



TWO PHASE FLOW, GAS- SOLID SYSTEM

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ABSTRACT

This article presents a resume of the flow of fluids with two phases, gas and solid circulating in parallel current through the interior of horizontal pipes illustrated with several examples..

KEYWORDS: *Two-phase flow, gas-solid, pressure drops, flow patterns, horizontal pipes.*

1.- INTRODUCTION

Systems where gases and solids come into contact in particles, grains or dust, have been used in the industry for almost a century. In its early days in 1926, the design of these systems was an art, whose ignorance could ruin the team, at best, or claim lives in the worst-case scenario. For these reasons, chemical engineering was given the task of investigating the behavior of gas-solid systems in order to understand their operation and design equipment such as catalytic reactors, pneumatic transport lines of solids, silos, hoppers, press filters, cyclones, continuous and batch dryers, mixers, heaters and particle coolers, among others, and thus turn the art of handling dusts and solid particles into a science.

Particulate solids behave like a liquid when they are contained in a container, the shape of which they take, but do not occupy the entire volume. Another similarity to liquids is to maintain a horizontal but irregular level in the vessel, in addition to having a hydrostatic pressure gradient along a column of particles, which is proportional to the density of the same and the height of the column. Because of these similarities, it is claimed that solid particles can flow like a liquid if they are dragged by a stream of gas or liquid. In the design of gas-solid systems, the engineer is concerned not only about the calculation of the pressure drop, but also about determining the gas speed needed for the operability of a particular flow system. Industrial practice has recommended operating speed intervals for each type of system, which have been represented in graphs similar to flow pattern maps for gas-liquid systems. To obtain an optimal design of the equipment, it is not enough to determine the speed of the gas and the pressure drop, it is also necessary to characterize the type of solid to be handled, according to its behavior in the face of a gas current. In addition, the flammability and explosive margins of gas-solid mixtures are also needed, as in the industry very powerful fires and explosions are known when handling fine powders, such as sugar, flours, pharmaceuticals, metal particles, pulverized coal, among many more. The accumulation of electrical loads in the equipment due to the friction of its walls with the particles, can lead to accidents, or at least disturbances in the proper operation of the equipment. Numerous theoretical and experimental research has been carried out to predict pressure drop and flow pattern, the latter dependent on gas velocity and solid type. As a result of these works, various correlations have been developed. Today, a general model has not been reached for all flow systems, as those developed are used for certain flow types and for certain operating conditions. Various groups of researchers continue to study the flow mechanisms of solid particles, to at least fully understand the operation of the different gas-solid biphasic flow management operations.

1.1.-Classification of solid particles

After observing the different flow patterns present in the particle beds, on which a gas flows, Geldart ^[1] classified the granular solids into four main groups, according to the particle diameter and the difference in densities between the solid and the gas. Geldart groups, in increasing order of particle size, are as follows:

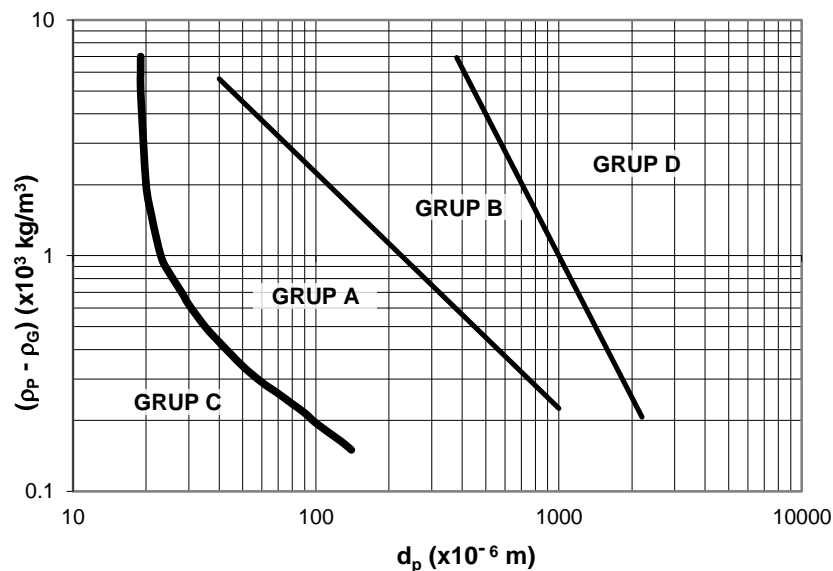
Group C: they are cohesive particles, whose fluidization is extremely difficult, as they result in the formation of channels through which the gas flows. In small diameter pipes, they form plugs that prevent the free flow of the gas phase. The difficulty in fluidization is due to the great attraction between the particles, the result of electrostatic charges, moisture or small particles joined by van der Waals forces. The particle diameter is generally less than 20 μm , and examples include starch, sugar, flours and cement.

Group A: they are airborne particles, whose fluidization is free of bubbles, so, when the speed of the gas increases, the bed expands considerably before the appearance of the first bubble. Fine particles act as a lubricant, allowing better fluidization and avoiding the accumulation of gas in the form of bubbles. The typical range of particle diameters ranges from 30 to 100 μm , and as a primary example you have the catalysts used for cracking.

Group B: they are sandy particles, whose fluidization is presented with continuous bubbling. These bubbles bind together (coalesce) as they ascend, increasing their size, and explode as they reach the surface of the bed. There is no maximum limit on the size of the bubbles, which is independent of the particle diameter. In shallow beds, the gas can be jet injected, as will be seen later, without the collapse of the gas-solid interface. The approximate range of particle diameters is 40 to 500 μm , and typical examples are taken of sand and table salt.

Group D: they are large and dense particles, the fluidization of which is commonly carried out by the injection of a jet of gas. These solids allow greater stability of the gas-solid interface in the jet, due to their larger size, resulting in very deep fluidized jet beds. Its higher density and diameter makes it difficult to develop a fluidization pattern such as that present in groups A and B, therefore, if the distribution of the gas to the bed is done improperly, a pipeline may occur, as in group C, or a violent and erratic bubbling. In this way, the injection of a gas jet is the most suitable mechanism to fluidize them. Its particle diameter is greater than 1 mm, and as examples are the peas, coffee beans, rice, wheat, fragments of coal (mineral coal) and metallic minerals. Its fluidization is commonly carried out by the injection of a jet of gas. These solids allow greater stability of the gas-solid interface in the jet, due to their larger size, resulting in very deep fluidized jet beds. Its higher density and diameter makes it difficult to develop a fluidization pattern such as that present in groups A and B, therefore, if the distribution of the gas to the bed is done improperly, a pipeline may occur, as in group C, or a violent and erratic bubbling. In this way, the injection of a gas jet is the most suitable mechanism to fluidize them. Its particle diameter is greater than 1 mm, and as examples are the peas, coffee beans, rice, wheat, fragments of coal (mineral coal) and metallic minerals.

Geldart proposed a diagram or map (Figure 1) where he graphed the transition boundaries between particle groups. This map is based on air fluidization data under atmospheric conditions. The ordering in Figure 1 is the difference in densities between solid particles and gas, and the abscissa is the average particle diameter.



1.- Figure.1.- Geldart particle classification map. (1973)

Geldart was the first to classify solid particles in a practical and objective way, as several authors had proposed arbitrary or based categories solely on the appearance of bubbles in fluidized beds. Geldart's criterion for classifying particles, on the other hand, is to group them according to their type of fluidization. He found four types of fluidization, while the other authors actually only distinguished two of them. Using fluidization data with air, nitrogen, carbon dioxide, helium, argon and freon 12, at different temperature and pressure conditions, Grace^[2] generalized the Geldart map by using dimensional groups as coordinates (Figure 2).

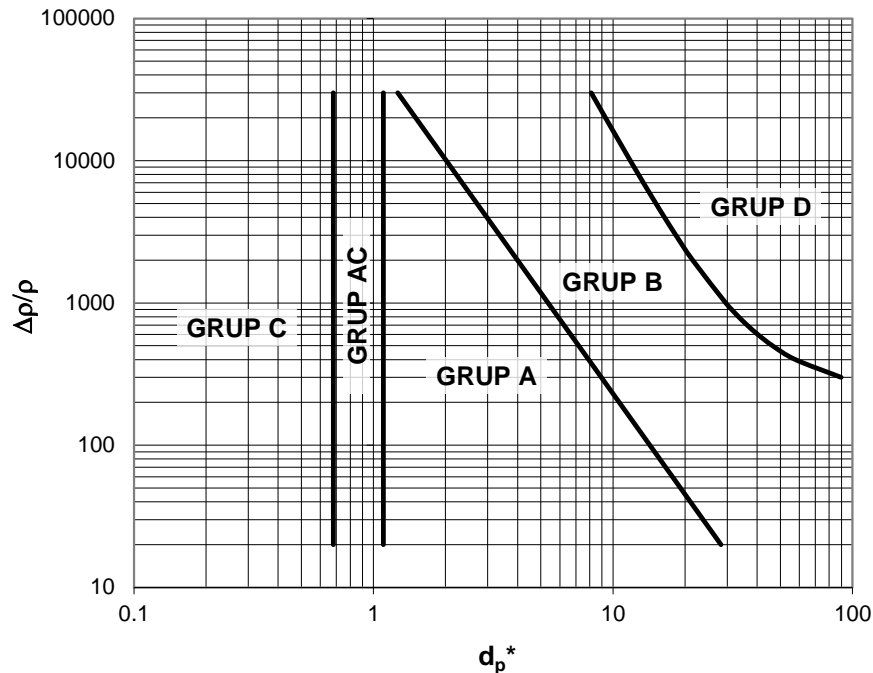


Figure 2.-Grace Particle Classification Map. (1986).

Where:

$\Delta\rho$ = difference in densities between the solid particle and the gas in kg/m³.

ρ = gas density in kg/m³.

d_p^* = particle dimensional diameter:

$$d_p^* = d_p \left[\frac{\rho \Delta\rho g}{\mu^2} \right]^{1/3} \quad (1)$$

d_p = particle diameter in m.

g = gravity acceleration = 9.81 m/s².

μ = gas viscosity kg/(m s).

Based on the Molerus^[3] criterion, which only takes into account the effect of van der Waals' forces on the cohesiveness of fine particles, Grace proposed a transitional region between groups C and A, which is often referred to as the AC Group, as the fluidization of these particles is like that of Group A, and when the gas flow is interrupted they give rise to the formation of plugs as group C. For the reader interested in learning more about classification and characterization of solid particles, it is recommended to consult the works of Kunii and Levenspiel, Leva, Valiente Barderas^[13], and Fan and Zhu.

2.- FLOW PATTERNS IN HORIZONTAL PIPING.

There are seven general types of flow patterns in horizontal pipes (Figure 3). Fixed in decreasing order of gas speed, flow patterns are as follows:

Homogeneous flow.- In this flow pattern, solid particles are completely suspended in the gas and are evenly distributed in the cross section of the pipe. It occurs at very high gas surface speeds, and very low particle

flows. It is also known as diluted suspension flow, uniform suspended flow, or homogeneous transport in diluted phase (dilute phase homogeneous conveying).

Heterogeneous flow.- By decreasing the gas speed, larger and heavier solid particles are transported by the gas phase in the lower portion of the pipe. Particle sedimentation is not yet present, as the gas speed is higher than the rate of sedimentation (saltation velocity), there is only one vertical gradient of solids concentration. It is also often called heterogeneous suspension flow, non-uniform suspended flow, or heterogeneous transport in diluted phase (dilute phase heterogeneous conveying).

Dune flow.- By decreasing the surface velocity of the gas phase to values below the rate of sedimentation, the particles begin to precipitate resulting in the formation of dunes in the lower portion of the pipe. In this type of flow, particles move from one dune to another in a periodic acceleration and deceleration motion. Depending on the speed of the gas, two types of dune flow are presented, which are:

Longitudinal dune flow: Immediately below the sedimentation rate, the particles form elongated dunes, parallel to the pipe, which advance in the direction of the flow. The width of these dunes is approximately 0.1 times the diameter of the pipe, and its length is 1 to 3 times the diameter of the pipe. It is also known as sediment flow or ribbon flow.

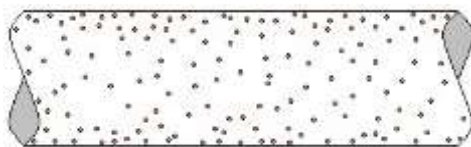
Transverse dune flow: At a lower gas speed, the particles form dunes perpendicular to the pipe, which advance in the direction of the flow. Its appearance is that of islands or clusters of well-defined particles. As the gas phase slows down, the length of the dunes decreases and their height increases. This flow pattern is the classic dune flow, also known as stratified flow.

Plug flow .- This flow pattern is characterized by excessive accumulation of particles on the dunes, resulting in the formation of solid plugs or pistons. The flow of both phases is intermittent, as it is in the form of alternating gas and solid plugs. It is presented only with particles of group C, which by their cohesiveness, form plugs with great ease. It is also often called plug flow.

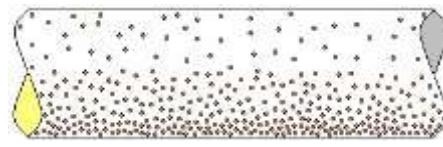
Slug flow.- By decreasing the speed of the gas, the particles accumulate in greater quantity in the dunes, increasing them in size to occupy the entire flow area of the pipe. This mechanism produces the formation of battering bodies, whose movement is in the flow direction, alternately with the flow of the gas phase. It is presented with particles from groups A, B and D.

Ripple flow .- At relatively lower gas surface speeds, particles occupy most of the pipe space, slowly advancing in the middle portion of the pipe and remaining stationary at the bottom of the pipe. At the top, the actual speed of the gas is higher due to the contraction of the flow area, so it drags particles, which form small waves or waves whose behavior is similar to that of the transverse dunes described above. It occurs only with particles in groups A and B.

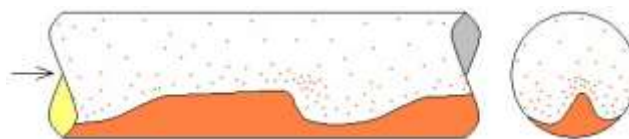
Flow with movil bed.- When the gas velocity decreases de particles that ocupied the tube flow slowly . They are present with particles of group A,B,and D. They are also know as flow in dense phase.



Homegeneus flow



Heterogeneous flow



Transverse dune flow

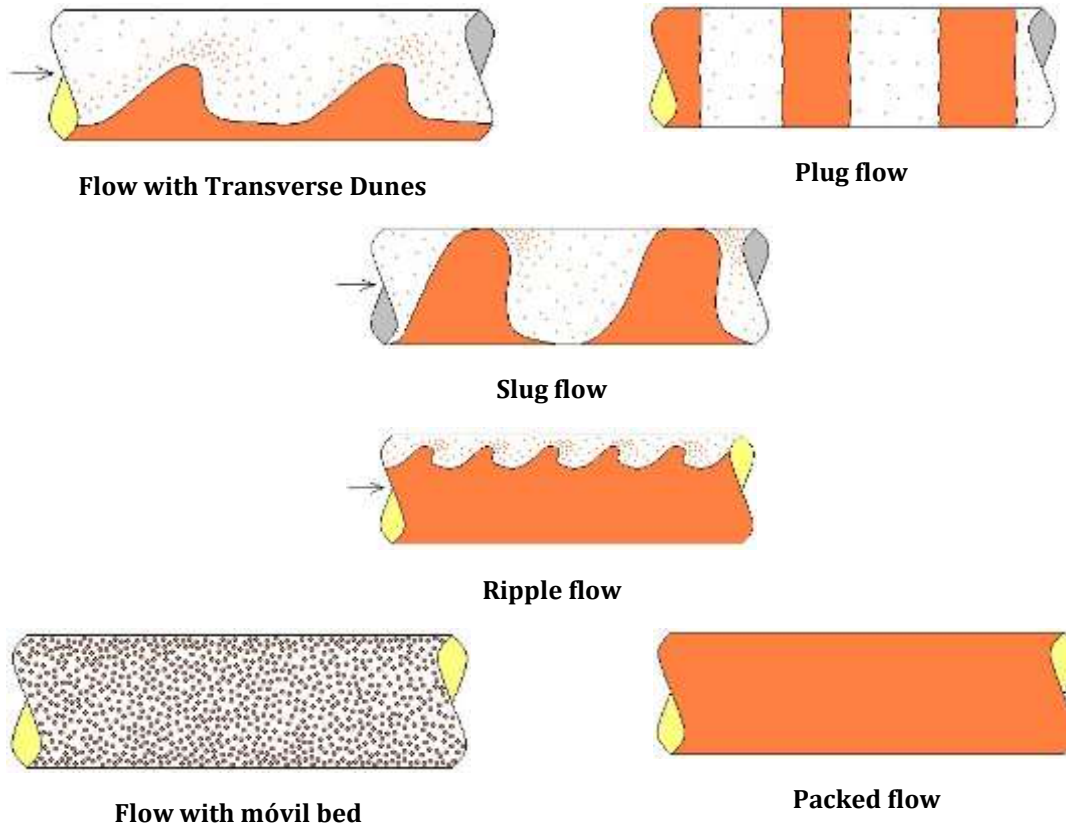


Figure 3.- Flow patterns to two phase gas-solid system in horizontal pipes.

3.- PREDICTION OF FLOW PATTERNS IN HORIZONTAL PIPES

To be able to size a solid particle transport line, and therefore design a gas-solid two-phase flow system, it is necessary to know first the flow pattern present in the line. Unlike the identification of horizontal flow patterns in gas-liquid systems, in gas-solid systems this identification does not depend on the technique used in experimentation, as the patterns are recognized according to the pressure drop profile in which they are presented. Although several researchers have conducted studies on the subject, only patterns of industrial interest have been well defined. There are many types of flow whose borders are unclear or unknown. Today, engineers and scientists continue to investigate the mechanisms by which different flow patterns develop, while new applications continue to be discovered for each of them. The first to develop a graph where horizontal flow patterns are recognized was Zenz^[4], who developed a qualitative scheme relating the pressure drop to the surface velocity of the gas phase. The Zenz graph is entirely experimental and is applicable only to a particular system, so it does not constitute a general map of flow patterns. Based on theoretical considerations and experimental data, Thomas^[5] developed the only generalized map of horizontal flow patterns (Figure 4). The data used corresponds mainly to water-solid systems, but data from air-solid systems are also included. Based on theoretical considerations and experimental data, Thomas⁵ developed the only generalized map of horizontal flow patterns (Figure 4). The data used corresponds mainly to water-solid systems, but data from air-solid systems are also included. This map was constructed for a quotient $(\rho_P - \rho_G)/\rho_G$ of 100, frequent average value in gas-solid systems, and applied for particles in groups A, B and D. Its borders are presented with great precision considering frictional effects for the identification of different flow patterns. In the flow with transverse dunes are included the ram and wave flows, which are presented very close to the border with the flow with mobile bed, preferably located in the region of the intermediate law.



The coordinates in the Thomas' map are:

$$\text{Abcisa} = \frac{d_p v_{f0} \rho_G}{\mu_G} \quad (2)$$

$$\text{Ordenada} = \frac{v_t}{v_{f0}} \quad (3)$$

Where:

d_p = diameter of the particle in m.

v_{f0} = frictional velocity at infinite dilution in m/s:

$$v_{f0} = v_{SG} \sqrt{\frac{f_f}{2}} \quad (4)$$

v_{SG} = gas superficial velocity in m/s.:

$$v_{SG} = \frac{Q_G}{3600 A} = \frac{W_G}{3600 \rho_G A} \quad (5)$$

f_f = Fanning friction factor de fricción de Fanning, gas flow only:

$$f_f = \frac{f_D}{4} \quad (6)$$

f_D = Darcy friction factor, from the Moody's diagram.

ρ_G = gas density kg/m^3 .

μ_G = gas viscosity kg/(m s) .

v_t = Particle terminal velocity m/s.

Note the use of the surface velocity of the gas instead of its actual speed and the absence of any parameter dependent on the amount of particles present in the pipe. Because solids do not flow by themselves in a horizontal pipe, their transport depends exclusively on the amount of drag gas, expressed in the form of surface speed, which considers that the gas occupies the entire flow area of the pipe. In this way, Thomas' map indicates the surface velocity of gas needed to induce some particle behavior and thus develop a certain flow pattern.

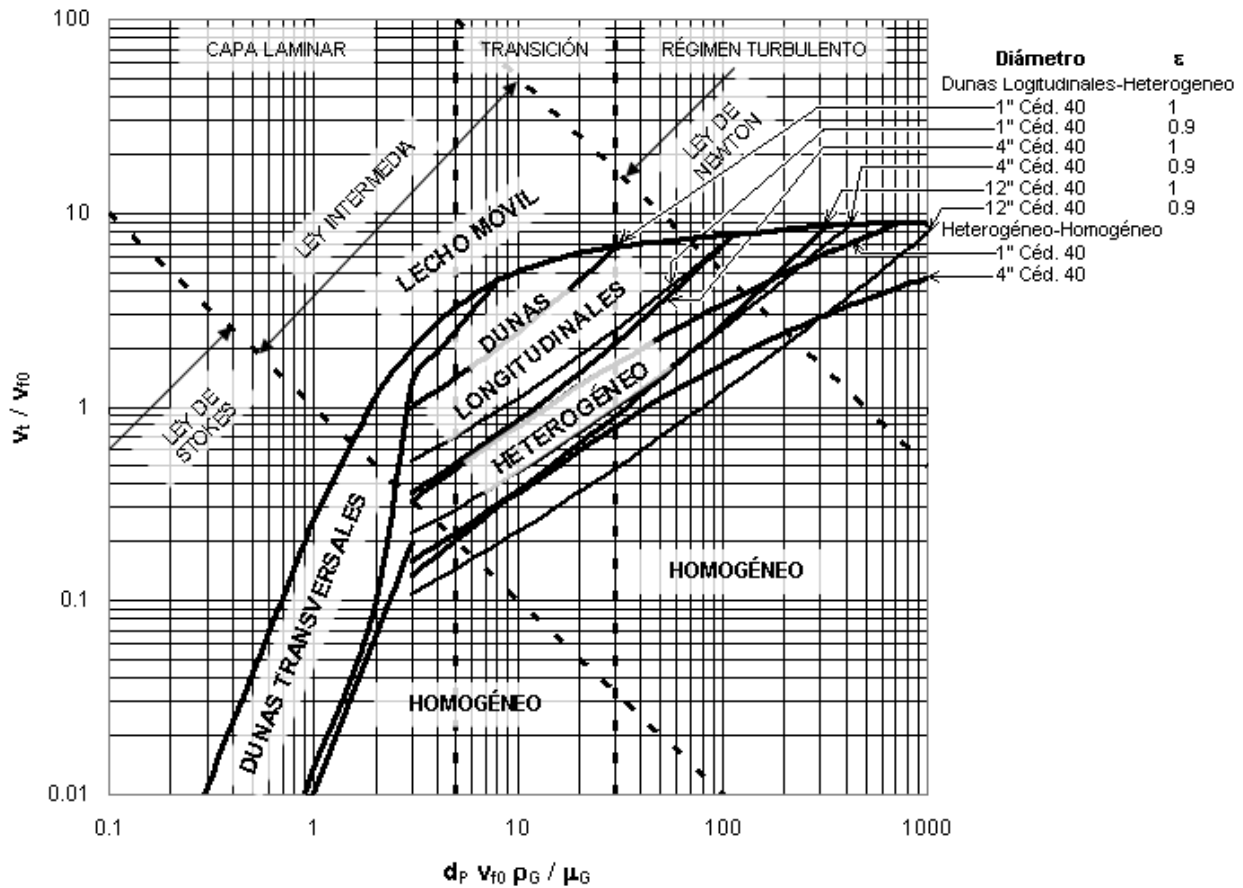


Figure 4.- Map of Thomas patterns for horizontal flow in gas-solid systems. (1964).

The transitional borders between mobile bed-transverse dunes, transverse dunes-longitudinal dunes and longitudinal-homogeneous dunes, are independent of the diameter of the pipe and the amount of particles, the latter expressed as a fraction of hollows. The position of the longitudinal-heterogeneous and heterogeneous-homogeneous dune borders varies from the diameter of the pipe, descending on the map as the diameter of the pipe increases. Also shown on the map is the dependence of the longitudinal-heterogeneous dunes border with respect to the fraction of hollows, which is given by the following equation:

$$\epsilon = \frac{Q_G}{Q_G + Q_P} = \frac{\frac{W_G}{\rho_G}}{\frac{W_G}{\rho_G} + \frac{W_P}{\rho_P}} \quad (7)$$

Where: Q_G and Q_P – are the volumetric flows of the gaseous and solid phases in m³/h. The terminal velocity depends on the sedimentation regime of the particles, which is determined by the number of particle terminal Reynolds:

$$(Re_p)_t = \frac{d_p v_t \rho_G}{\mu_G}$$

If $(Re_p)_t < 1$, Stokes' law is complied with:

$$v_t = \frac{g(\rho_P - \rho_G)d_p^2}{18\mu_G} \left[\frac{m}{s} \right] \quad (9)$$



Where: g = acceleration of gravity $\times 9.81 \text{ m/s}^2$. ρ_P - intermediate law is enforced:
solid particle density in kg/m^3 . If $1 < (Re_p)t < 500$, the

$$v_t = \frac{0.153 g^{0.71} d_p^{1.14} (\rho_P - \rho_G)^{0.71}}{\rho_G^{0.29} \mu_G^{0.43}} \left[\frac{\text{m}}{\text{s}} \right] \quad (10)$$

If $(Re_p)t > 500$, Newton's law is followed:

$$v_t = 1.74 \sqrt{\frac{g d_p (\rho_P - \rho_G)}{\rho_G}} \left[\frac{\text{m}}{\text{s}} \right] \quad (11)$$

A quick way to obtain this speed is found in appendix LVI of Valiente Barderas' book^[9] and on page 81 of Kunii and Levenspiel's work, which uses the dimensional particle diameter and the graph in Figure 5, shown below, where their coordinates are given by:

$$d_p^* = d_p \left[\frac{\rho \Delta \rho g}{\mu^2} \right]^{1/3} \quad (12)$$

$$u_t^* = v_t \left[\frac{\rho^2}{\mu \Delta \rho g} \right]^{1/3} \quad (13)$$

Where:

d_p^* - particle dimensional diameter ; u_t^* - dimensional terminal speed ; d_p - particle diameter in m. v_t - terminal velocity in m/s . ; ρ gas density in kg/m^3 . ; $\Delta \rho$ difference in densities between the two phases in kg/m^3 . ; μ gas viscosity in kg/(m s) . ; g - acceleration of gravity - 9.81 m/s^2 .

Although the terminal velocity also depends on the sphericity of the particles, whether it is unknown it can be assumed that the particles are spherical. The terminal velocity obtained in this way is the highest possible for a particle falling into the sine of a fluid, whether liquid or gas.

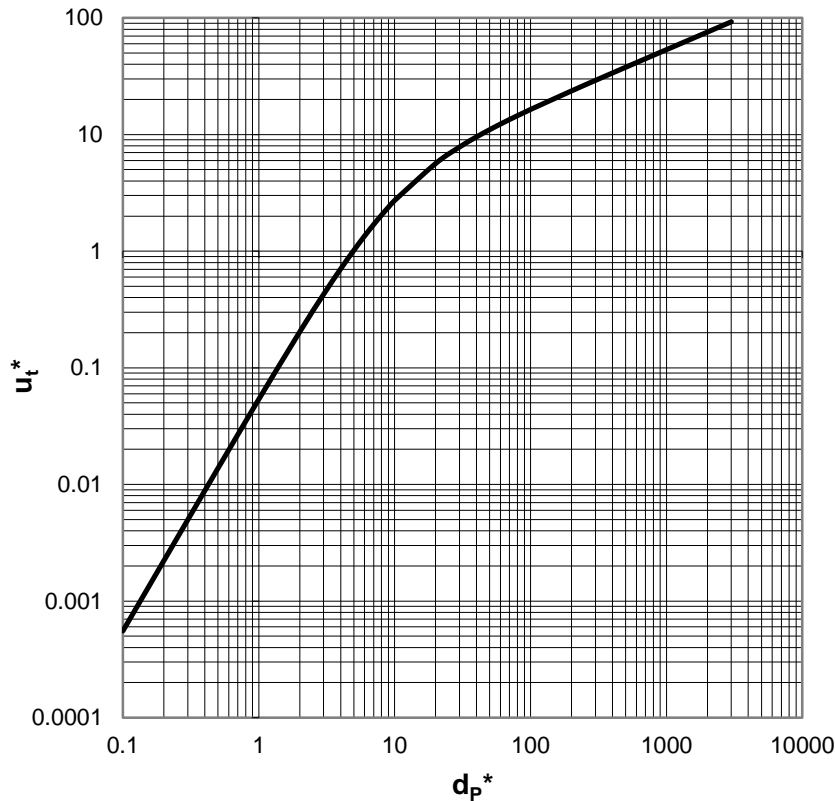


Figure 5.- Terminal speed of spherical particles in fluids.

Thomas method for determining horizontal flow patterns:

1.- Obtain the terminal velocity of the particles using the graph in Figure 5:

$$v_t = (u_t^*) \left[\frac{\mu \Delta \rho g}{\rho^2} \right]^{1/3} \left[\frac{m}{s} \right] \quad (14)$$

2.- Calculate the superficial Particle Reynolds.

$$Re_{SP} = \frac{d_p v_{SG} \rho_G}{\mu_G} \quad (15)$$

3.- Determine the friction factor of Fanning equation (6) , using the superficial particle Reynolds and Moody diagram, or, using the equations of Hagen-Poiseuille or Chen depending on the flow regimen (laminar or turbulent, respectively).

Laminar:
$$f_D = \frac{64}{Re}$$

Turbulent:

$$\frac{1}{\sqrt{f_D}} = -2 \log \left[\frac{\epsilon}{3.7065 D} - \frac{5.0452}{Re} \log \left(\frac{1}{2.8257} \left(\frac{\epsilon}{D} \right)^{1.1098} + \frac{5.8506}{Re^{0.8981}} \right) \right]$$

4.- Calculate the frictional velocity at infinite dilution with equation 4.

5.- Obtain the coordinates of Thomas with equations 2 and 3, and determine the flow pattern present in the pipe with the map in Figure 4. The inclined dotted lines present in Figure 4 separate the different sedimentation regimes

from the particles, while the vertical lines indicate the limits on the diameter of the particles relative to the thickness of the film with laminar flow, defined by von Karman by the following parameter:

$$\delta = \frac{5\mu_G}{\rho_G v_{f0}} \quad (14)$$

Where:

δ = The thickness of the laminar film postulated by von Karman. If $d_p < \delta$, the particle flow rate is laminar. On Thomas' map, this regime corresponds to:

$$\frac{d_p v_{f0} \rho_G}{\mu_G} < 5 \quad (15)$$

If $\delta < d_p < 6 \delta$, the particle flow rate is transitional. On the map mentioned above, this regime is presented between:

$$5 < \frac{d_p v_{f0} \rho_G}{\mu_G} < 30 \quad (16)$$

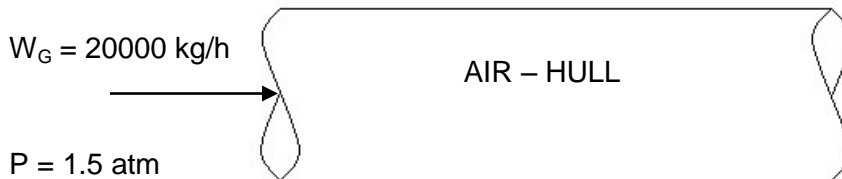
If $d_p > 6 \delta$, the particle flow rate is turbulent. On Thomas' map, this regime corresponds to:

$$\frac{d_p v_{f0} \rho_G}{\mu_G} > 30 \quad (17)$$

Example 1

What will be the expected flow pattern in a 4-inch horizontal pipe through which 20000 kg/h of air flows at a pressure of 1.5 atm and 25°C temperature? Hull particles (mineral coal) are transported through the tube, with a diameter of 200 μm , with a density of 640 kg/m^3

1.-Traduction



2.-Discussion

To find the flow pattern, Thomas's parameters must be known and then use its flow pattern map (Figure 22).

2.1.-Thomas coordinates.

$$\text{Abscisa} = \frac{d_p v_{f0} \rho_G}{\mu_G} \quad \text{Ordenada} = \frac{v_t}{v_{f0}}$$

3.-Calculs.

3.1.-Terminal particle velocity

$\mu_G = 0.0183 \text{ cp} = 1.83 \times 10^{-5} \text{ kg}/(\text{m s})$ @ 25°C, 1.5 atm

$$\rho_G = \frac{1.5 \text{ atm} \left(29 \frac{\text{kg}}{\text{kgmol}} \right)}{0.082 \frac{\text{m}^3 \text{ atm}}{\text{kgmol K}} (25 + 273.15) \text{ K}} = 1.78 \frac{\text{kg}}{\text{m}^3}$$

$$d_p^* = \left(200 \times 10^{-6} \text{ m} \right) \left[\frac{\left(1.78 \frac{\text{kg}}{\text{m}^3} \right) (640 - 1.78) \frac{\text{kg}}{\text{m}^3} \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}{\left(1.83 \times 10^{-5} \frac{\text{kg}}{\text{ms}} \right)^2} \right]^{1/3} = 6.43$$

This adimensional diameter value is obtained from the terminal velocity plot of particles in fluids (Figure 5):

$$v_t = (1.5) \left[\frac{\left(1.83 \times 10^{-5} \frac{\text{kg}}{\text{ms}} \right) (640 - 1.78) \frac{\text{kg}}{\text{m}^3} \left(9.81 \frac{\text{m}}{\text{s}^2} \right)}{\left(1.78 \frac{\text{kg}}{\text{m}^3} \right)^2} \right]^{\frac{1}{3}} = 0.50 \frac{\text{m}}{\text{s}}$$

3.2.-Frictional speed to infinite dilution

For a pipe of 4" nominal diameter, ced. 40, its internal diameter is
 $D = 4.026 \text{ in} = 0.1023 \text{ m}$

$$A = \frac{\pi}{4} D^2 = 0.008213 \text{ m}^2$$

$$v_{SG} = \frac{20000 \frac{\text{kg}}{\text{h}}}{3600 \frac{\text{s}}{\text{h}} \left(1.78 \frac{\text{kg}}{\text{m}^3} \right) (0.008213 \text{ m}^2)} = 380.02 \frac{\text{m}}{\text{s}}$$

$$Re_{SP} = \frac{(200 \times 10^{-6} \text{ m}) \left(380.02 \frac{\text{m}}{\text{s}} \right) \left(1.78 \frac{\text{kg}}{\text{m}^3} \right)}{1.83 \times 10^{-5} \frac{\text{kg}}{\text{ms}}} = 7393 \quad \text{Turbulent}$$

$$e/D = 0.00045$$

$$f_D = 0.034$$

$$f_f = \frac{0.034}{4} = 0.0085$$

$$v_{f0} = 380.02 \frac{\text{m}}{\text{s}} \sqrt{\frac{0.0085}{2}} = 24.77 \frac{\text{m}}{\text{s}}$$

3.3.-Thomas' coordinates

$$\text{Abscisa} = \frac{(200 \times 10^{-6} \text{ m}) \left(24.77 \frac{\text{m}}{\text{s}} \right) \left(1.78 \frac{\text{kg}}{\text{m}^3} \right)}{1.83 \times 10^{-5} \frac{\text{kg}}{\text{ms}}} = 481.9$$

$$\text{Ordenada} = \frac{0.50 \frac{\text{m}}{\text{s}}}{24.77 \frac{\text{m}}{\text{s}}} = 0.020$$

With these coordinates, the flow pattern corresponding to the intersection of these values is located on the Thomas map in Figure 4, the latter being observed in the homogeneous flow region.

4.-RESULT

The flow obtained is homogeneous in diluted phase.

4.- PREDICTION OF PRESSURE DROP IN HORIZONTAL PIPES

To size pneumatic particle transport lines requires pressure drop calculation along the line. Engineering has developed correlations and methodologies applicable to each flow pattern. To date, there is no general theoretical model capable of correctly predicting pressure drop, so the semi-empirical correlations most used in the industry for

their simplicity and precision are set out below. Pneumatic transport in diluted phase In general, pressure drop in horizontal pipes receives contributions by acceleration and friction. For homogeneous and heterogeneous flows in diluted phase the following expression is given:

$$\Delta P_{2F} = \frac{\epsilon \rho_G v_G^2}{2g_C} + \frac{(1-\epsilon)\rho_P v_P^2}{2g_C} + F_{gw} L + F_{pw} L \quad \left[\frac{\text{kgf}}{\text{m}^2} \right] \quad (18)$$

Where:

ΔP_{2F} = Two phase total pressure drop in kgf/m^2 .

ϵ = Void fraction, gas phase, holdup.

v_G = Real gas velocity m/s:

$$v_G = \frac{v_{SG}}{\epsilon} \quad \left[\frac{\text{m}}{\text{s}} \right] \quad (19)$$

v_P = real solid velocity in m/s:

$$v_P = \frac{G_P}{\rho_P(1-\epsilon)} \quad \left[\frac{\text{m}}{\text{s}} \right] \quad (20)$$

G_P = solid phase mass velocity in $\text{kg/m}^2\text{s}$:

$$G_P = \frac{W_P}{A} \quad \left[\frac{\text{kg}}{\text{m}^2 \text{ s}} \right] \quad (21)$$

W_P = particles mass flow kg/s .

A = pipe flow area m^2 .

$g_C = 9.81 \text{ m kg/(s}^2 \text{ kgf)}$

F_{gw} = friction force between the gas and tube wall kgf/m^3 .

F_{pw} = friction force between the particles and the tube wall in kgf/m^3 .

L = tube long m.

The first term corresponds to the gas acceleration pressure drop, the second to the acceleration pressure drop of the particles, the third is the loss of frictional pressure between the gas and the pipe wall, and the fourth is the frictional pressure drop between the particles and the pipe wall. The term corresponding to the friction between the gas and the wall can be determined by the Fanning equation:

$$F_{gw} = \frac{2f_G \rho_G v_G^2}{Dg_C} \quad \left[\frac{\text{kgf}}{\text{m}^2} \right] \quad (22)$$

Where: f_G is the gas Fanning friction factor; D - diameter of the pipe in m.

To determine the last term of the total pressure drop, Hinkle^[6] proposed the following equation:

$$F_{pw} = \frac{2f_p(1-\epsilon)\rho_P v_P^2}{Dg_C} = \frac{2f_p G_P v_P}{Dg_C} \quad \left[\frac{\text{kgf}}{\text{m}^2} \right] \quad (23)$$

Where:

f_p = friction factor for the solid particles:

$$f_p = \frac{3}{8} C_D \left(\frac{\rho_G}{\rho_P} \right) \left(\frac{D}{d_p} \right) \left[\frac{v_G - v_P}{v_P} \right]^2 \quad (24)$$

C_D = drag coefficient for the particles.

The drag coefficient is a function of the Particle Reynolds of the particle between phases and the sphericity of the particles. To obtain this coefficient, the graph shown in Figure 6 is used, where

$$(Re_P)_{slip} = \frac{d_P (v_G - v_P) \rho_G}{\mu_G} \quad (25)$$

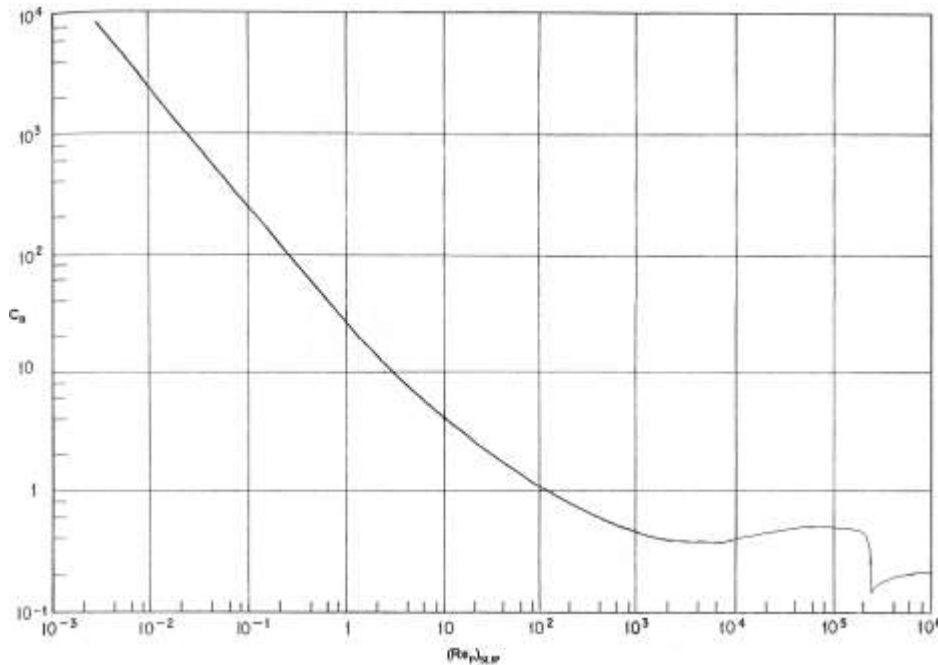


Figure 6.- Drag coefficient based on the sliding particle Reynolds.

From experimental data, Hinkle obtained a correlation to predict the actual speed of the particles, which is:

$$v_P = v_{SG} \left(1 - 0.0638 d_P^{0.3} \rho_P^{0.5} \right) \left[\frac{m}{s} \right] \quad (26)$$

Using Hinkle data, Yang7 modified the solids friction factor equation for greater accuracy in calculating total pressure drop for homogeneous flow. The friction factor is then given by:

$$f_P = 0.117 \frac{(1-\epsilon)}{\epsilon^3} \left[(1-\epsilon) \frac{(Re_P)_t}{(Re_P)_{slip}} \frac{v_G}{\sqrt{gD}} \right]^{-1.15} \quad (27)$$

Where: $(Re_P)_t$ - Reynolds terminal particle given by equation 8.

$(Re_P)_{slip}$ - Reynolds of the sliding particle phase-between given by Equation 25 : $(Re_P)_{slip} = \frac{d_P (v_G - v_P) \rho_G}{\mu_G}$ (25)

4.1- Hinkle method:

1.- Determine the flow pattern using Thomas' map (Figure 4). If the flow is homogeneous or heterogeneous, this method should be continued.

2.- Calculate the actual speed of the particles with equation 26.

3.- Get the fraction of gaps or gas holdup with the following equation:

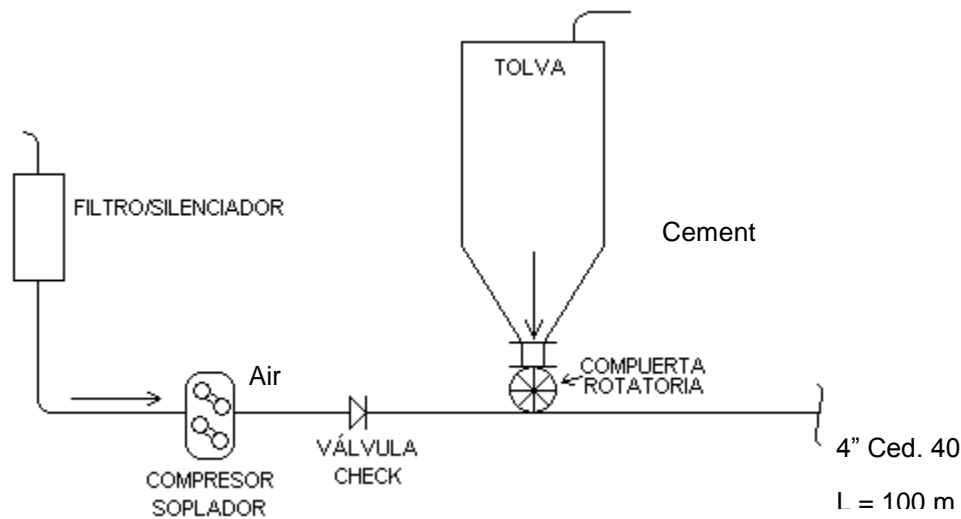
$$\epsilon = 1 - \frac{G_P}{\rho_P v_P} \quad (28)$$

- 4.- Calculate the actual speed of the gas with equation 19, using it to obtain the friction factor of the gas.
- 5.- Determine the frictional pressure drop between the gas and the wall with equation 22.
- 6.- Calculate the friction factor of the solids with equations 24 or 27.
- 7.- Get friction pressure drop between particles and wall with equation 23.
- 8.- Determine the total pressure drop with equation 18.

Example 2

What is the total pressure drop in a 4-inch horizontal line 40 through which 500 kg/h of air passes, at a pressure of 3 atm and a temperature of 15°C? This pipe transports cement in a diluted form, which is fed to the system by a hopper at a rate of 5700 kg/h, and whose particles have a diameter of 81 μm and a density of 1240 kg/m³. The length of the pipe is 100 m.

1.- Translation



2.- Planning

2.1.- Discussion.

To determine the pressure drop, it is necessary to first identify the flow pattern using Thomas' map. Subsequently, pressure losses are calculated using the Hinkle method.

3.- Calculations

3.1.- Flow pattern.

The air properties at 3 atm and 15°C are:

$$\rho = 3.68 \text{ kg/m}^3 ; \mu = 0.0175 \text{ cp} = 1.75 \times 10^{-5} \text{ kg/(m s)} ; dp^* = 4.26$$

With the adimensional particle diameter the terminal velocity is obtained in figure. 5

$$u_t^* = 0.8$$

$$v_t = 0.20 \text{ m/s}$$

$$D = 4.026 \text{ in} = 0.1023 \text{ m}$$

$$A = 0.008213 \text{ m}^2$$

$$v_{SG} = 4.60 \text{ m/s}$$

$$Re_{SP} = 78$$

laminar flow

$$f_D = \frac{64}{78} = 0.820$$

$$f_f = 0.205$$

$$v_{f0} = 1.47 \text{ m/s}$$

$$\text{Abscisa} = 25.0$$

$$\text{Ordenad} = 0.136$$

The flow pattern is homogeneous.

3.2.- Void fraction.



$$v_p = 4.60 \frac{m}{s} \left[1 - 0.0638 (81 \times 10^{-6} m)^{0.3} \left(1240 \frac{kg}{m^3} \right)^{0.5} \right] = 3.99 \frac{m}{s}$$

$$G_p = \frac{5700 \frac{kg}{h}}{3600 \frac{s}{h} (0.008213 m^2)} = 192.8 \frac{kg}{m^2 s}$$

$$\varepsilon = 1 - \frac{192.8 \frac{kg}{m^2 s}}{1240 \frac{kg}{m^3} \left(3.99 \frac{m}{s} \right)} = 0.961$$

3.3.-Friction pressure drop between gas and Wall.

$$v_G = \frac{4.60 \frac{m}{s}}{0.961} = 4.79 \frac{m}{s}$$

$$Re_G = \frac{(0.1023 m) \left(4.79 \frac{m}{s} \right) \left(3.68 \frac{kg}{m^3} \right)}{1.75 \times 10^{-5} \frac{kg}{ms}} = 103044$$

Turbulent flow

From the Moody:

$$e/D = 0.00045$$

$$f_D = 0.020$$

$$f_G = f_f = 0.005$$

$$F_{gw} = \frac{2(0.005) \left(3.68 \frac{kg}{m^3} \right) \left(4.79 \frac{m}{s} \right)^2}{0.1023 m \left(9.81 \frac{mkg}{s^2 kgf} \right)} = 0.841 \frac{kgf}{m^2}$$

3.4.-Friction pressure drop between particles and wall.

$$(Re_P)_t = \frac{(81 \times 10^{-6} m) \left(0.20 \frac{m}{s} \right) \left(3.68 \frac{kg}{m^3} \right)}{1.75 \times 10^{-5} \frac{kg}{ms}} = 3.41$$

$$(Re_P)_{slip} = \frac{(81 \times 10^{-6} m) (4.79 - 3.99) \frac{m}{s} \left(3.68 \frac{kg}{m^3} \right)}{1.75 \times 10^{-5} \frac{kg}{ms}} = 13.6$$



$$f_p = 0.117 \frac{(1-0.961)}{(0.961)^3} \left[(1-0.961) \frac{3.41}{13.6} \frac{4.79 \frac{m}{s}}{\sqrt{9.81 \frac{m}{s^2} (0.1023m)}} \right]^{-1.15} = 0.174$$

$$F_{pw} = \frac{2(0.174) \left(192.8 \frac{kg}{m^2 s} \right) \left(3.99 \frac{m}{s} \right)}{(0.1023m) \left(9.81 \frac{mkg}{s^2 kgf} \right)} = 266.8 \frac{kgf}{m^2}$$

3.5.- Total drop pressure.

$$\Delta P_{2F} = \frac{0.961 \left(3.68 \frac{kg}{m^3} \right) \left(4.79 \frac{m}{s} \right)^2}{2 \left(9.81 \frac{mkg}{s^2 kgf} \right)} + \frac{(1-0.961) \left(1240 \frac{kg}{m^3} \right) \left(3.99 \frac{m}{s} \right)^2}{2 \left(9.81 \frac{mkg}{s^2 kgf} \right)} + \left(0.841 \frac{kgf}{m^3} \right) (100m) + \left(266.8 \frac{kgf}{m^3} \right) (100m) = 26807.5 \frac{kgf}{m^2}$$

4.-RESULT

The total pressure drop on the pneumatic transport line is 26807.5 kgf/m².

Pneumatic transport in dense phase

In the case of flow patterns with dunes (transverse and longitudinal), piston, battering ram, wave and with movable bed, Klinzing and Mathur^[12] proposed two equations for total pressure drop, depending on the actual particle Reynolds:

$$Re_p = \frac{d_p v_G \rho_G}{\mu_G} \quad (29)$$

If $Re_p < 1$, the gaseous phase flows around the particles as in a porous medium, applying Darcy's law for this type of flow :

$$\frac{\Delta P_{2F}}{L} = \frac{\mu_G (v_G - v_P)}{K g_C} + \frac{f_P v_P^2 \rho_B}{D g_C} \left[\frac{kgf}{m^2} \right] \quad (30)$$

Where:

K = permeability in m²:

$$K = 3.28 \times 10^{-14} \left(\frac{W_P}{W_G} \right)^{0.48} \frac{d_p^{0.43}}{D^{0.73}} \left[m^2 \right] \quad (31)$$

W_P y W_G = Massic flows of the solid and gas phases kg/s.

ρ_B = particle bulk density kg/m³:

$$\rho_B = (1 - \varepsilon) \rho_P \left[\frac{kg}{m^3} \right] \quad (32)$$

The first term corresponds to the pressure drop by flow in porous medium, and the second is the friction pressure drop. The recommended friction factor is that obtained by Yang^[8] for dense phase flow:

$$f_p = 0.0410 \frac{(1-\varepsilon)}{\varepsilon^3} \left[(1-\varepsilon) \frac{(Re_p)_t}{(Re_p)_{slip}} \right]^{-1.021} \quad (33)$$

If $Re_p > 1$, Klinzing and Mathur developed a pressure drop equation for turbulent dense flow, which is:

$$\frac{\Delta P_{2F}}{L} = \frac{\alpha}{g_c} (v_G - v_p)^2 \left[\frac{\text{kgf/m}^2}{\text{m}} \right] \quad (34)$$

Where: α = Klinzing-Mathur parameter:

$$\alpha = 6.59 \times 10^{-4} \left(\frac{W_p}{W_g} \right)^{3.15} \frac{D^{0.36}}{d_p^{0.84}} \quad (35)$$

These researchers used the following expression for the actual speed of particles in this type of pneumatic transport:

$$v_p = v_{SG} \left(1 - 0.68 d_p^{0.93} \rho_p^{0.5} \rho_g^{-0.2} D^{-0.54} \right) \left[\frac{\text{m}}{\text{s}} \right] \quad (36)$$

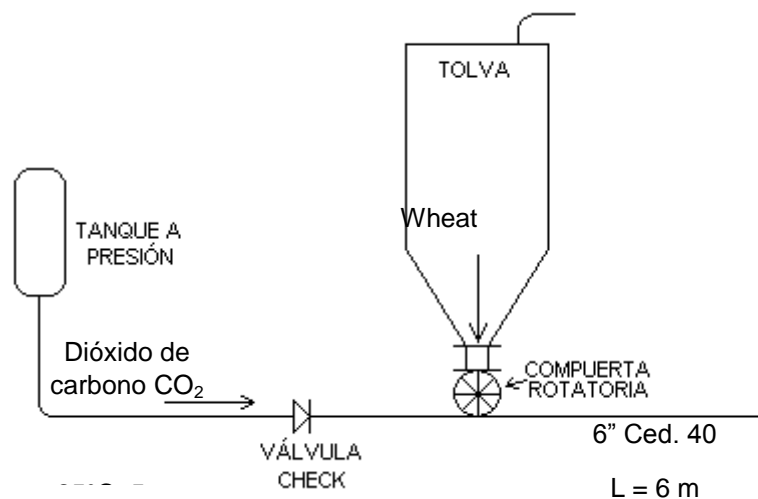
Klinzing-Mathur method:

- 1.- Determine the flow pattern using Thomas' map (Figure 4). If the flow is with longitudinal dunes, transverse dunes or moving bed, proceed with this method.
- 2.- Calculate the actual particle speed with equation 36.
- 3.- Get the fraction of gaps with equation 28.
- 4.- Calculate the actual speed of the gas with equation 19.
- 5.- Calculate the actual particle Reynolds with equation 29.
- 6.- Determine the total pressure drop with equations 30 or 34, depending on the actual particle Reynolds.

Example 3

Determine the total pressure drop in a 6-inch horizontal pipe 40 and 6 m in length, through which 1200 kg/h of carbon dioxide flows, at a temperature of 25°C and 5 atm pressure. 50000 kg/h of wheat are transported along the line. The grains have an average diameter of 4.8 mm and their density is 750 kg/m³.

1.-TRANSLATION



2.-Planning

2.1.-Discussion

The flow pattern is determined with the Thomas map in Figure 4, and the pressure drop is calculated using the Klinzing-Mathur method.



3.-Calculations.

3.1.-Flow pattern

The properties of carbon dioxide at 5 atm and 25°C are:

$$\rho_G = 9.00 \text{ kg/m}^3; \mu_G = 0.0148 \text{ cp} = 1.48 \times 10^{-5} \text{ kg/(m s)}; d_p^* = 321$$

With the particle adimensional diameter in the terminal speed plot we get (Figure 5):

$$u_t^* = 30; v_t = 3.30 \text{ m/s}; D = 6.065 \text{ in} = 0.1541 \text{ m}; A = 0.018639 \text{ m}^2; v_{SG} = 1.99 \text{ m/s}$$

$$Re_{SP} = 5809 \quad \text{turbulent regime}$$

From the Moody's graphic:

$$e/D = 0.00030; f_D = 0.036; f_f = 0.009; v_{f0} = 0.13 \text{ m/s}$$

$$\text{Abscissa} = 379.5; \text{Ordenada} = 25.4$$

The pattern obtained is that of flow with mobile bed.

3.2.- Void fraction.

$$v_p = 1.99 \frac{\text{m}}{\text{s}} \left[1 - 0.68 (4.8 \times 10^{-3} \text{ m})^{0.93} \left(750 \frac{\text{kg}}{\text{m}^3} \right)^{0.5} \left(9.00 \frac{\text{kg}}{\text{m}^3} \right)^{-0.2} (0.1541 \text{ m})^{-0.54} \right] = 1.53 \frac{\text{m}}{\text{s}}$$

$$G_p = 745.15 \text{ kg/(m}^2 \text{ s)}$$

$$\varepsilon = 1 - \frac{745.15 \frac{\text{kg}}{\text{m}^2 \text{ s}}}{750 \frac{\text{kg}}{\text{m}^3} \left(1.53 \frac{\text{m}}{\text{s}} \right)} = 0.35$$

3.3.- Total pressure drop.

$$v_G = \frac{1.99 \frac{\text{m}}{\text{s}}}{0.35} = 5.69 \frac{\text{m}}{\text{s}}$$

$$Re_p = \frac{(4.8 \times 10^{-3} \text{ m}) \left(5.69 \frac{\text{m}}{\text{s}} \right) \left(9 \frac{\text{kg}}{\text{m}^3} \right)}{1.48 \times 10^{-5} \frac{\text{kg}}{\text{m s}}} = 16609 > 1$$

$$\alpha = 6.59 \times 10^{-4} \left(\frac{50000 \frac{\text{kg}}{\text{h}}}{1200 \frac{\text{kg}}{\text{h}}} \right)^{3.15} \frac{(0.1541 \text{ m})^{0.36}}{(4.8 \times 10^{-3} \text{ m})^{0.84}} = 3772.1$$

$$\Delta P_{2F} = \frac{3772.1}{9.81 \frac{\text{m kg}}{\text{s}^2 \text{ kgf}}} \left(5.69 \frac{\text{m}}{\text{s}} - 1.53 \frac{\text{m}}{\text{s}} \right)^2 (6 \text{ m}) = 39925.7 \frac{\text{kgf}}{\text{m}^2}$$

4.-RESULT

The total pressure drop on the pneumatic transport line of wheat grains is 39925.7 kgf/m².

The Hinkle and Klinzing - Mathur methods predict pressure drops to two gas-solid phases in horizontal pipes with an error of 20%. Both methods are very simple, compared to others whose procedure is iterative and doubtful response. There are other semi-empirical correlations to find pressure drops in horizontal flow to two gas-solid phases. The interested reader can consult the correlations of Mehta-Smith-Comings^[8], Vogt-White^[9], Rose-Duckworth^[10], Chari^[11], among others.

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