ortiz^[4]

$$v_{SG} = \frac{325000 \frac{\text{kg}}{\text{h}}}{3600 \frac{\text{s}}{\text{h}} \left(32 \frac{\text{kg}}{\text{m}^3} \right) (0.1443 \text{ m}^2)} = 19.55 \frac{\text{m}}{\text{s}}$$

$$v_{SL} = \frac{275000 \frac{\text{kg}}{\text{h}}}{3600 \frac{\text{s}}{\text{h}} \left(537 \frac{\text{kg}}{\text{m}^3} \right) (0.1443 \text{ m}^2)} = 0.986 \frac{\text{m}}{\text{s}}$$

The flow is foam and is very close to the borders with scrambled and annular flows.

From the Map of Oshinowo-Charles^[5]:



$$v_{2F} = 0.986 \frac{\text{m}}{\text{s}} + 19.55 \frac{\text{m}}{\text{s}} = 20.54 \frac{\text{m}}{\text{s}}$$



$$Fr_{2F} = \frac{\left(20.54 \frac{m}{s}\right)^2}{9.81 \frac{m}{s^2} (0.4287 m)} = 100.28$$

The properties of water are obtained from tables, for which it is considered a temperature of 20 °C, because it is an average ambient temperature.

$$\Lambda = \left(\frac{0.1cp}{1cp}\right) \left[\left(\frac{997 \frac{kg}{m^3}}{537 \frac{kg}{m^3}} \right) \left(\frac{72.75 \frac{dina}{cm}}{5.7 \frac{dina}{cm}} \right)^3 \right]^{\frac{1}{4}} = 0.788$$

$$Ox = \frac{100.28}{\sqrt{0.788}} = 112.96$$

$$R_v = \frac{325000 \frac{kg}{h} \left(537 \frac{kg}{m^3}\right)}{275000 \frac{kg}{h} \left(32 \frac{kg}{m^3}\right)} = 19.83$$

$$Oy = \sqrt{19.83} = 4.45$$

The flow is foaming bullet and is located near the border with the foam flow. In conclusion, the flow pattern determined by these maps is a frothy flow in the transition of bullet and scrambled flows.

3.2.-Pressure drop

The Davis method, then applied, does not need the calculation of the flow pattern present in the vertical line. Only the flow pattern was determined in order to show the use of vertical flow pattern maps.

Pressure drop in the liquid phase:

$$Re_{SL} = \frac{0.4287 m \left(0.986 \frac{m}{s}\right) \left(537 \frac{kg}{m^3}\right)}{\left(0.001 \frac{kg}{ms} / cp\right) (0.1cp)} = 2.27 \times 10^6 \text{ turbulent}$$

From Moody's graph:

$$e/D = 0.00013$$

$$f_D = 0.0125$$



$$\Delta P_L = \frac{0.0125 \left(0.986 \frac{\text{m}}{\text{s}} \right)^2 \left(537 \frac{\text{kg}}{\text{m}^3} \right) (1\text{m})}{2 \left(9.81 \frac{\text{mkg}}{\text{s}^2 \text{kgf}} \right) (0.4287\text{m})} = 0.776 \frac{\text{kgf}}{\text{m}^2}$$

Pressure drop in the gas phase:

$$\text{Re}_{SG} = \frac{0.4287\text{m} \left(19.55 \frac{\text{m}}{\text{s}} \right) \left(32 \frac{\text{kg}}{\text{m}^3} \right)}{\left(0.001 \frac{\text{kg}}{\text{ms}} \right) (0.01\text{cp})} = 2.68 \times 10^7 \text{ turbulent.}$$

From Moody's graph:

$$e/D = 0.00013$$

$$f_D = 0.0122$$

$$\Delta P_G = 6.379 \times 10^{-7} \left(325000 \frac{\text{kg}}{\text{h}} \right)^2 (1\text{m}) \left(\frac{0.0122}{\left(0.4287\text{m} \right)^5 \left(32 \frac{\text{kg}}{\text{m}^3} \right)} \right) = 1774 \frac{\text{kgf}}{\text{m}^2}$$

Parameter X:

$$X = \left(\frac{0.776}{1774} \right)^{0.5} = 0.0209$$

Parameter X_D :

$$v_M = \frac{\frac{275000 \frac{\text{kg}}{\text{h}}}{537 \frac{\text{kg}}{\text{m}^3}} + \frac{325000 \frac{\text{kg}}{\text{h}}}{32 \frac{\text{kg}}{\text{h}}}}{3600 (0.1443 \text{m}^2)} = 20.54 \frac{\text{m}}{\text{s}}$$

$$\text{Fr} = \frac{\left(20.54 \frac{\text{m}}{\text{s}} \right)^2}{9.81 \frac{\text{m}}{\text{s}^2} (0.4287\text{m})} = 100.28$$

$$X_D = 0.19 (0.0209) (100.28)^{0.185} = 0.00932$$

Two-phase drop pressure:



$$\phi_G = \exp\{1.4659 + 0.49138 (\ln 0.00932) + 0.04887 (\ln 0.00932)^2 - 0.000349 (\ln 0.00932)^3\}$$

$$\phi_G = 1.313$$

$$\Delta P_{2F} = (1.313)^2 \left(1774 \frac{\text{kgf}}{\text{m}^2} \right) = 3059 \frac{\text{kgf}}{\text{m}^2}$$

4.-RESULT

Friction pressure drop is 3059 kgf/m² per tube meter.

It is worth noting the non-dependence of Davis correlation on the flow pattern developed in the pipe, as can be seen in the method applied to this example.

Homogeneous models

Based on the homogeneous flow model^[6], Hughmark and Pressburg^[7] proposed the following balance equation to assess two-phase flow pressure losses in vertical pipes:

$$\frac{L(W_L + W_G)}{\frac{W_L}{\rho_L} + \frac{W_G}{\rho_G}} - (P_1 - P_2) + \Delta P_{2F} = 0 \quad (5)$$

The first term of this equation corresponds to the pressure change due to potential energy. The second term corresponds to the total pressure drop for flow to two vertical upward phases. The third term is the pressure drop due to friction. The latter is produced by two mechanisms:

- 1.- The friction exerted by the fluid on the walls of the tube.
- 2.- The turbulence between the two phases, which is a function of the slip velocity:

$$V_{SLIP} = V_G - V_L \quad (6)$$

Where: VSLIP - sliding velocity between the two phases. V_G - actual velocity of the gas phase. V_L - actual velocity of the liquid phase.

Hughmark and Pressburg defined the following parameter to relate the physical properties of the liquid to the total speed mass of the mixture:

$$\Psi = \frac{1}{\mu_L^{0.147} \sigma_L^{0.194} G_T^{0.70}} \quad (7)$$

Where: μ_L - viscosity of the liquid in cp. ; σ_L = -surface tension of the liquid in dina/cm. G_T - total mix mass velocity mass in lb/(ft² s).

Assuming that the only the liquid flows through the pipe, these researchers developed the graph in Figure 1, in which they linked the frictional pressure drop to two phases with the sliding speed and the parameter . Hughmark and Pressburg also present a graph to calculate the liquid holdup for the flow to two phases vertical upwards (Figure 2). To produce this graph, they defined a parameter where they relate the physical properties of the phases and their more than flows:

$$X = \left(\frac{W_L}{W_G} \right)^{0.9} \frac{\mu_L^{0.19} \sigma_L^{0.205} \rho_G^{0.70} \mu_G^{2.75}}{G_T^{0.435} \rho_L^{0.72}} \quad (8)$$

Where: W_L and W_G - massive flows of the liquid and gas, respectively, in lb/s.

ρ_L and ρ_G - liquid and gas densities, respectively, in lb/ft³.

Hughmark-Pressburg method:

- 1.- Calculate pressure drop due to potential energy difference:



$$\Delta P_{\text{potencial}} = \frac{L(W_L + W_G)}{\frac{W_L}{\rho_L} + \frac{W_G}{\rho_G}} \left[\frac{\text{kgf}/\text{m}^2}{\text{m}} \right] \quad (9)$$

2.- Calculate the X parameter of Hughmark - Pressburg with equation 8.

3.- Obtain the liquid holdup using the graph in Figure 2.

4.- Calculate the actual speed of each of the phases:

$$v_L = \frac{W_L}{1097.28 R_L \rho_L A} \left[\frac{\text{ft}}{\text{s}} \right] \quad (10)$$

$$v_G = \frac{W_G}{1097.28(1-R_L)\rho_G A} \left[\frac{\text{ft}}{\text{s}} \right] \quad (11)$$

Where: W_L and W_G - massive flows of liquid and gas in kg/h.

ρ_L y ρ_G = liquid and gas densities in kg/m^3 . A - transverse area of the pipe in m^2

5.- Calculate the sliding velocity between phases with equation 6.

6.- Calculate the Parameter ψ of Hughmark-Pressburg with Equation 7.

7.- Obtain $\frac{\Delta P_{2F} - \Delta P_L}{L}$ by the graph in Figure 1, using the sliding velocity between the phases and the parameter ψ .

The coordinate units are : the abscissa is in ft/s and the ordered in $(\text{lbf} / \text{ft}^2) / \text{ft}$.

8.- Calculate the pressure drop of the liquid phase, assuming that it occupies the entire volume of the pipe:

$$\frac{\Delta P_L}{L} = 3.182 \times 10^{-3} \frac{f_D G_T^2}{D \rho_L} \left[\frac{\text{lbf}/\text{ft}^2}{\text{ft}} \right] \quad (12)$$

Where: G_T - total mass velocity of the mixture in $\text{kg}/(\text{m}^2 \text{ s})$; D - internal diameter of the pipe in m. ρ_L = density of the liquid in kg/m^3 .

9.- Calculate friction pressure drop:

$$\frac{\Delta P_{2F}}{L} = \frac{\Delta P_L}{L} + \left(\frac{\Delta P_{2F} - \Delta P_L}{L} \right) \left[\frac{\text{lbf}/\text{ft}^2}{\text{ft}} \right] \quad (13)$$

To convert $\frac{\text{lbf}}{\text{ft}^2}$ to $\frac{\text{kgf}}{\text{m}^2}$, multiply by 16.0185.

10.- Get total pressure drop:

$$\Delta P_{\text{Total}} = (P_1 - P_2) = \Delta P_{\text{potencial}} + L \left(\frac{\Delta P_{2F}}{L} \right) \left[\frac{\text{kgf}}{\text{m}^2} \right] \quad (14)$$

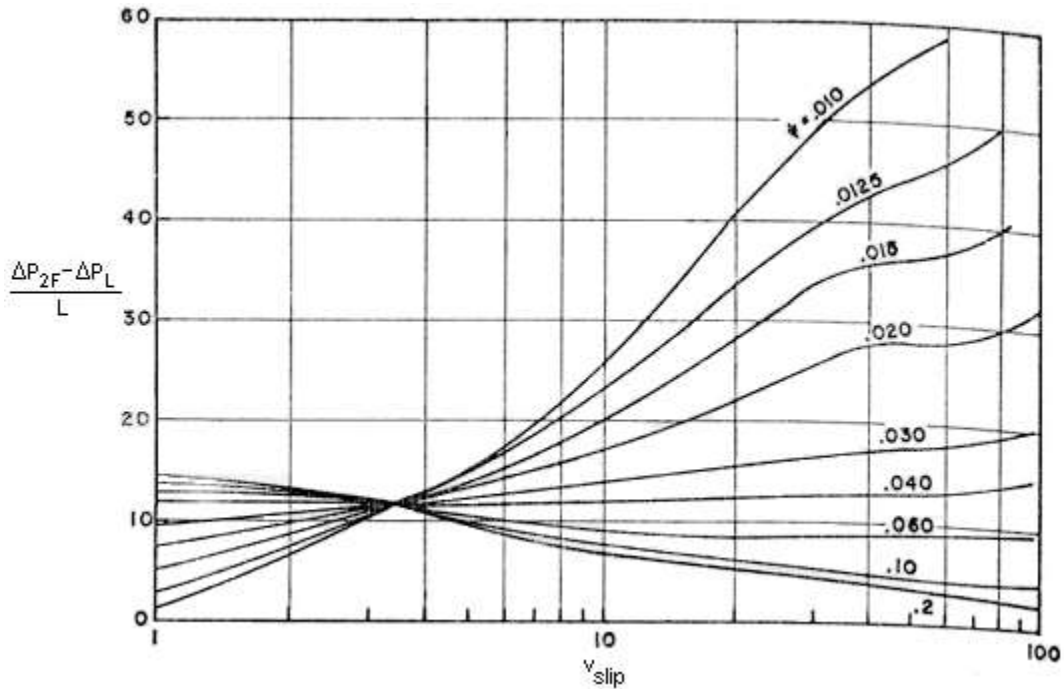
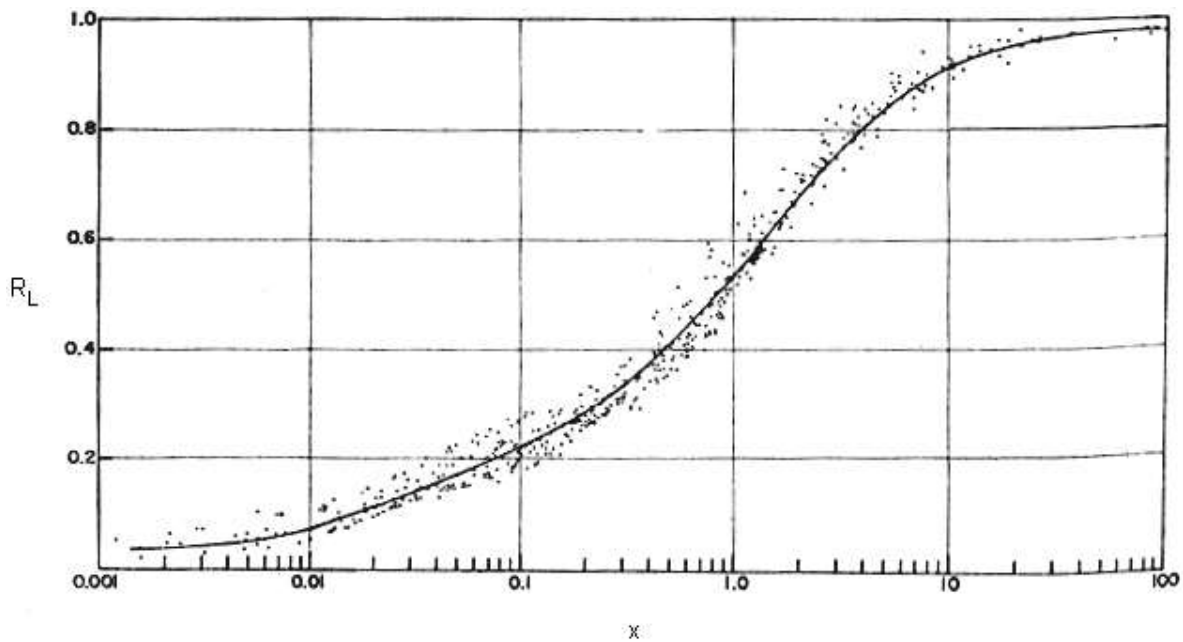


Figure 1.- Hughmark-Pressburg graph for friction pressure drop. (1961)



Figure

2.- Hughmark-Pressburg graph for liquid holdup. (1961)

Example 2

If the above example were resolved by the method proposed by Hughmark and Pressburg, what would be the pressure drop?

3.-CALCULATIONS

3.1.-Holdup liquid



$$G_T = \frac{275000 \frac{\text{kg}}{\text{h}} + 325000 \frac{\text{kg}}{\text{h}}}{3600(0.1443 \text{m}^2)} = 1155 \frac{\text{kg}}{\text{m}^2 \text{s}} = 4158004 \frac{\text{kg}}{\text{m}^2 \text{h}} = 236.56 \frac{\text{lb}}{\text{ft}^2 \text{s}}$$

$$\rho_L = 537 \text{ kg/m}^3 = 33.5 \text{ lb/ft}^3$$

$$\rho_G = 32 \text{ kg/m}^3 = 2 \text{ lb/ft}^3$$

$$x = \frac{\left(\frac{275000 \frac{\text{kg}}{\text{h}}}{325000 \frac{\text{kg}}{\text{h}}} \right)^{0.9} (0.1 \text{cp})^{0.19} \left(5.7 \frac{\text{dina}}{\text{cm}} \right)^{0.205} \left(2 \frac{\text{lb}}{\text{ft}^3} \right)^{0.70} (0.01 \text{cp})^{2.75}}{\left(236.56 \frac{\text{lb}}{\text{ft}^2 \text{s}} \right)^{0.435} \left(33.5 \frac{\text{lb}}{\text{ft}^3} \right)^{0.72}} = 3.02 \times 10^{-8}$$

In Figure 2, the R_L in the R_L chart X is asymptotic to very small values of the X parameter. Therefore: $R_L = 0.05$

3.2.-Sliding velocity between phases

$$v_L = \frac{275000 \frac{\text{kg}}{\text{h}}}{1097.28(0.05) \left(537 \frac{\text{kg}}{\text{m}^3} \right) (0.1443 \text{m}^2)} = 64.69 \frac{\text{ft}}{\text{s}}$$

$$v_G = \frac{325000 \frac{\text{kg}}{\text{h}}}{1097.28(1-0.05) \left(32 \frac{\text{kg}}{\text{m}^3} \right) (0.1443 \text{m}^2)} = 67.52 \frac{\text{ft}}{\text{s}}$$

$$v_{\text{SLIP}} = 67.52 \frac{\text{ft}}{\text{s}} - 64.69 \frac{\text{ft}}{\text{s}} = 2.83 \frac{\text{ft}}{\text{s}}$$

3.3.-Friction pressure drop.

$$\psi = \frac{1}{(0.1 \text{cp})^{0.147} \left(5.7 \frac{\text{dina}}{\text{cm}} \right)^{0.194} \left(236.56 \frac{\text{lb}}{\text{ft}^2 \text{s}} \right)^{0.70}} = 0.0218$$

From fig. 1:

$$\frac{\Delta P_{2F} - \Delta P_L}{L} = 12 \frac{\text{lbf}}{\text{ft}^2}$$



$$Re_L = \frac{DG_T}{\mu_L} = \frac{0.4287 \text{ m} \left(1155 \frac{\text{kg}}{\text{m}^2 \text{ s}} \right)}{0.1 \text{ cp} \left(0.001 \frac{\text{kg}}{\text{m s}} \right)} = 4.951 \times 10^6$$

From Moody:

$$e/D = 0.00013$$

$$f_D = 0.0123$$

$$\frac{\Delta P_L}{L} = 3.182 \times 10^{-3} \frac{0.0123 \left(1155 \frac{\text{kg}}{\text{m}^2 \text{ s}} \right)^2}{0.4287 \text{ m} \left(537 \frac{\text{kg}}{\text{m}^3} \right)} = 0.2268 \frac{\text{lbf}}{\text{ft}^2}$$

$$\frac{\Delta P_{2F}}{L} = 0.2268 \frac{\text{lbf}}{\text{ft}^2} + 12 \frac{\text{lbf}}{\text{ft}^2} = 12.2268 \frac{\text{lbf}}{\text{ft}^2} = 195.85 \frac{\text{kgf}}{\text{m}^2}$$

4.-RESULTS

The friction pressure drop is 195.85 kgf/m² per tube meter, value just over ten times less than that obtained with Davis' method.

Note that the Hughmark-Pressburg correlation does not take into account the flow pattern present in the pipe. After Hughmark and Pressburg, Orkiszewski^[8], based on the work of Griffith and Wallis^[9], developed a correlation to predict pressure drop in oil wells and in pipes with upward vertical two-phase flow. In studying this phenomenon, Orkiszewski found a strong dependence on pressure drop in two factors: the difference in speeds of both phases (the sliding velocity between phases), and the geometry of the two phases (the flow pattern). The upstream patterns considered by Orkiszewski in its correlation are:

1.- Bubble flow. ; 2.- Bullet flow. ; 3.- Transition flow. ; 4.- Annular-fog flow.

According to this author, the total pressure drop is given by:

$$\left[\frac{\Delta P_{2F}}{L} \right] = \frac{\tau_f + \rho_{2F} (\text{sen} \theta) \frac{g}{g_C}}{1 - AC} \quad (15)$$

Where:

L - height of the pipe run.

τ_f = friction pressure drop.

ρ_{2F} = density of the mixture.

θ = the inclination angle of the pipe.

AC - Acceleration Pressure Drop:

$$AC = \frac{G_T v_{SG}}{g_C P_{av}} \quad (16)$$

G_T = mixture mass velocity :



$$G_T = \frac{W_L + W_G}{3600 A} \quad (17)$$

v_{SG} = gas surface velocity .

P_{av} = average pressure in the pipe.

The inclination angle for upstream is 90 degrees, and for downflow it is 270 degrees or - 90 degrees. To determine the flow pattern, Orkiszewski defined the following dimensional parameters:

$$Gv = \frac{Q_G}{A} \left(\frac{\rho_L}{g \sigma_L g_C} \right)^{1/4} \quad (18)$$

$$Lb = 1.071 - 0.2218 \frac{v_{NS}^2}{D} \geq 0.13 \quad (19)$$

$$Ls = 50 + 36 Gv \frac{Q_L}{Q_G} \quad (20)$$

$$Lm = 75 + 84 \left(Gv \frac{Q_L}{Q_G} \right)^{0.75} \quad (21)$$

Where:

Gv = dimension speed of the gas. ;

Lb - number of the upward bubble flow, defines the border between the bubble and bullet flows. ;

Ls - number of the upward bullet flow, defines the border between bullet flows and transition.

The number of the ascending mist flow, defines the border between transition and mist flows.

Q_G and Q_L - volumetric flow of the gas and liquid phases in m^3/h .

A - transverse area of the pipe in m^2 .

ρ_L - density of the liquid in kg/m^3 . σ_L = surface tension of the liquid in kgf/m .

g - Acceleration of Gravity - $9.81 m/s^2$

$g_C = 9.81 m kg/(s^2 kgf)$; D - internal diameter of the pipe in ft.

v_{NS} = mixing velocity in ft/s, equation 22:

$$v_{NS} = v_{SL} + v_{SG} \quad (22)$$

The flow patterns were defined by Orkiszewski as follows:

Bubble flow: $\frac{v_{SG}}{v_{NS}} < Lb \quad (23)$

Bullet flow: $\frac{v_{SG}}{v_{NS}} > Lb \text{ y } Gv < Ls \quad (24)$

Transition flow: $Lm > Gv > Ls \quad (25)$

Mist flow: $Gv > Lm \quad (26)$

From previous work and his own, this researcher was able to establish a series of correlations to calculate friction pressure drop and mix density, for each of the flow patterns. Bubble flow:

For this regimen, the density of the mixture and the liquid holdup are given by:

$$\rho_{2F} = R_L \rho_L + (1 - R_L) \rho_G \quad (27)$$



$$R_L = 0.5 - 0.625 v_{NS} + \left[(0.5 + 0.625 v_{NS})^2 - 1.25 v_{SG} \right]^{1/2} \quad (28)$$

Where:

v_{NS} = mixing velocity in ft/s. (Equation 22)

v_{SG} = surface velocity of the gas phase in ft/s.

The friction pressure drop is given by Darcy's equation:

$$\tau_f = \frac{f_{2F} \rho_L \left(\frac{v_{SL}}{R_L} \right)^2}{2g_C D} \left[\frac{\text{kgf}/\text{m}^2}{\text{m}} \right] \quad (29)$$

The friction factor for two-phase flow is calculated using the Chen equation, or is obtained by the Moody graph, where the Reynolds number is defined as:

$$Re_B = \frac{D v_{SL} \rho_L}{\mu_L R_L} \quad (30)$$

Bullet Flow:

The density of the mixture to two phases is given in this case by:

$$\rho_{2F} = \frac{G_T + \rho_L v_r}{v_{NS} + v_r} + \Gamma \rho_L \quad (31)$$

Where:

v_r = velocity of Taylor's bubbles rise in m/s.

Γ = dimension coefficient of liquid distribution;

G_T - mass mixing velocity in kg/(s m²). (Equation 17); ρ_L - density of the liquid phase in kg/m³. v_{NS} = Mixing velocity in m/s. (Equation 22)

The Reynolds number for this flow is:

$$Re_S = \frac{D v_{NS} \rho_L}{\mu_L} \quad (32)$$

To get the ascent speed of Taylor's bubbles, DeGance and Atherton^[6] defined the following parameters:

$$N_1 = 0.572 \times 10^5 \left[-0.35 + \left(0.1225 + \frac{0.04931 v_{NS}}{D^{0.5}} \right)^{1/2} \right] \quad (33)$$

$$N_2 = 0.5721 \times 10^5 \left[-0.546 + \left(0.2981 + \frac{0.01849 v_{NS}}{D^{0.5}} \right)^{1/2} \right] \quad (34)$$

Where;

$$\text{If } Re_S > N_1: \quad v_r = (1.985 + 4.958 \times 10^{-5} Re_S) D^{0.5} \left[\frac{\text{ft}}{\text{s}} \right] \quad (35)$$

$$\text{If } Re_S < N_2: \quad v_r = (3.097 + 4.958 \times 10^{-5} Re_S) D^{0.5} \left[\frac{\text{ft}}{\text{s}} \right] \quad (36)$$

$$\text{If } N_2 < Re_S < N_1: \quad v_r = 0.5 \left[\gamma + \left(\gamma^2 + \frac{13.59 \mu_L}{\rho_L D^{0.5}} \right)^{0.5} \right] \left[\frac{\text{ft}}{\text{s}} \right] \quad (37)$$



$$\gamma = (1.423 + 4.958 \times 10^{-5} \text{Re}_S) D^{0.5} \left[\frac{\text{ft}}{\text{s}} \right] \quad (38)$$

En donde:

- D = inner diameter of the pipe ft.
- μ_L = Liquid viscosity centipoise.
- ρ_L = liquid density lb/ft³.

The liquid distribution coefficient (Γ) depends on the type of continuous liquid phase, and is determined by the corresponding equation:

Continuos líquid phase	v_{NS} (m/s)	Equation
Water	< 3	39
Water	> 3	40
Petroleum	< 3	41
Petroleum	> 3	42

$$\Gamma = \frac{0.013(\log \mu_L)}{D^{1.38}} - 0.681 + 0.232(\log v_{NS}) - 0.428(\log D) \quad (39)$$

$$\Gamma = \frac{0.045(\log \mu_L)}{D^{0.799}} - 0.709 - 0.162(\log v_{NS}) - 0.888(\log D) \quad (40)$$

$$\Gamma = \frac{0.0127[\log(\mu_L + 1)]}{D^{1.415}} - 0.284 + 0.167(\log v_{NS}) + 0.113(\log D) \quad (41)$$

$$\Gamma = \frac{0.0274[\log(\mu_L + 1)]}{D^{1.371}} + 0.161 + 0.569(\log D) - \log v_{NS} \left(\frac{0.01[\log(\mu_L + 1)]}{D^{1.571}} + 0.397 + 0.63(\log D) \right) \quad (42)$$

Where:

- μ_L = viscosity of the liquid phase in centipoise. v_{NS} = mixing speed in ft/s. (Equation 46)
- D - internal diameter of the pipe in ft.

In order to eliminate the discontinuities in pressure between flow regimes, the coefficient Γ is subject to the following restrictions:

If $v_{NS} < 3$ m/s: $\Gamma \geq -0.065 v_{NS} \quad (43)$

IF $v_{NS} > 3$ m/s: $\Gamma \geq \frac{v_r (G_T - v_{NS})}{(v_r + v_{NS})(v_r + v_{NS} + 1)} \quad (44)$

Where: G_T - mass mixing velocity in lb/(s ft²).

v_{NS} = mixing speed in ft/s. (Equation 46)

Friction pressure drop is given by the following equation:

$$\tau_f = \frac{f_{2F} \rho_L v_{NS}^2}{2g_C D} \left(\frac{v_{SL} + v_r}{v_{NS} + v_r} + \Gamma \right) \left[\frac{\text{kgf}}{\text{m}} \right] \quad (45)$$

Where:

- ρ_L - density of the liquid phase in kg/m³. ; v_{NS} - mixing speed at m/s.



$g_c = 9.81 \text{ m kg/(s}^2 \text{ kgf)}$; V_{SL} - surface speed of the liquid phase in m/s.

V_r - velocity of ascent of Taylor's bubbles in m/s. ; D - internal diameter of the pipe in m.

The friction factor is obtained by the Moody chart (see the article published by the authors^[10] "Prediction of pressure drop in horizontal pipes with gas-liquid flow – EPRA Journal of multidisciplinary Research vol.6,issue 5,may 2020, p.315) ,or calculated with Chen's equation, using the Res.

Transition Flow:

The density of the mixture in this regimen is the average of the mixing densities of the bullet and mist flows, and is expressed as follows:

$$\rho_{2F} = \frac{L_m - G_v}{L_m - L_s} [\rho_{2F}]_{bala} + \frac{G_v - L_s}{L_m - L_s} [\rho_{2F}]_{neblina} \quad (46)$$

Similarly, frictional pressure drop is the average friction loss of bullet and mist flows:

$$\tau_f = \frac{L_m - G_v}{L_m - L_s} [\tau_f]_{bala} + \frac{G_v - L_s}{L_m - L_s} [\tau_f]_{neblina} \quad (47)$$

Where $[\tau_f]_{neblina}$ it is calculated using the following expression for the volumetric flow of the gas:

$$Q_G = A(L_m) \left(\frac{\rho_L}{g \sigma_L g_C} \right)^{-1/4} \quad (48)$$

Mist Flow:

The calculation of the parameters for this flow pattern is similar to that of the bubble flow, as the mixing density and frictional pressure drop are given by:

$$\rho_{2F} = R_L \rho_L + (1 - R_L) \rho_G \quad (27)$$

$$\tau_f = \frac{f_{2F} \rho_G \left(\frac{v_{SG}}{1 - R_L} \right)^2}{2 g_C D} \left[\frac{\text{kgf/m}^2}{\text{m}} \right] \quad (49)$$

Due to the absence of a slippage between the phases, the liquid holdup is in this case:

$$R_L = \frac{Q_L}{Q_L + Q_G} \quad (50)$$

To obtain the friction factor, the Moody plot or Chen equation is used, using a Reynolds number defined by:

$$Re_M = \frac{D v_{SG} \rho_G}{\mu_G} \quad (51)$$

Relative roughness is determined by the Weber number:

$$We = \left(\frac{v_{SG} \mu_L}{\sigma_L g_C} \right)^2 \frac{\rho_G}{\rho_L} \quad (52)$$

If $We < 0.005$:

$$\frac{\varepsilon}{D} = 34 \frac{\sigma_L g_C}{\rho_G v_{SG}^2 D} \quad (53)$$



If $We > 0.005$:

$$\frac{\varepsilon}{D} = 174.8 \frac{\sigma_L g_C We^{0.302}}{\rho_G v_{SG}^2 D} \quad (54)$$

For this flow :

$$10^{-3} \leq \frac{\varepsilon}{D} \leq 0.5$$

Orkiszewski method:

1. - Determine the flow pattern present in the pipe, calculating the dimensional parameters with equations 18 to 21 and using definitions 23 to 26.
2. - Calculate the friction pressure drop using the equations corresponding to the flow pattern determined in step 1. In the case of bullet flow, to calculate the Vr velocity follow these steps:
 - 2.1. - Calculate the Reynolds number with equation 32.
 - 2.2. - Calculate parameters N_1 and N_2 with equations 33 and 34.
 - 2.3. - Calculate Vr with equations 35 to 38.
3. - Get the pressure drop by acceleration with equation 16.
4. - Calculate the total pressure drop with equation 15.

Example 3

By a vertical steel pipe of 6 inches 40 ascends a mixture of hydrocarbons at 42 atm and 13°C. The mixture consists of 63 kg/s of liquid and 107 kg/s of steam. The density of the liquid is 806 kg/m³ and the vapour density of 36 kg/m³. The viscosity of the liquid is 0.8148 cp, and the vapour viscosity is 0.0115 cp; the surface tension of the liquid is 18 dinas/cm. Find the pressure drop per meter of pipe.

1. Translation.



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- 2.- Planning.

- 2.1.-Discussion .

The problem will be solved by the Orkiszewski method.

- 3.-CALCULATIONS

- 3.1.-Flow pattern

$$D = 6.065 \text{ in} = 0.1541 \text{ m} = 0.5054 \text{ ft}$$

$$A = 0.01864 \text{ m}^2$$

$$\sigma_L = 18 \text{ dina/cm} = 1.84 \times 10^{-3} \text{ kgf/m}$$

$$Q_G = 2.97 \text{ m}^3/\text{s}$$

$$Q_L = 0.0782 \text{ m}^3/\text{s}$$



$$v_{SL} = 4.19 \text{ m/s}$$

$$v_{SG} = 159.33 \text{ m/s}$$

$$v_{NS} = 163.52 \text{ m/s} = 536.48 \text{ ft/s}$$

$$Gv = \frac{2.97 \frac{\text{m}^3}{\text{s}}}{0.01864 \text{ m}^2} \left(\frac{806 \frac{\text{kg}}{\text{m}^3}}{9.81 \frac{\text{m}}{\text{s}} \left(1.84 \times 10^{-3} \frac{\text{kgf}}{\text{m}} \right) \left(9.81 \frac{\text{mkg}}{\text{s}^2 \text{kgf}} \right)} \right)^{1/4} = 1308.75$$

$$Lb = 1.071 - 0.2218 \frac{\left(536.48 \frac{\text{ft}}{\text{s}} \right)^2}{0.5054 \text{ ft}} = -126307.7 \not\geq 0.13$$

$$Lb = 0.13$$

$$Ls = 50 + 36 \left(1308.75 \right) \frac{0.0782 \frac{\text{m}^3}{\text{s}}}{2.97 \frac{\text{m}^3}{\text{s}}} = 1290.54$$

$$Lm = 75 + 84 \left(1308.75 \frac{0.0782 \frac{\text{m}^3}{\text{s}}}{2.97 \frac{\text{m}^3}{\text{s}}} \right)^{0.75} = 1269.70$$

Flow pattern:

$$\frac{159.33 \frac{\text{m}}{\text{s}}}{163.52 \frac{\text{m}}{\text{s}}} = 0.974 \not\leq 0.13$$

$$1308.75 \not\leq 1290.54$$

$$1269.70 \not\geq 1308.75 > 1290.54$$

$$1308.75 > 1269.70$$

The flow pattern obtained is mist. Using the Oshinowo-Charles map to compare:

$$Ox = 12332.48$$

$$Oy = 6.167$$

The pattern is the limit case of annular flow when the gas flows at high speed, that is, the flow is mist.



3.2.-Friction pressure drop.

For mist flow:

$$R_L = \frac{0.0782 \frac{\text{m}^3}{\text{s}}}{0.0782 \frac{\text{m}^3}{\text{s}} + 2.97 \frac{\text{m}^3}{\text{s}}} = 0.0257$$

$$\rho_{2F} = 0.0257 \left(806 \frac{\text{kg}}{\text{m}^3} \right) + (1 - 0.0257) \left(36 \frac{\text{kg}}{\text{m}^3} \right) = 55.75 \frac{\text{kg}}{\text{m}^3}$$

$$Re_M = \frac{0.1541 \text{m} \left(159.33 \frac{\text{m}}{\text{s}} \right) \left(36 \frac{\text{kg}}{\text{m}^3} \right)}{0.0115 \text{cp} \left(0.001 \frac{\text{kg}/\text{ms}}{\text{cp}} \right)} = 76860792$$

$$We = \left(\frac{159.33 \frac{\text{m}}{\text{s}} (0.8148 \text{cp}) \left(0.001 \frac{\text{kg}/\text{ms}}{\text{cp}} \right)}{1.84 \times 10^{-3} \frac{\text{kgf}}{\text{m}} \left(9.81 \frac{\text{mkg}}{\text{s}^2 \text{kgf}} \right)} \right)^2 \frac{36 \frac{\text{kg}}{\text{m}^3}}{806 \frac{\text{kg}}{\text{m}^3}} = 2.31 > 0.005$$

$$\frac{\varepsilon}{D} = 174.8 \frac{1.84 \times 10^{-3} \frac{\text{kgf}}{\text{m}} \left(9.81 \frac{\text{mkg}}{\text{s}^2 \text{kgf}} \right) (2.31)^{0.302}}{36 \frac{\text{kg}}{\text{m}^3} \left(159.33 \frac{\text{m}}{\text{s}} \right)^2 (0.1541 \text{m})} = 2.885 \times 10^{-5} \neq 10^{-3}$$

$$\frac{\varepsilon}{D} = 10^{-3}$$

$$f_{2F} = 0.0195$$

$$\tau_f = \frac{0.0195 \left(36 \frac{\text{kg}}{\text{m}^3} \right) \left(\frac{159.33 \frac{\text{m}}{\text{s}}}{1 - 0.0257} \right)^2}{2 \left(9.81 \frac{\text{mkg}}{\text{s}^2 \text{kgf}} \right) (0.1541 \text{m})} = 6209.34 \frac{\text{kgf}}{\text{m}^2}$$

3.3.- Total pressure drop.



$$G_T = 9120.17 \text{ kg}/(\text{m}^2 \text{ s})$$

$$P_{av} = 42 \text{ atm} = 433974 \text{ kgf}/\text{m}^2$$

$$AC = \frac{9120.17 \frac{\text{kg}}{\text{m}^2 \text{ s}} \left(159.33 \frac{\text{m}}{\text{s}} \right)}{9.81 \frac{\text{mkg}}{\text{s}^2 \text{ kgf}} \left(433974 \frac{\text{kgf}}{\text{m}^2} \right)} = 0.341$$

$$\left[\frac{\Delta P_{2F}}{L} \right] = \frac{6209.34 \frac{\text{kgf}}{\text{m}^3} + 55.75 \frac{\text{kg}}{\text{m}^3} \text{sen}(90^\circ) \left(1 \frac{\text{kgf}}{\text{kg}} \right)}{1 - 0.341} = 9506.97 \frac{\text{kgf}}{\text{m}^2 \text{ m}}$$

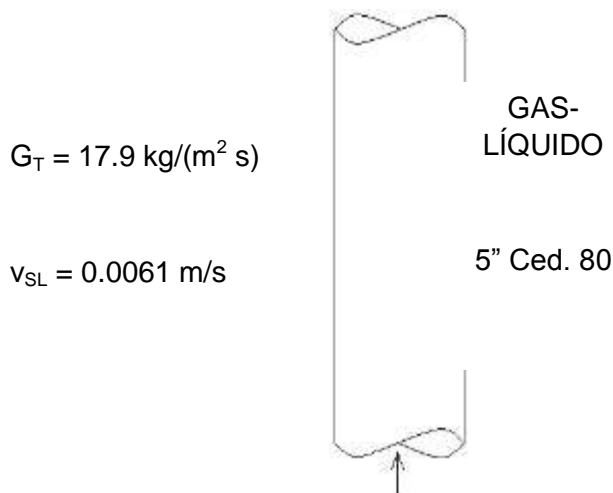
4.-RESULT

The total pressure drop is 9506.97 kgf/m² per meter of pipe length, equivalent to a drop of 0.92 atm per meter of tube.

Example 4

A mixture of hydrocarbons ascends through a 5-inch 80-inch steel pipe. The pressure is 40 atm and the temperature is 13°C. The total mass velocity of the mixture is 17.9 kg/m²s. The surface speed of the liquid is 0.0061 m/s and the gas speed is 0.338 m/s, being the liquid density of 810 kg/m³, and its viscosity of 0.8086 cp. The surface tension of the liquid phase is 18 dina/cm. The vapour density is 36 kg/m³ and its viscosity is 0.0115 cp. Non-slip or non-slip speed is 0.344 m/s. Find the expected pressure drop per meter in tube length.

1. Translation.



2.-Planning

2.1.- Discussion .

The calculation of the pressure drop will be done using the Orkiszewski method.

3.-CALCULATIONS.

3.1. - Flow Pattern.

$$D = 4.813 \text{ in} = 0.1223 \text{ m} = 0.4011 \text{ ft}$$



$$A = 0.011738 \text{ m}^2$$

$$\sigma_L = 18 \text{ dina/cm} = 1.84 \times 10^{-3} \text{ kgf/m}$$

$$v_{NS} = 0.344 \text{ m/s} = 1.13 \text{ ft/s}$$

$$Gv = 0.338 \frac{\text{m}}{\text{s}} \left(\frac{810 \frac{\text{kg}}{\text{m}^3}}{9.81 \frac{\text{m}}{\text{s}} \left(1.84 \times 10^{-3} \frac{\text{kgf}}{\text{m}} \right) \left(9.81 \frac{\text{mkg}}{\text{s}^2 \text{kgf}} \right)} \right)^{1/4} = 2.78$$

$$Lb = 1.071 - 0.2218 \frac{\left(1.13 \frac{\text{ft}}{\text{s}} \right)^2}{0.4011 \text{ft}} = 0.365 \geq 0.13$$

$$Ls = 50 + 36(2.78) \frac{0.0061 \frac{\text{m}}{\text{s}}}{0.338 \frac{\text{m}}{\text{s}}} = 51.8$$

$$Lm = 75 + 84 \left(2.78 \frac{0.0061 \frac{\text{m}}{\text{s}}}{0.338 \frac{\text{m}}{\text{s}}} \right)^{0.75} = 83.9$$

Flow pattern:

$$\frac{0.338 \frac{\text{m}}{\text{s}}}{0.344 \frac{\text{m}}{\text{s}}} = 0.983 \not\leq 0.365$$

$$2.78 < 51.8$$

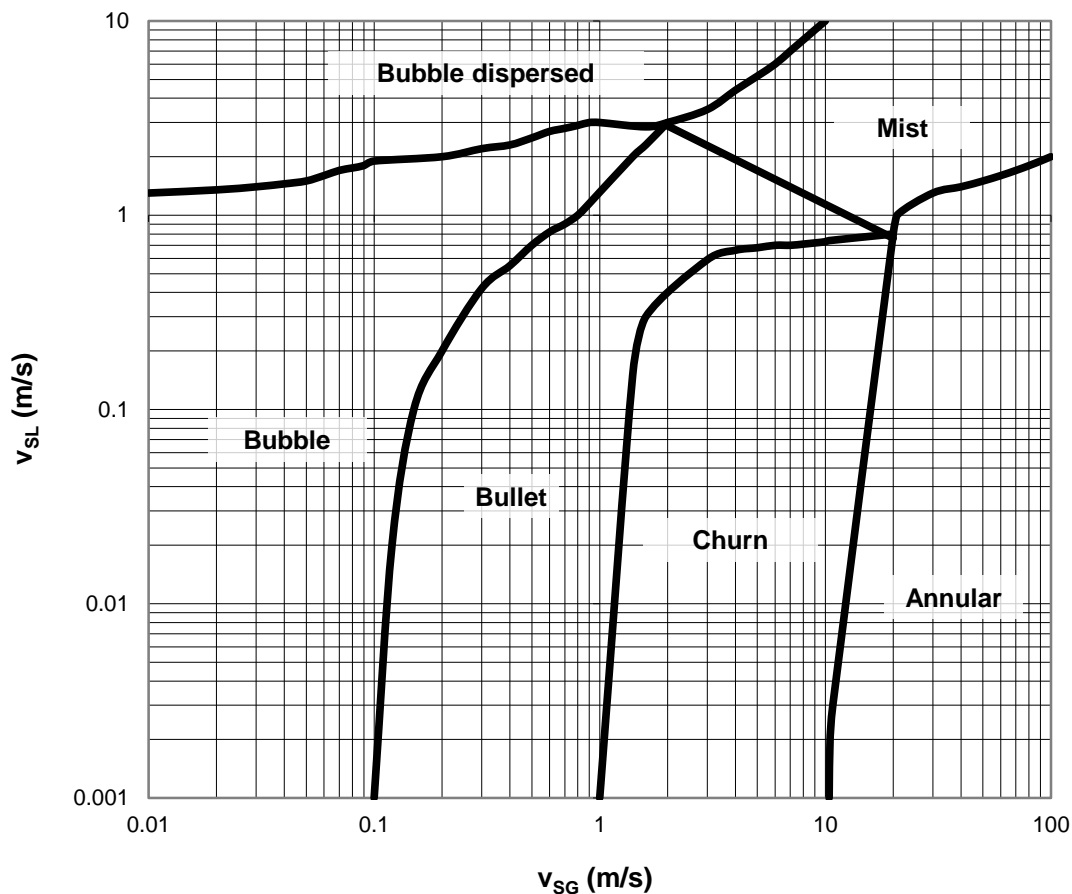
The flow pattern obtained is bullet.

Using Oshinowo-Charles parameters and map:

$$Ox = 0.0633$$

$$Oy = 7.44$$

The given flow pattern is bullet dispersed. Using Gonzalez Ortiz's map, bullet flow is determined, which matches Oshinowo-Charles's map and Orkiszewski's definitions.



Map of González Ortiz.^[11]

3.2.-Friction pressure drop .
Taylor's bubble ascent rate:

$$Re_s = \frac{0.1223 \text{ m} \left(0.344 \frac{\text{m}}{\text{s}} \right) \left(810 \frac{\text{kg}}{\text{m}^3} \right)}{0.8086 \text{ cp} \left(0.001 \frac{\text{kg}}{\text{cp}} \frac{\text{ms}}{\text{cp}} \right)} = 42144$$

$$N_1 = 0.572 \times 10^5 \left[-0.35 + \left(0.1225 + \frac{0.04931 \left(1.13 \frac{\text{ft}}{\text{s}} \right)^{1/2}}{(0.4011 \text{ ft})^{0.5}} \right) \right] = 6222.31$$



$$N_2 = 0.5721 \times 10^5 \left[-0.546 + \left(0.2981 + \frac{0.01849 \left(1.13 \frac{\text{ft}}{\text{s}} \right)^{1/2}}{(0.4011 \text{ft})^{0.5}} \right) \right] = 1682.24$$

$$Re_s = 42144 > N_1 = 6222.31$$

$$v_r = \left[1.985 + 4.958 \times 10^{-5} (42144) \right] (0.4011 \text{ft})^{0.5} = 2.58 \frac{\text{ft}}{\text{s}} = 0.787 \frac{\text{m}}{\text{s}}$$

Parameter \square :

$$v_{NS} = 0.344 \text{ m/s} < 3 \text{ m/s}$$

Equation 41 for petroleum corresponds to the hydrocarbon mixture:

$$\Gamma = \frac{0.0127 [\log(0.8086 \text{ cp} + 1)]}{(0.1223 \text{ m})^{1.415}} - 0.284 + 0.167 \left[\log \left(0.344 \frac{\text{m}}{\text{s}} \right) \right] + 0.113 [\log(0.1223 \text{ m})]$$

$$\Gamma = -0.4006 \neq -0.065 \left(0.344 \frac{\text{m}}{\text{s}} \right) = -0.0224$$

$$\Gamma = -0.0224$$

Friction drop:

From Moody^[10]

$$\square/D = 0.00035$$

$$f_{2F} = 0.023$$

$$\tau_f = \frac{0.023 \left(810 \frac{\text{kg}}{\text{m}^3} \right) \left(0.344 \frac{\text{m}}{\text{s}} \right)^2}{2 \left(9.81 \frac{\text{mkg}}{\text{s}^2 \text{kgf}} \right) (0.1223 \text{ m})} \left(\frac{0.0061 \frac{\text{m}}{\text{s}} + 0.787 \frac{\text{m}}{\text{s}}}{0.344 \frac{\text{m}}{\text{s}} + 0.787 \frac{\text{m}}{\text{s}}} + (-0.0224) \right) = 0.624 \frac{\text{kgf}}{\text{m}^2}$$

3.3.- Total pressure drop.

$$\rho_{2F} = \frac{17.9 \frac{\text{kg}}{\text{m}^2 \text{ s}} + 810 \frac{\text{kg}}{\text{m}^3} \left(0.787 \frac{\text{m}}{\text{s}} \right)}{0.344 \frac{\text{m}}{\text{s}} + 0.787 \frac{\text{m}}{\text{s}}} + (-0.0224) \left(810 \frac{\text{kg}}{\text{m}^3} \right) = 561.3 \frac{\text{kg}}{\text{m}^3}$$

$$P_{av} = 40 \text{ atm} = 413309 \text{ kgf/m}^2$$



$$AC = \frac{17.9 \frac{\text{kg}}{\text{m}^2 \text{s}} \left(0.338 \frac{\text{m}}{\text{s}} \right)}{9.81 \frac{\text{mkg}}{\text{s}^2 \text{kgf}} \left(413309 \frac{\text{kgf}}{\text{m}^2} \right)} = 1.49 \times 10^{-6}$$

$$\left[\frac{\Delta P_{2F}}{L} \right] = \frac{0.624 \frac{\text{kgf}}{\text{m}^3} + 561.3 \frac{\text{kg}}{\text{m}^3} \text{sen}(90^\circ) \left(1 \frac{\text{kgf}}{\text{kg}} \right)}{1 - 1.49 \times 10^{-6}} = 561.94 \frac{\text{kgf}}{\text{m}^2}$$

4.-RESULT

Pressure drop is 561.94 kgf/m² per meter of pipe length, equivalent to 0.054 atm per meter of pipe, or 0.799 psi per meter.

The correlation of Orkiszewski is the best for calculating the pressure drops in flow to two vertical upward phases, since its accuracy is in the order of 10%. However, Davis' correlation is most commonly used because of its simplicity and similarity to Lockhart-Martinelli's correlation.

In addition to the correlations presented here, many others can be found in the literature on the subject; for the interested reader is advised to review those of Hagedorn^[12], Aziz^[13], Beggs^[14] and Oshinowo-Charles^[15].

GENERAL CONSIDERATIONS

Depending on the flow pattern, the liquid contained in the tube can be accelerated to the speed of the gas phase. In certain cases, this speed is higher than is desirable in process pipes. High speeds produce a phenomenon known as erosion-corrosion, where the corrosion rate of the tube material is accelerated due to the erosive force of the liquid at high speeds.

An index based on loads or velocity heads indicates whether erosion-corrosion can be significant at a particular rate, and is used to determine the range of density and mixing speed within which erosion-corrosion does not occur. This index is^[15]:

$$\rho_M v_M^2 \leq 15000 \tag{55}$$

Where the density of the mixture is: $\rho_M = \frac{W_L + W_G}{\frac{W_L}{\rho_L} + \frac{W_G}{\rho_G}} \left[\frac{\text{kg}}{\text{m}^3} \right]$ (56)

And the velocity of the mixture is given by equation 46:

$$v_M = v_{SL} + v_{SG} \left[\frac{\text{m}}{\text{s}} \right] \tag{46}$$

CONCLUSIONS

For the general case, the mixing speed should be less than 15 m/s, as experience has shown that erosion occurs when that value is exceeded. In addition to keeping the speed-density product within the acceptable range, the appropriate flow rate should also be maintained on the lines. Above all, the battering flow should be avoided, because it causes serious mechanical and process problems, the latter due to the intermittent inlet of liquid and gas to a team. The dispersed flow or mist^[16] is an almost homogeneous mixture of liquid phase in the gas phase, and therefore behaves similarly to a compressible fluid. However, despite the goodness of this flow for pipe design, flash tanks and distillation columns cause separation problems, due to liquid drag. Once this flow pattern is acquired in a system, it is virtually impossible to separate the phases, as it requires achieving speeds impossible to obtain in the vast majority of flow systems. Stratified and wave flows are used only for long horizontal pipes. Plug and bullet



flows ^[16] are rare at design time due to their intermittency. The ring or film is undesirable because it causes erosion in the walls of the pipes.

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