TWO-PHASE FLOW, SOLID LIQUID SYSTEM

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ABSTRACT

Chemical engineers have at different times to work with two-phase flow, where two phases move at the same time by pipes. The authors have presented in other articles [30,31,32,33,34] the two phase flow with gas-liquid, gas-solid, in this final article the authors summarize the flow patterns that are presented and the pressure drops presented in the two –phase flow solid-liquid systems.

KEYWORDS: *Two-phase flow, solid-liquid, flow patterns, pressure drops.*

1. INTRODUCTION

The phenomenon of concurrent flow of granular or particulate liquids and solids (known in English as slurry flow) should not be ignored by chemical engineering, as its field of applications in the industry is vast and extensive. As in gas-solid systems, the design of equipment whose main component is the flow of liquids and solids in parallel, was an unknown art for any engineer outside these systems. In early 1906, liquid-solid flow systems comprised almost exclusively city and industrial drains, which were designed and built by civil engineers. With the design and construction of large coal-by-water pipelines in the early 1950s in various parts of the world, liquid-solid biphasic flow research intensified, and in this way chemical engineering was able to glimpse a better future by converting this discipline into science.

In addition to pipes for hydraulic transport of solid materials, there are other equipment where parallel flow conditions of liquids and solid particles are presented, such as catalytic reactors, press filters, ion exchange equipment, decanters, mixers, heaters and coolers, pumps for suspension handling, among others.

In the design of liquid-solid systems, the engineer's concern is focused on determining pressure drop, which depends on the flow pattern, the speed of the conveyor liquid, and the properties of both phases. The speed of the liquid phase is a critical variable as it determines the speed of solid particles. Poor selection of liquid velocity causes erosion of the internal walls of pipes and equipment with liquid-solid biphasic flow, and consequently the failure of them. Unlike the flow of gas-solid mixtures, there is no risk of explosion or fire, but the phenomenon of erosion leads to corrosion of tubes and equipment.

Although both a gas and a liquid can be used to carry solid particles, the behavior of both fluid-solid biphasic flows has important differences. In gas-solid flow, particle-to-particle interactions and the pipe wall dominate the interactions between fluid and particles. In liquid-solid flow, fluid-particle and particle interactions dominate over those between particles and the pipe wall. It is also worth noting the great similarity between the two phenomena, highlighting the flow patterns and the series of variables on which their behavior depends.

In order to optimize the design of these systems and improve their operability, it is necessary to know in greater detail the phenomenon of liquid-solid flow. Despite research in this field, it has not yet been possible to develop a general model, as the correlations existing to date depend on the flow pattern. However, thanks to similarities to the gas-solid flow, a large number of these correlations are applicable for both types of biphasic flow, which can lead us in the future to a general two-phase flow model.

1.1.- CLASSIFICATION OF SOLID PARTICLES

The fluidization of particle beds can occur in two ways, homogeneously or with the appearance of bubbles, depending on the size of the particles and the density of the particles. According to these fluidization patterns, Gibilaro, Hossain and Foscolo[1] classified the particles into three groups, presenting them on a graph (Figure 1).
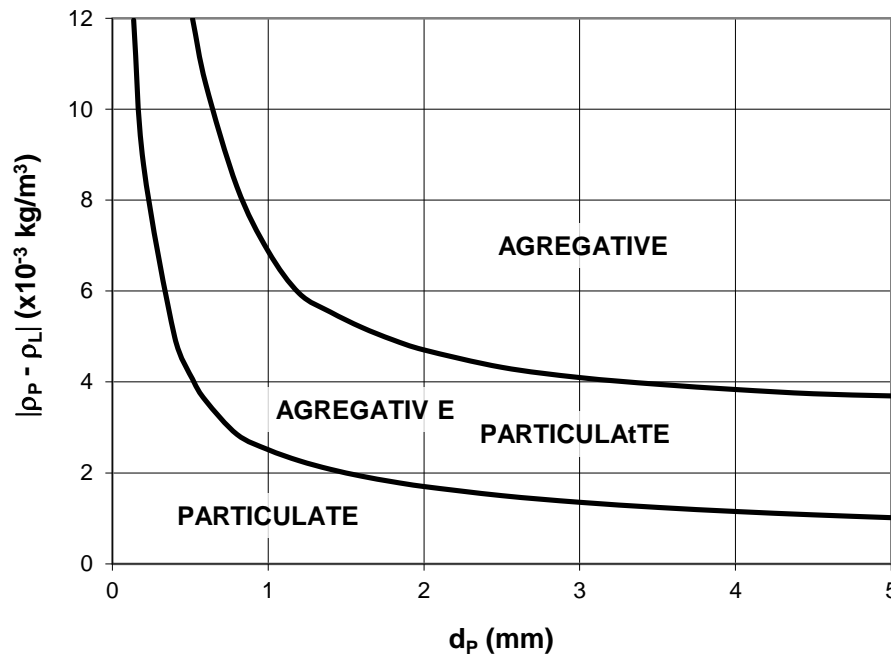

Figure 1.- Particle classification map of Gibilaro-Hossain-Foscolo.[1] (1986).

This map was built based on water fluidization data under atmospheric conditions. For other liquids and other pressure and temperature conditions, Di Felice [2] developed a classification map (Figure 2), based on the classification criteria proposed by him, Gibilaro and Foscolo [3], and whose coordinates are dimensional. The map coordinates of Di Felice are:

$$d_P{}^* = d_P \left[\frac{\rho_L (\rho_P - \rho_L) g}{\mu_L^2} \right]^{1/3} = Ar^{1/3} \qquad (1)$$

$$De = \frac{\rho_L}{\rho_P} \qquad (2)$$

Where:

$d_P{}^*$ = adimensional diameter of the particule ..

De = density ratio of the phases.

d_P = particle diameter m.

ρ_L = liquid density kg/m^3.

ρ_P = solid particle density kg/m^3.

g = gravity= 9.81 m/s^2.

μ_L = liquid viscosity kg/(m s).

Ar = Arquímedes´ number, also called Galileo´snumber (Ga):

$$Ar = \frac{d_P^3 \rho_L (\rho_P - \rho_L) g}{\mu_L^2} = (d_P *)^3 \qquad (3)$$

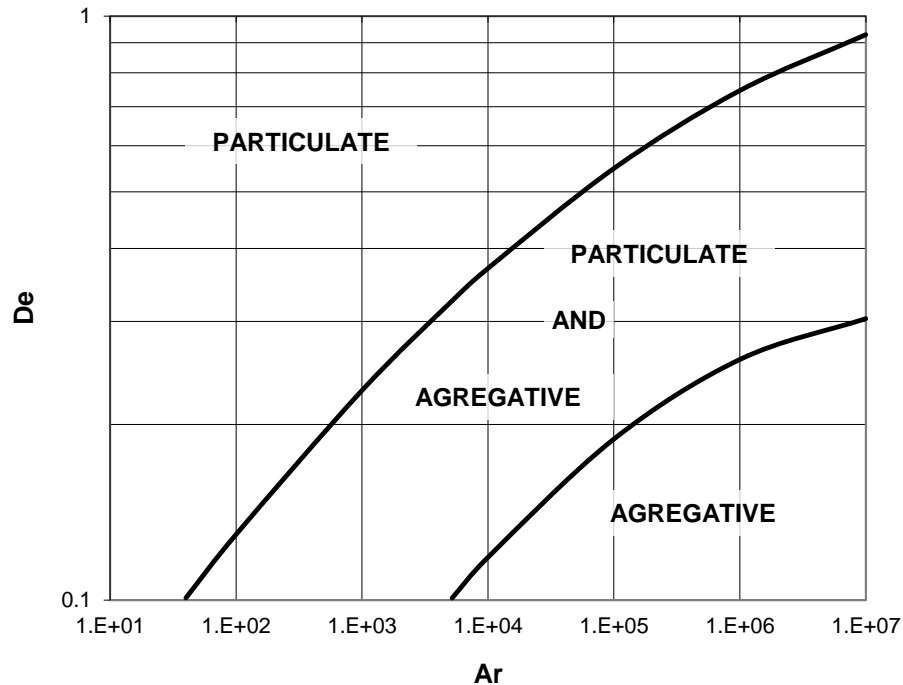$$Ar = \frac{d_P^3 \rho_L (\rho_P - \rho_L) g}{\mu_L^2} = (d_P *)^3 \qquad (3)$$

Figure 2.- Di Felice particle classification map.[2] (1995).

Readers interested in learning more about solid particle characterization are advised to consult the works of Brave Barderas [22], Leva [23], Kunii and Levenspiel [24], Fan and Zhu [25], and Shook and Roco [26

1.2 .-FLOW PATTERNS IN HORIZONTAL PIPING
There are five liquid-solid biphasic flow patterns in horizontal pipes (Figure 3), which are arranged in decreasing order of liquid speed are as follows:
Homogeneous flow
Solid particles are completely suspended in the liquid and are evenly distributed throughout the pipe flow area. The presence of particles affects the rheology of the liquid. If the concentration of the solids is less than 5% by volume, the flow behavior is of a Newtonian type, and if the concentration is higher, the flow behaves like a non-Newtonian fluid. This flow rate occurs at very high surface liquid speeds. By remebing a single homogeneous phase, it is often referred to as pseudohomogeneous flow.
Heterogeneous flow
 By decreasing the speed of the liquid, the larger and heaviest particles descend to be transported by the liquid phase in the lower portion of the tube. Particles are still suspended, so there is no sediment at the bottom of the pipe. It is also known as heterogeneous slurry.
Dune flow.
 Further decreasing the speed of the liquid phase, to a value less than the rate of sedimentation, the particles begin to precipitate, thus forming sediments whose transport is carried out in the form of dunes or mounds. The liquid velocity determines the type of dunes present, which are similar to those in the flow to two phases gas-solid:
Longitudinal dune flow:

Immediately below the sedimentation rate, the particles form elongated dunes, parallel to the pipe, which advance in the direction of the flow. The width of these dunes is approximately 0.1 times the diameter of the pipe, and its length is 1 to 3 times the diameter of the pipe. It is also known as sediment flow, belt flow, or saltation flow.
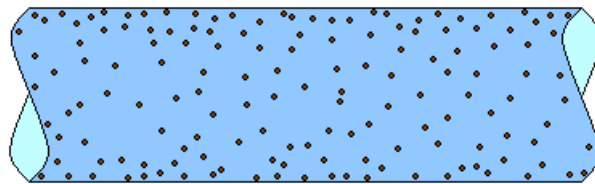
Transverse dune flow

At a lower liquid speed, the particles form dunes perpendicular to the pipe, which advance in the direction of the flow. Its appearance is that of islands or clusters of well-defined particles. As the speed of the liquid phase decreases, the length of the dunes decreases and their height increases. This flow pattern is the classic dune flow, also known as stratified flow.
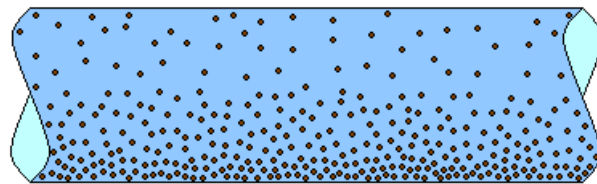
Ripple flow

At lower liquid speeds, particles form a stationary bed at the bottom of the pipe. In the middle portion, the particles advance slowly, sliding over the stationary bed. At the top, the liquid is flowing freely at a higher speed relative to the stationary bed, carrying particles that form waves or waves that move in the direction of the flow as transverse dunes. It is often called flow with stationary bed.
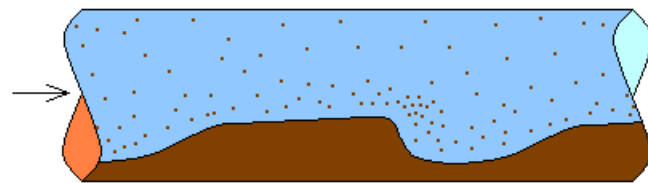
Moving bed flow

By further decreasing the surface speed of the liquid phase, the particles fully occupy the flow area of the pipe, thus the upper portion flowing slowly and the lower portion remains stationary. If the liquid speed drops further, the movement of the particles ceases causing the line to be blocked. It is also called continuous dense phase flow.
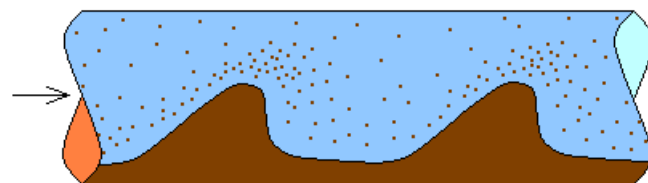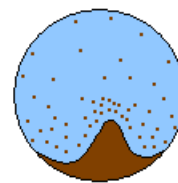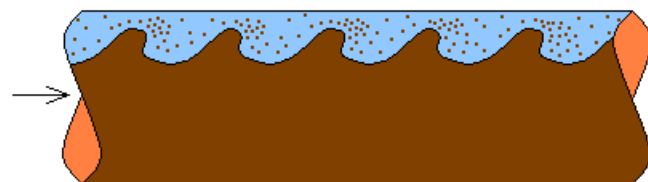

Homogeneous flow
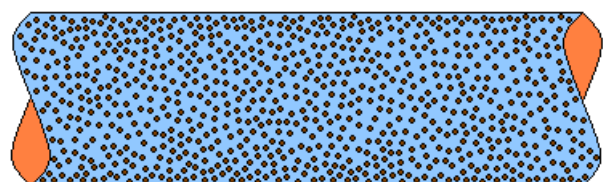
Heterogeneous flow

Longitudinal dune flow:

Transverse dune flow

Ripple flow

Moving bed flow

Figure 3.- Flow patterns to two phases liquid-solid system in horizontal pipes.

These flow patterns are classified according to the concentration of the solids in the flow area of the pipe, as follows:
1.2.1.- Diluted phase flow:
The concentration of particles is relatively low, as the surface velocity of the liquid is higher than that of sedimentation of solids, which are dispersed in the liquid phase; it is also known as diluted phase hydraulic transport: homogeneous and heterogeneous flows.
Dense phase flow
The surface velocity of the liquid is less than that of sedimentation of the particles, causing a accumulation of particles in the pipe; it is also known as dense phase hydraulic transport: flows with dunes, wave and with mobile bed.

2.- PREDICTION OF FLOW PATTERNS IN HORIZONTAL PIPES
The determination of the flow pattern to two liquid-solid phases present in a transport line is the first step for sizing. Similar to the identification of horizontal flow patterns in gas-solid systems, in liquid-solid systems this identification is independent of the technique used by researchers, as flow patterns are recognized by the pressure drop profile in which they are presented.

One of the first maps of horizontal flow patterns was developed by Newitt and his collaborators [4], based on the work of Durand and Condolios [5], and using data on the transport of sand, gravel, manganese oxide and coal with water. Subsequently, other maps have appeared in the literature, including that of Turian and Yuan [6], with the disadvantage of being specific to a particular system and certain flow conditions.

The only widespread map to date is that developed by Thomas [7] (Figure 4), which is based on theoretical considerations and experimental data. As seen in gas-solid systems, the data used by Thomas mainly correspond to water-solid systems, although some data belonging to air-solid systems was used.

Below is Thomas' map corresponding to the liquid-solid systems, along with its methodology for determining the horizontal flow pattern. The distribution of the patterns in this graph is similar to that of the Thomas map for gas-solid systems. The map shown below was constructed for a quotient $(\rho_P - \rho_L)/\rho_L$ of 1.65, which is a common average value in liquid-solid systems and was originally used by Thomas when drawing up his map.
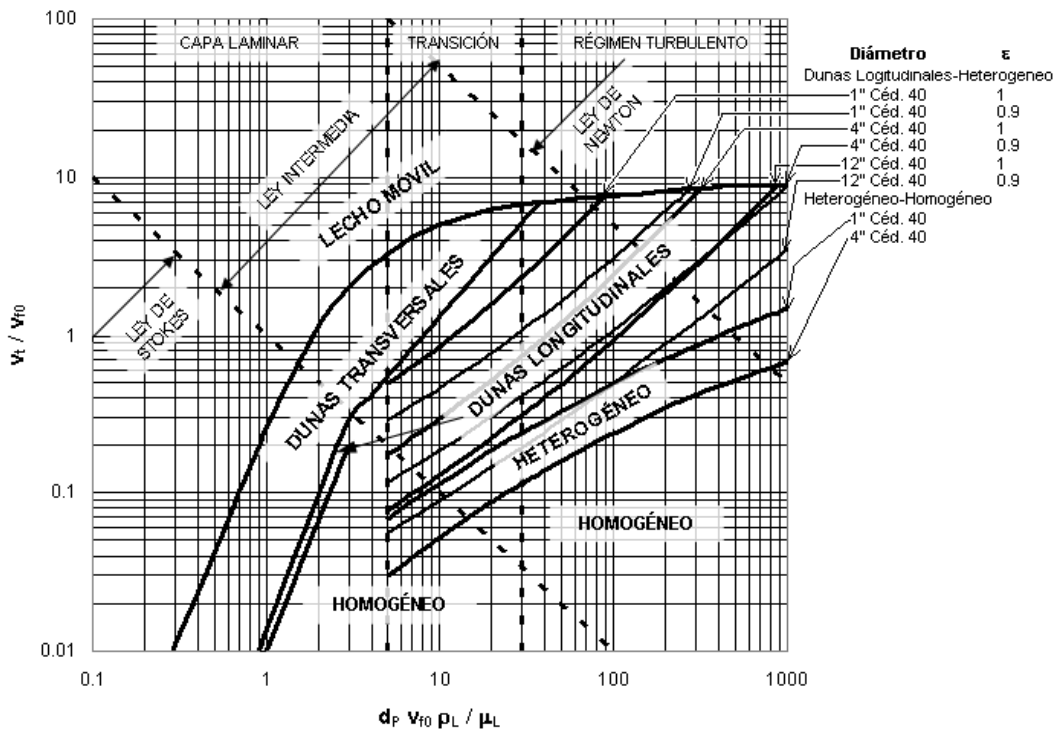

Figure 4.- Map of Thomas patterns for horizontal flow in liquid-solid systems[6]. (1964).

The fraction of gaps or liquid holdup is given by the following equation:

$$\varepsilon = \frac{Q_L}{Q_L + Q_P} = \frac{\dfrac{W_L}{\rho_L}}{\dfrac{W_L}{\rho_L} + \dfrac{W_P}{\rho_P}} \qquad (4)$$

Where:
Q_L and Q_P - volumetric flows of the liquid and solid phases in m^3/h.

2.1 Thomas method for determining horizontal flow patterns:

1.- Obtain the terminal velocity of the particles using the graph in Figure 5, and equation A:

$$v_t = (u_t{}^*)\left[\frac{\mu \Delta \rho g}{\rho^2}\right]^{\frac{1}{3}} \quad \left[\frac{m}{s}\right] \qquad (A)$$
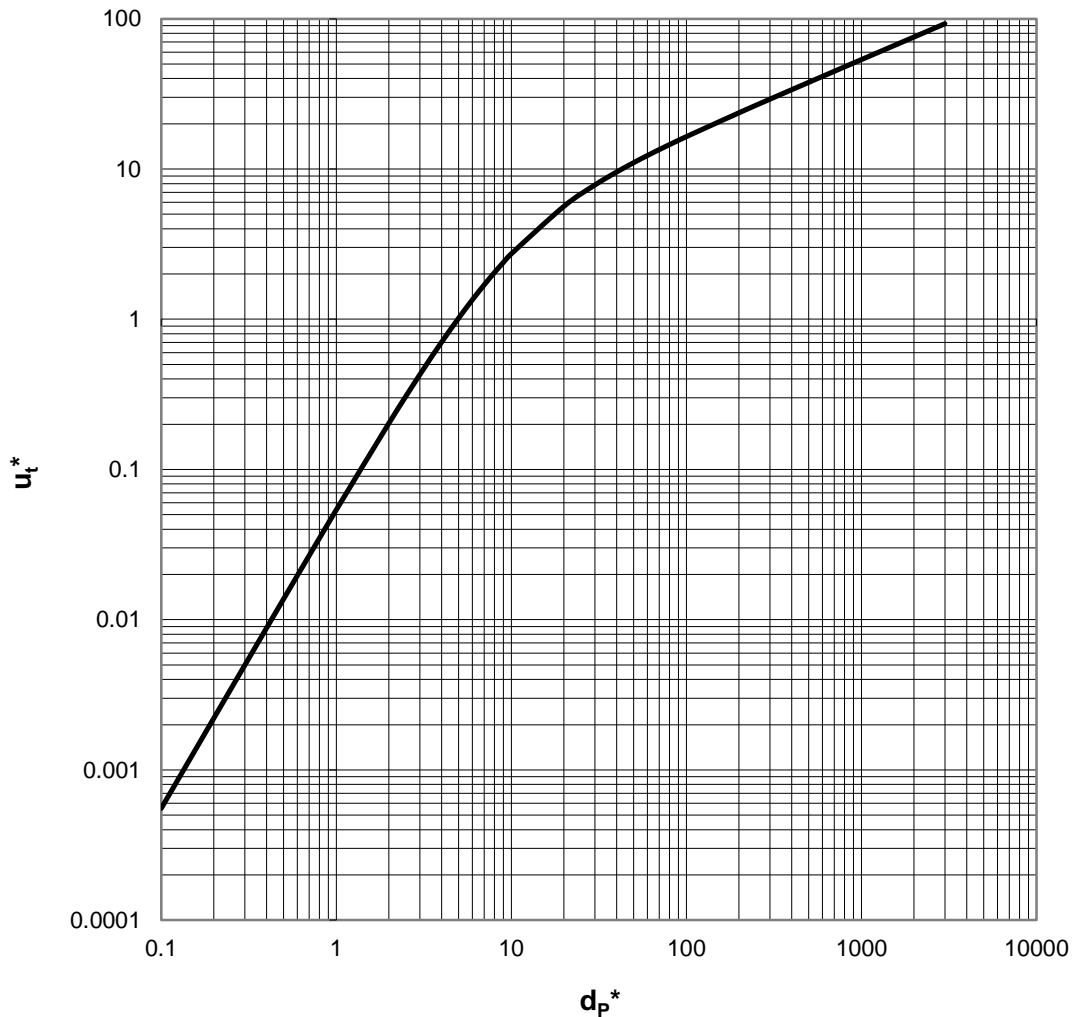

Figure 5.- Terminal speed of spherical particles in fluids.

2.- Calculate the superficial particle Reynolds:

$$Re_{SP} = \frac{d_P \, v_{SL} \, \rho_L}{\mu_L} \qquad (5)$$

.3.- Determine the friction factor of Fanning , using the superficial particle Reynolds and Moody diagram or, using the equations of Hagen-Poiseuille or Chen depending on the flow regimen (laminar or turbulent, respectively):

$$f_f = \frac{f_D}{4} \qquad (B)$$

Laminar flow:
$$f_D = \frac{64}{Re} \qquad (C)$$

Turbulent flow:

$$\frac{1}{\sqrt{f_D}} = -2\log\left[\frac{\varepsilon}{3.7065\,D} - \frac{5.0452}{Re}\log\left(\frac{1}{2.8257}\left(\frac{\varepsilon}{D}\right)^{1.1098} + \frac{5.8506}{Re^{0.8981}} \right) \right] \qquad (D)$$

4.- Calculate the frictional velocity at infinite dilution with the following equation:

$$v_{f0} = v_{SL}\sqrt{\frac{f_f}{2}} \qquad (6)$$

5.- Obtain the coordinates of Thomas' map with the following equations, and determine the flow pattern present in the pipe:

$$Abscisa = \frac{d_P \, v_{f0} \, \rho_L}{\mu_L} \qquad (7)$$

$$Ordenada = \frac{v_t}{v_{f0}} \qquad (8)$$

Example 1

Determine the flow pattern present in a 4-inch horizontal line 40 by which 1500 kg/h of water flows, at a temperature of 25°C, transporting sand particles with an average diameter of 175 μm and its density of 2650 kg/m^3.

1.-TRANSLATION

$W_L = 1500$ kg/h   AGUA – ARENA

2.-Planning.

2.1.-Discussion

To find the flow pattern, the Parameters of Thomas must be known, which will be used in your flow pattern map in Figure 4.

2.2.-Thomas coordinates

$$Abscisa = \frac{d_P \, v_{f0} \, \rho_L}{\mu_L} \qquad\qquad Ordenada = \frac{v_t}{v_{f0}}$$

3.-CALCULATIONS

3.1.-Terminal particle speed .

The properties of water at 25°C are:

$\rho_L = 997.08$ kg/m³ $\mu_L = 0.8937$ cp $= 8.937 \times 10^{-4}$ kg/(m s)

$$d_P^* = \left(175 \times 10^{-6} \text{ m}\right)\left[\frac{\left(997.08 \frac{kg}{m^3}\right)(2650 - 997.08)\frac{kg}{m^3}\left(9.81 \frac{m}{s^2}\right)}{\left(8.937 \times 10^{-4} \frac{kg}{ms}\right)^2}\right]^{\frac{1}{3}} = 4.77$$

This adimensional diameter value is obtained from the terminal velocity plot of particles in fluids (Figure 5):

$u_t^* = 0.95$

$$v_t = \left(0.95\right)\left[\frac{\left(8.937 \times 10^{-4} \frac{kg}{ms}\right)(2650 - 997.08)\frac{kg}{m^3}\left(9.81 \frac{m}{s^2}\right)}{\left(997.08 \frac{kg}{m^3}\right)^2}\right]^{\frac{1}{3}} = 0.023 \frac{m}{s}$$

3.2.-Frictional speed to infinite dilution

For a pipe of 4" nominal diameter ced 40, its internal diameter is:

$D = 4.026$ in $= 0.1023$ m

$$A = \frac{\pi}{4}D^2 = 0.008213 \text{ m}^2$$

$$v_{SL} = \frac{1500 \frac{kg}{h}}{3600 \frac{s}{h}\left(997.08 \frac{kg}{m^3}\right)(0.008213 \text{ m}^2)} = 0.051 \frac{m}{s}$$

$$Re_{SP} = \frac{\left(175 \times 10^{-6} \text{ m}\right)\left(0.051 \frac{m}{s}\right)\left(997.08 \frac{kg}{m^3}\right)}{8.937 \times 10^{-4} \frac{kg}{ms}} = 9.96 \qquad \text{laminar flow}$$

$$f_D = \frac{64}{9.96} = 6.43$$

$$f_f = \frac{6.43}{4} = 1.61$$

$$v_{f0} = 0.051 \frac{m}{s}\sqrt{\frac{1.61}{2}} = 0.046 \frac{m}{s}$$

3.3.- Thomas coordinates.

$$\text{Abscisa} = \frac{\left(175 \times 10^{-6}\, m\right)\left(0.046\, \frac{m}{s}\right)\left(997.08\, \frac{kg}{m^3}\right)}{8.937 \times 10^{-4}\, \frac{kg}{ms}} = 8.98$$

$$\text{Ordenada} = \frac{0.023\, \frac{m}{s}}{0.046\, \frac{m}{s}} = 0.50$$

With these coordinates, the flow pattern corresponding to the intersection of these values is located on Thomas's map (Figure 4), the latter being observed in the flow region with longitudinal dunes.

4.-RESULT

The pattern developed in the pipe corresponds to the flow with longitudinal dunes.

3.- PREDICTION OF PRESSURE DROP IN HORIZONTAL PIPES

To complete the process of sizing hydraulic particle transport pipes, it is necessary to determine the pressure drop in them. There is currently no general model capable of correctly predicting pressure losses in liquid-solid systems. Although empirical correlations have been developed to calculate pressure drops, none of them apply to other systems outside the range of experimental data with which the correlation was built. Correlations based on theoretical considerations better predict pressure losses in pipes, including molerus and Wellmann [8], who using dimensional analysis and based on a broad spectrum of experimental data, developed the best semi-empirical model to date for pressure drops in horizontal pipes.

3.1.- Molerus-Wellman correlation

In general, the total pressure drop in liquid-solid systems is the sum of pressure drops for each of the phases, and is expressed as follows:

$$\Delta P_{2F} = \Delta P_L + \Delta P_P$$

Where:

ΔP_{2F} = Two-Phase total pressure drop kgf/m^2.

ΔP_L = Liquid phase pressure drop kgf/m^2.

ΔP_P = Solid phase pressure drop kgf/m^2.

The first term of this equation can be determined by the Darcy equation:

$$\Delta P_L = \frac{f_L \rho_L v_M^2 L}{2 D g_C} \quad \left[\frac{kgf}{m^2}\right] \tag{9}$$

f_L = Darcy friction factor for the liquid phase.

v_M = Mixture mean velocity. m/s:

$$v_M = \frac{v_{SL} + v_{SP}}{2} \tag{10}$$

L = lenght of the pipe en m.

D = pipe diameter m.

g_C = 9.81 m kg/(s² kgf).

v_{SL} = liquid phase surfase velocity m/s:

$$v_{SL} = \frac{W_L}{3600\, \rho_L\, A} \quad \left[\frac{m}{s}\right] \tag{11}$$

v_{SP} = solid phase surfase velocity m/s:

$$v_{SP} = \frac{W_P}{3600 \, \rho_P \, A} \left[\frac{m}{s} \right] \qquad (12)$$

Molerus and Wellmann developed an expression to determine the pressure drop for the solid phase:

$$\Delta P_P = \frac{(x^*)\varphi(\rho_P - \rho_L)gL}{g_C}\left(\frac{v_M}{v_t}\right)^2 \left[\frac{kgf}{m^2}\right] \qquad (13)$$

Where: x* = adimensional pressure drop (Molerus-Wellmann parameter).
Φ = fraction volume of solids relative to the total volume of mixture:

$$\varphi = \frac{\dfrac{W_P}{\rho_P}}{\dfrac{W_L}{\rho_L} + \dfrac{W_P}{\rho_P}} \qquad (14)$$

To get the value of the x* parameter, Molerus and Wellmann defined three variables, the result of their dimensional analysis:

$$Fr_t = \frac{v_t}{\sqrt{gD(S-1)}} \qquad (15)$$

$$Fr_P = \frac{v_M}{\sqrt{gd_P(S-1)}} \qquad (16)$$

$$x_0 = \frac{\left(\dfrac{v_{SLIP}}{v_M}\right)^2_0}{1 - \left(\dfrac{v_{SLIP}}{v_M}\right)_0} \qquad (17)$$

Where: Frt =number of Froude terminal. Fr_P = Particle Froude number. S - Density Ratio:

$$S = \frac{\rho_P}{\rho_L} = \frac{1}{De} \qquad (18)$$

Xo - dimension pressure drop independent of concentration. $(V_{slip} / V_M)0$ - dimensional sliding speed independent of the concentration of the solids. V_{slip} = sliding speed between phases (slip velocity) in m/s:

$$v_{SLIP} = v_L - v_P \left[\frac{m}{s}\right] \qquad (19)$$

V_L = actual liquid speed at m/s. V_P - actual particle speed in m/s.

The slip velocity is, as in all two-phase flow systems, the main parameter in the transport mechanism and energy dissipation, since the drag force exerted by the fluid on the particle (or drop in the case of gas-liquid systems) depends on the relative speed between the two phases. The fluid always advances faster than the particles, causing some friction between the phases and, therefore, a loss of kinetic energy.

In order to determine the slip speed, Molerus and Wellmann produced a graph where this speed is a function of the terminal and particle Froude numbers, as shown in Figure 6. The x* parameter depends on the volumetric fraction . φ . as follows:

$0 \le \varphi \le 0.25$: $\qquad\qquad\qquad x^* = x_0 \qquad (20)$

$\varphi > 0.25$: $\qquad\qquad\qquad x^* = x_0 + 0.1Fr_t^2(\varphi - 0.25) \qquad (21)$

3.2.- Molerus-Wellmann method:

1.- Calculate the terminal and particle Froude numbers with equations 15 and 16.

2.- Determine the dimensional sliding speed using the graph in Figure 6.

3.- Calculate the parameters x_0 and φ with equations 17 and 14, respectively.

4.- Get the x* parameter with equations 20 and 21.

5.- Calculate the pressure drop of the solid phase with equation 13.

6.- Calculate the pressure drop of the liquid phase with equation 9.

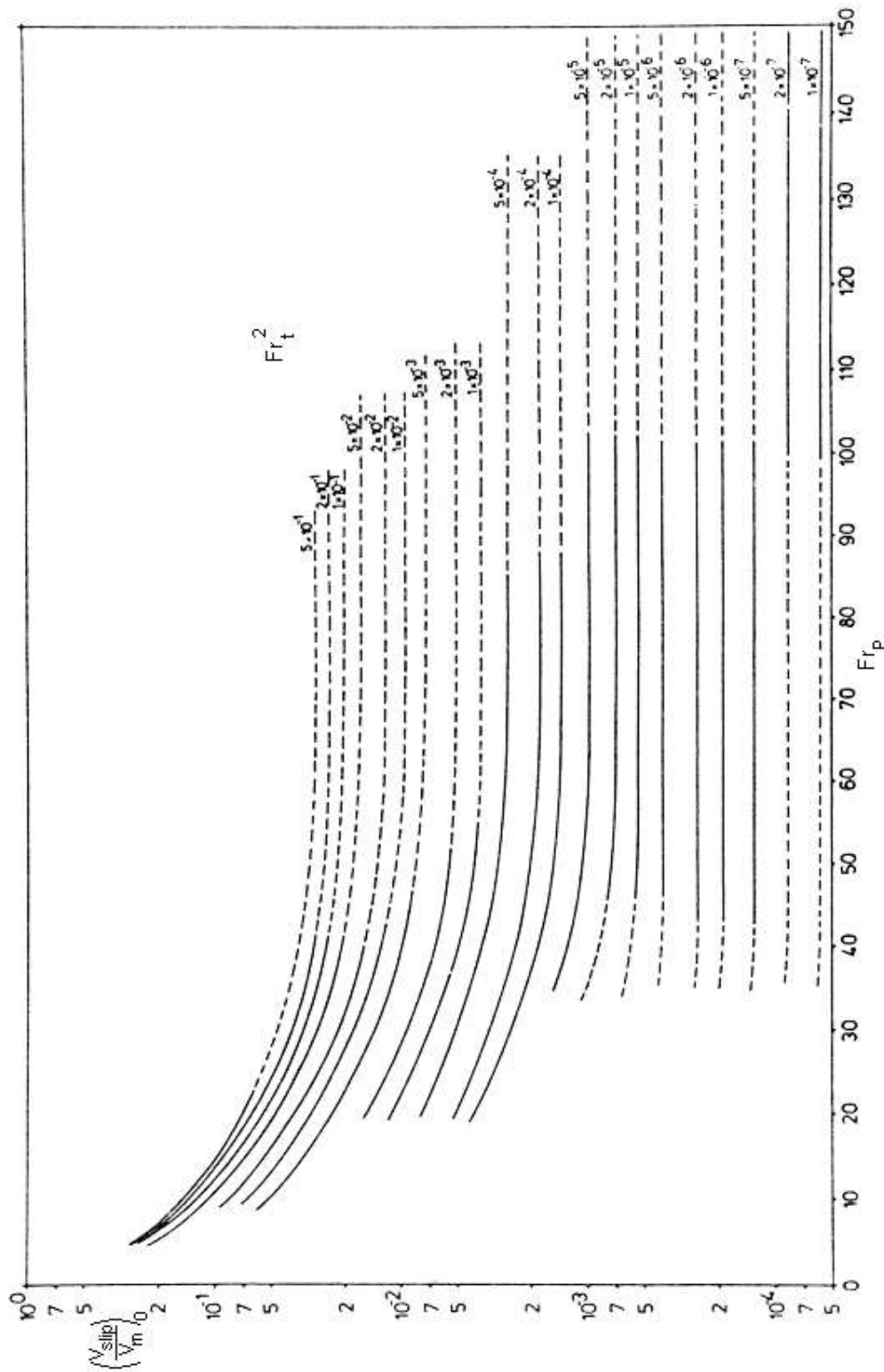7.- Determine the total pressure drop with equation 8.

Figure 6.- Molerus-Wellmann graph for determining the dimensional slip speed. (1981)

Example 2

Get total pressure drop in a 6-inch horizontal pipe that transports mineral carbon particles at a rate of 30000 kg/h. These particles are dragged by 100000 kg/h of water at 20°C, having an average diameter of 5200 μm and a density of 1270 kg/m³. The length of the pipe is 350 m.

1.-TRANSLATION

W_L = 100000 kg/h

Water – Coal

W_P = 30000 kg/h

6" Ced. 40   L = 350 m
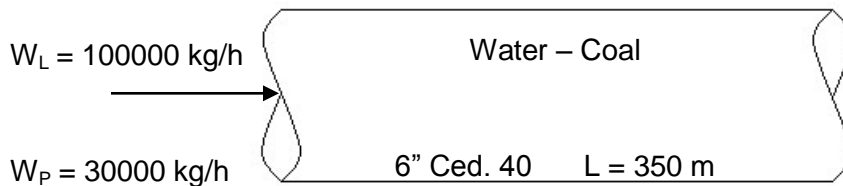
2.-Planning.

2.1.-Discussion

To obtain the pressure drop in the line the Molerus-Wellmann method is used, which does not need the determination of the flow pattern.

3.-CÁLCULS

3.1.-Dimensional sliding speed

The water properties at 20°C are:

ρ_L = 998.23 kg/m³

μ_L = 1.005 cp = 1.005x10⁻³ kg/(m s)

d_P^* = 71.82

This adimensional diameter value is obtained from the terminal velocity plot of particles in fluids (Figure 5):

u_t^* = 13

v_t = 0.181 m/s

$$S = \frac{1270 \frac{kg}{m^3}}{998.23 \frac{kg}{m^3}} = 1.27$$

For a 6" nominal diameter pipe 40, its internal diameter is:

D = 6.065 in = 0.1541 m

A = 0.0186388 m²

$$v_{SL} = \frac{100000\,\frac{kg}{h}}{3600\,\frac{s}{h}\left(998.23\,\frac{kg}{m^3}\right)\left(0.0186388\,m^2\right)} = 1.49\,\frac{m}{s}$$

$$v_{SP} = \frac{30000\,\frac{kg}{h}}{3600\,\frac{s}{h}\left(1270\,\frac{kg}{m^3}\right)\left(0.0186388\,m^2\right)} = 0.35\,\frac{m}{s}$$

$$v_M = \frac{1.49\,\frac{m}{s} + 0.35\,\frac{m}{s}}{2} = 0.92\,\frac{m}{s}$$

$$Fr_P = \frac{0.92\,\frac{m}{s}}{\sqrt{9.81\,\frac{m}{s^2}\left(5200\times10^{-6}\,m\right)(1.27-1)}} = 7.84$$

$$Fr_t = \frac{0.181\,\frac{m}{s}}{\sqrt{9.81\,\frac{m}{s^2}\left(0.1541m\right)(1.27-1)}} = 0.283$$

$$Fr_t^2 = (0.283)^2 = 8\times10^{-2}$$

With the Particle Froude (Fr $_P$) and the square of the Froude terminal (Fr$_t^2$), , from the Molerus-Wellmann chart (Figure 6) you get:

$$\left(\frac{v_{SLIP}}{v_M}\right)_0 = 0.15$$

3.2.-Solid phase pressure drop.

$$x_0 = \frac{(0.15)^2}{1-0.15} = 0.0265$$

$$\varphi = \frac{\dfrac{30000\,\dfrac{kg}{h}}{1270\,\dfrac{kg}{m^3}}}{\dfrac{100000\,\dfrac{kg}{h}}{998.23\,\dfrac{kg}{m^3}} + \dfrac{30000\,\dfrac{kg}{h}}{1270\,\dfrac{kg}{m^3}}} = 0.191 < 0.25$$

x* = 0.0265

$$\Delta P_P = 0.0265\,(0.191)(1270 - 998.23)\frac{kg}{m^3}(350\,m)\left(\frac{0.92\,\dfrac{m}{s}}{0.181\,\dfrac{m}{s}}\right)^2 = 12438.5\,\frac{kgf}{m^2}$$

3.3.-Liquid phase pressure drop.

$$Re_{SL} = \frac{0.1541m\left(1.49\,\dfrac{m}{s}\right)\left(998.23\,\dfrac{kg}{m^3}\right)}{1.005 \times 10^{-3}\,\dfrac{kg}{m\,s}} = 228062 \qquad \text{turbulent flow}$$

$\varepsilon/D = 0.0003$

$f_L = f_D = 0.0175$

$$\Delta P_L = \frac{0.0175\left(998.23\,\dfrac{kg}{m^3}\right)\left(0.92\,\dfrac{m}{s}\right)^2(350\,m)}{2(0.1541m)\left(9.81\dfrac{kg\,m}{kgf\,s^2}\right)} = 1711.6\,\frac{kgf}{m^2}$$

3.4.-Total pressure drop.

$$\Delta P_{2F} = 1711.6\,\frac{kgf}{m^2} + 12438.5\,\frac{kgf}{m^2} = 14150.1\,\frac{kgf}{m^2}$$

4.-RESULT

The total pressure drop is 14150.1 kgf/m^2.

Considerations

This method does not consider the flow pattern present in the line, as the theoretical basis used by these researchers implicitly carries the appearance or absence of sediment at the bottom of the pipe. The Molerus-Wellmann method predicts pressure drops to two liquid-solid phases in horizontal pipes with a maximum error of $\pm\ 10\%$, and is based on experimental data whose intervals are as follows: :25 mm $\leq$ D $\leq$ 315 mm (1'' $\leq$ D $\leq$ 12''), 12 μm, $\leq d_P \leq$ 5200 μm, 1270 kg/m$^3 \leq \rho_P \leq$ 5250 kg/m^3.

There are other methodologies for calculating pressure drops in horizontal flow to two liquid-solid phases. The interested reader can consult the correlations of Durand-Condolios [5], Newitt-Richardson-Abbott-Turtle [4], Condolios-Chapus [9], Rose-Duckworth [10], Turian-Yuan [6], among others.

4.-FLOW PATTERNS IN VERTICAL PIPES

Unlike the gas-solid flow, the existing patterns in the vertical and descending liquid-solid biphasic flow are practically the same, due to the great influence of the viscosity and density of the liquid on the fluid-particle interactions, which dominate along with particle-particle interactions over those existing between the particles and the pipe wall , as mentioned above. The only difference between the two types of flow is the actual movement of the particles, for this reason the upstream patterns will be exposed separately from those corresponding to the downstream.

4.1.- Upstream vertical flow patterns

There are three patterns of which one is two-phase flow and the other two are fluidization (Figure 7). The latter, along with the fixed bed, are exposed only to provide a broader view of the phenomenon. Fixed in increasing order of liquid velocity, flow patterns are:

Fixed bed

As seen in gas-solid systems, this flow rate is not a real flow pattern to two liquid-solid phases, nor fluidization, as the particles remain motionless while the liquid ascends through the gaps between them. This type of bed is presented at surface fluid rates lower than the minimum fluidization velocity. It is also called packed bed.

Particulate fluidization

The bed expands as the surface velocity of the liquid increases. The liquid phase flows through the interstitials between the particles, which increase in size evenly with expansion. In this way, there are no inhomogeneities in the bed, such as bubbles or liquid bullets inside. It is given at surface rates of liquid greater than the minimum fluidization and lower than the terminal speed of descent of the particles (terminal settling velocity).

Aggregative fluidization

In this pattern of fluidization, inhomogeneities appear in the bed, such as the formation of liquid bubbles within the bed of solid particles, giving it an appearance of being boiling. Liquid bubbles are similar to those of their gaseous counterpart, in the fluidization with bubbling in gas-solid systems, as they have a station of particles in their immediate lower portion. Liquid bullets can also occur, similar to bullet flow in gas-solid systems, or the appearance of stretch marks or liquid bands, which move upwards. This fluidization regime occurs only with particles belonging to the group whose fluidization is of this type, and at surface rates of liquid greater than the minimum fluidization but less than the terminal rate of descent of the particles.

Hydraulic transport

This is the only real two-phase flow pattern observed in vertical uplines, as it is characterized by the dragging of particles from the bottom portion of the pipe to the top. At low liquid speeds, solid particles are evenly dispersed throughout the pipe flow area. At higher liquid rates, particles tend to flow preferably through the center of the tube, causing the formation of a liquid ring, which contains a small amount of particles in its sinus. It occurs at higher surface liquid speeds at the terminal rate of particle descent.

Fixed bed   Particulate fluidization   Aggregative fluidization   Hydraulic transport
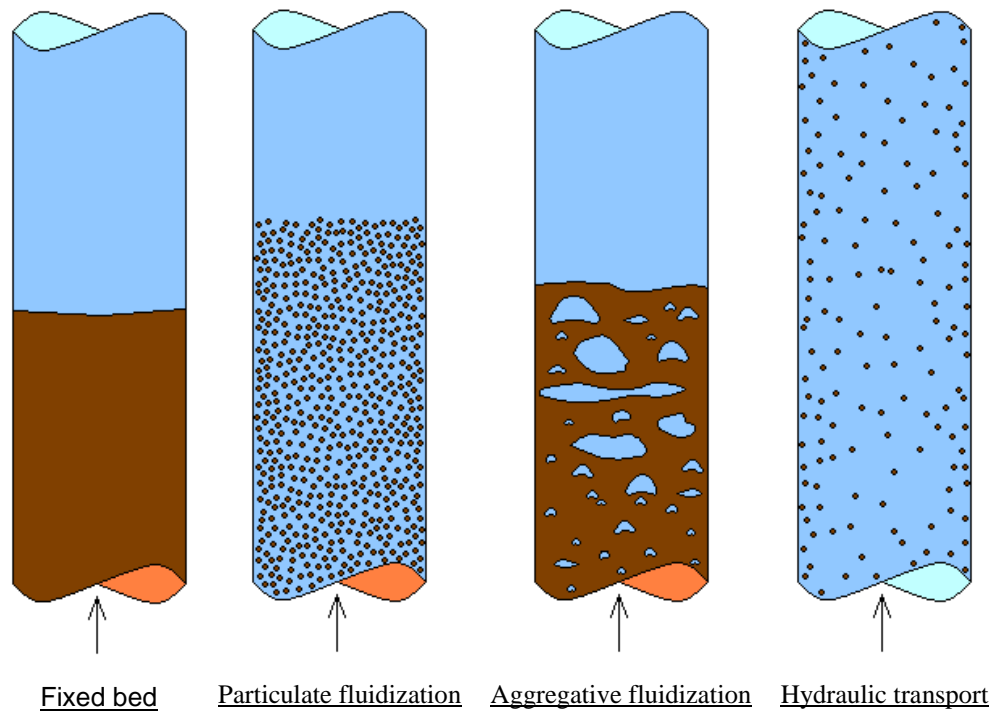
Figure 7.- Upstream patterns to two phases liquid-solid system in vertical pipes.

4.2.- Downstream vertical flow patterns

This type of flow is rare but can be found in lines coming from storage tanks and drains. Fixed in increasing order of sliding speed between phases, flow patterns are as follows (Figure 8):

Packed bed flow

The particle descent is as a packed bed, where the liquid flows through the interstitials between the particles. Occurs at lower sliding speeds at the minimum relative fluidization speed.

Fluidized bed flow

This flow pattern is characterized by the decrease of solid particles as a bed with particulate fluidization. It is similar to its counterpart in gas-solid systems, and occurs at faster sliding speeds than the minimum relative fluidization speed.

Aggregate fluidized bed flow

Is characterized by the downstream flow of a particle mass as a bed with aggregative fluidization. It is similar to the bubbling fluidized bedflow of gas-solid systems, but occurs only with particles whose fluidization is aggregative, at sliding speeds greater than the minimum relative fluidization rate.

Packed bed flow   Fluidized bed flow   Aggregate fluidized bed flow

.
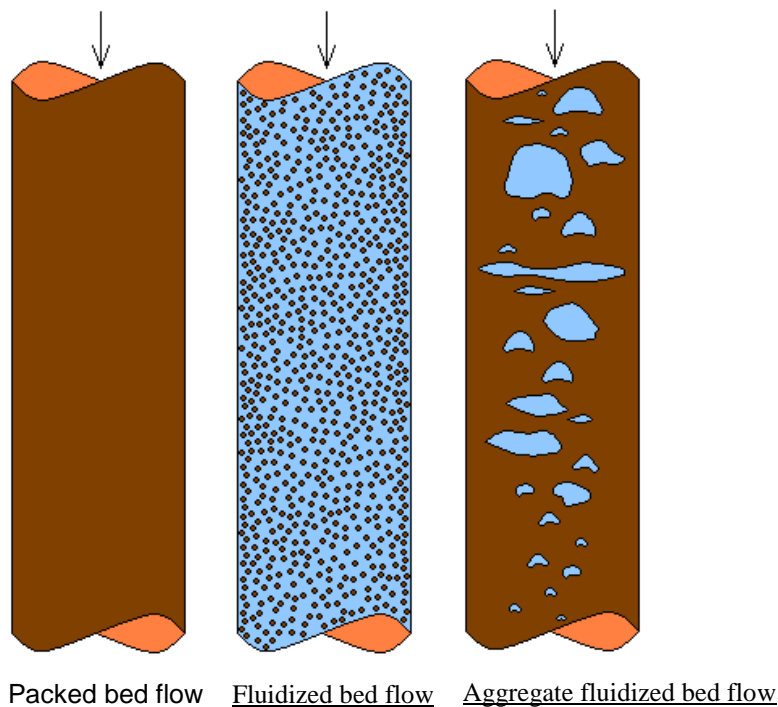
Fig. 8.- Downward flow patterns to two phases liquid-solid system in vertical pipes.

5.- PREDICTION OF FLOW PATTERNS IN VERTICAL PIPES

Thanks to the enormous similarity between the gas-solid and liquid-solid systems, the study and characterization carried out through the use of the same type of variables and correlations used in the case of gas-solid flow. One of the first attempts in this regard was made by Creasy[11], who proposed a map with dimensional coordinates where he made a distinction between particulate and aggregative fluidizations. Later, Molerus [12] produced a map where the fixed and fluidized beds are located, whose coordinates are well-known dimensional groups.

Resuming the Molerus map, Grace [13] built her own by using the particle's dimensional diameter and the dimensional velocity of the fluid. This map (Figure 9) is similar to that corresponding for gas-solid upstream vertical flow, and as the most general is shown below.

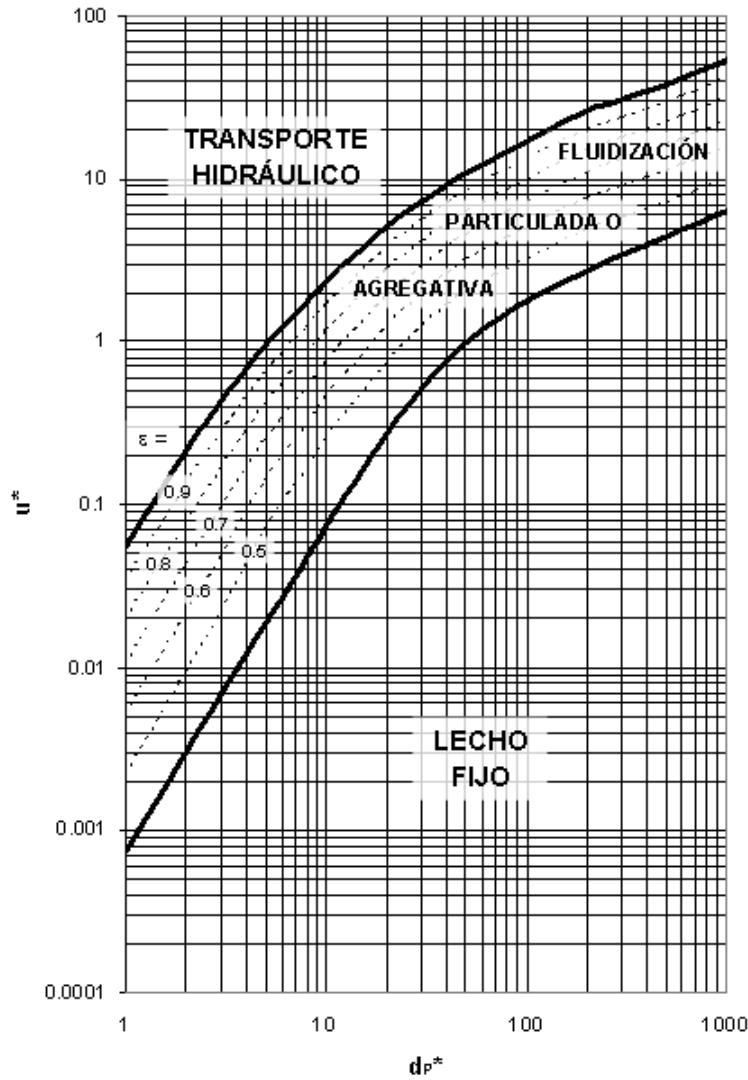**Figure 9.- Grace pattern map for upstream vertical flow in liquid-solid systems. (1986)**

Where:

d_P^* = Particle adimensional diameter .

u^* = adimensional surface liquid velocity:

$$u^* = v_{SL}\left[\frac{\rho_L^2}{\mu_L\left(\rho_P - \rho_L\right)g}\right]^{\frac{1}{3}} = Ly^{\frac{1}{3}} = \frac{Re_{SP}}{Ar^{\frac{1}{3}}} \qquad (22)$$

Ly = Lyaschenko number or similarity number (M):

$$\mathrm{Ly} = \frac{v_{SL}^3\, \rho_L^2}{\mu_L\left(\rho_P - \rho_L\right)g} \qquad\qquad (\,23\,)$$

To determine the type of fluidization (particulate or aggregative), any of the particle classification maps (Figures 1 and 2) should be used, depending on the type of fluid used in the system. In this map, the transition boundaries between flow or fluidization patterns do not depend on the mass velocity of the solid phase, nor on the diameter of the pipe. The transition criterion for the boundary between the fixed bed and the fluidized bed is the minimum fluidization speed, and the transition criterion for the boundary between the fluidized bed and hydraulic transport is the terminal speed.

Below is a generalized map of downstream vertical flow patterns (Figure 10), based on Grace's transition criteria and those proposed by Rhodes for fluid-solid systems. The ordering on this map is the difference in surface speeds (or sliding speed) between the two phases, and is defined by the following equation:

$$\left|v_{SL} - v_{SP}\right|^* = \left|v_{SL} - v_{SP}\right|\left[\frac{\rho_L^2}{\mu_L\left(\rho_P - \rho_L\right)g}\right]^{1/3} \qquad\qquad (\,24\,)$$

Similar to Figure 9, to distinguish bed flows with particulate fluidization and bed with aggregative fluidization, any of the particle classification maps shown in Figures 1 and 2 should be used. If the system is water-solid, Figure 1 is used, and if it is any other system, the Di Felice map is used (Figure 2). In the map in Figure 10, the transition criterion between the flow in the packed bed and the fluidized bed corresponds to the minimum speed of fluidization, but whose ordering is given by equation 24.
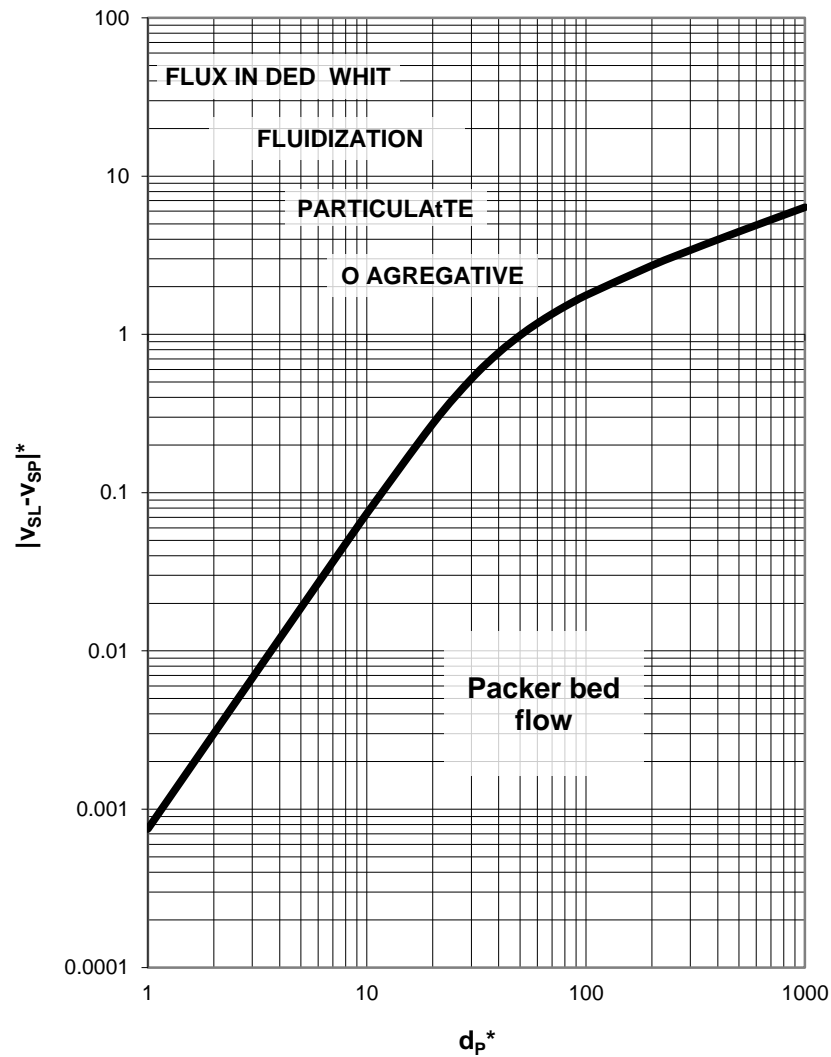
Figure 10.- Generalized pattern map for downstream vertical flow in liquid-solid systems. (2004)

Example 3

What will be the expected flow pattern in a 4-inch vertical pipe 40 by which 1000 kg/h of water rises through a bed formed by wheat grains? The water temperature is 25°C, the average diameter of the grains is 4.8 mm and the density of these is 750 kg/m^3.

1.-Translation

W_L = 1000 kg/h   AGUA – TRIGO
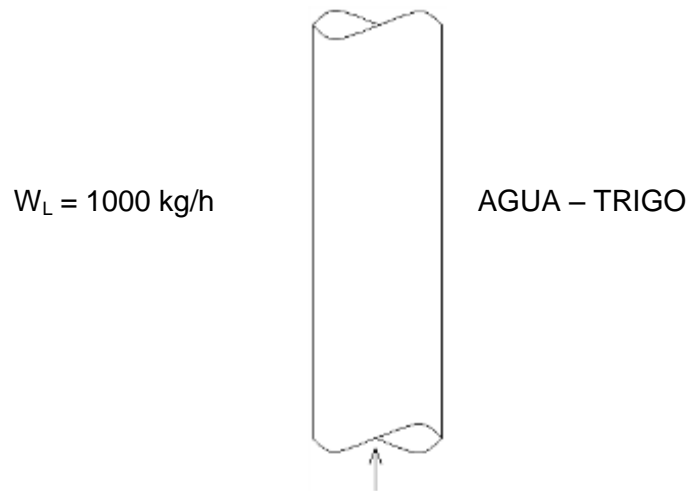
2.-Planning.

2.1.-Discussion The flow pattern is determined using the Grace map for liquid-solid systems (Figure 9).

2.2.-Grace coordinates

$$d_P{}^* = d_P \left[\frac{\rho_L \left(\rho_P - \rho_L \right) g}{\mu_L^2} \right]^{\frac{1}{3}} \qquad u^* = v_{SL} \left[\frac{\rho_L^2}{\mu_L \left(\rho_P - \rho_L \right) g} \right]^{\frac{1}{3}}$$

3.-CÁLCULS

3.1.-Suface liquid velocity.

The properties of water at 25°C are:

ρ_L = 997.08 kg/m^3

μ_L = 0.8937 cp = 8.937x10^{-4} kg/(m s)

For a pipe of 4" nominal diameter card 40, its internal diameter is: D = 4.026 in = 0.1023 m

A = 0.008213 m^2

$$v_{SL} = \frac{1000 \, \frac{kg}{h}}{3600 \, \frac{s}{h} \left(997.08 \, \frac{kg}{m^3} \right) \left(0.008213 \, m^2 \right)} = 0.0339 \, \frac{m}{s}$$

3.2.-Flow pattern.

$$d_P{}^* = \left(4.8 \times 10^{-3}\,m\right)\left[\frac{997.08\,\frac{kg}{m^3}\left(997.08 - 750\right)\frac{kg}{m^3}\left(9.81\,\frac{m}{s^2}\right)}{\left(8.937 \times 10^{-4}\,\frac{kg}{ms}\right)^2}\right]^{1/3} = 69.43$$

$$u^* = \left(0.0339\,\frac{m}{s}\right)\left[\frac{\left(997.08\,\frac{kg}{m^3}\right)^2}{\left(8.937 \times 10^{-4}\,\frac{kg}{ms}\right)\left(997.08 - 750\right)\frac{kg}{m^3}\left(9.81\,\frac{m}{s^2}\right)}\right]^{1/3} = 2.61$$

In the Grace map in Figure 9, these coordinates intersect in the region corresponding to the fluidization. In order to determine the type of fluidization present in the wheat bed, the particle sorting Map will then be used (Figure 2):

$$Ar = \left(69.43\right)^3 = 334689$$

$$De = \frac{997.08\,\frac{kg}{m^3}}{750\,\frac{kg}{m^3}} = 1.33$$

According to the map of Di Felice, the fluidization of the bed is particulate type.

4.-RESULT

The pattern developed in the bed is that of particulate fluidization.

Example 4

Determine the flow pattern present in a 6-inch vertical pipe by 40 inches by which 500 kg/h of mineral coal and 300 kg/h of water descend to 25°C. Carbon particles have an average diameter of 12.7 mm and a density of 720 kg/m^3.

1.-TRANSLATION

W_P = 500 kg/h

W_L = 300 kg/h

Water – Coal

2.-Planning.

2.1.-Discussion

To obtain the flow pattern the generalized pattern map for downstream vertical flow is used (Figure10).

3.-CALCULATIONS

3.1.-Surface speed of the liquid phase

The properties of water at 25°C are:

ρ_L = 997.08 kg/m³

μ_L = 0.8937 cp = 8.937x10⁻⁴ kg/(m s)

For a 6" nominal diameter pipe 40, its internal diameter is:

D = 6.065 in = 0.1541 m

A = 0.0186388 m²

$$v_{SL} = \frac{300\,\frac{kg}{h}}{3600\,\frac{s}{h}\left(997.08\,\frac{kg}{m^3}\right)\left(0.0186388\,m^2\right)} = 0.0045\,\frac{m}{s}$$

.2.-Solid phase surface velocity.

$$v_{SP} = \frac{500\,\frac{kg}{h}}{3600\,\frac{s}{h}\left(720\,\frac{kg}{m^3}\right)\left(0.0186388\,m^2\right)} = 0.0103\,\frac{m}{s}$$

3.3.-Flow pattern.

$$d_P{}^* = \left(12.7 \times 10^{-3}\, m\right) \left[\frac{997.08\,\frac{kg}{m^3}\left(997.08 - 720\right)\frac{kg}{m^3}\left(9.81\,\frac{m}{s^2}\right)}{\left(8.937 \times 10^{-4}\,\frac{kg}{ms}\right)^2}\right]^{\frac{1}{3}} = 190.84$$

$$\left|v_{SL} - v_{SP}\right|^* = \left|0.0045\,\frac{m}{s} - 0.0103\,\frac{m}{s}\right| \left[\frac{\left(997.08\,\frac{kg}{m^3}\right)^2}{\left(8.937 \times 10^{-4}\,\frac{kg}{ms}\right)\left(997.08 - 720\right)\frac{kg}{m^3}\left(9.81\,\frac{m}{s^2}\right)}\right]^{\frac{1}{3}} = 0.43 \text{ In}$$

the generalized downstream vertical flow map in Figure 10, these coordinates intersect in the region corresponding to the packed bed flow.

4.-RESULT

The pattern is the flow in packed bed.

6.- PREDICTION OF PRESSURE DROP IN VERTICAL PIPES

Pressure drop in vertical pipes with flow to two liquid-solid phases, as in the gas-solid flow, depends on the flow pattern. The semi-empirical correlations most commonly used in the design of pipes and equipment will then be exposed, according to the flow pattern. Hydraulic transport In vertical pipes, according to Kopko, Barton and McCormick [14], the total pressure drop is given by the following equation:

$$\Delta P_{2F} = \frac{\varepsilon \rho_L\, v_L^2}{2\,g_C} + \frac{(1-\varepsilon)\rho_P\, v_P^2}{2\,g_C} + \frac{\varepsilon \rho_L\, L\, g\, sen\theta}{g_C} + \frac{(1-\varepsilon)\rho_P\, L\, g\, sen\theta}{g_C} + \frac{f_L\, v_L^2\, \rho_L}{2D\,g_C} \quad \left[\frac{kgf}{m^2}\right]$$

(25)

Where:

ε = void fraction or a liquid holdup. ; θ = the tilt angle of the pipe.

The first two terms correspond to the acceleration pressure drop, the next two terms are elevation pressure drops, and the last is frictional pressure loss between the liquid phase and the pipe walls. The inclination angle for upstream is 90 degrees and for downflow is -90 degrees or 270 degrees. It should be noted the absence of a term corresponding to the drop in frictional pressure between the solid particles and the walls of the pipe. Newitt and collaborators [15] studied total friction pressure losses and concluded that frictional losses are slightly higher than those caused by friction between the liquid and the tube wall, when the liquid speed is low and the terminal velocity of the particles is in transitional or turbulent regime. For very high liquid speeds, the frictional pressure drop between the particles and the pipe wall is identical to the frictional pressure drop between the liquid and the wall

Newitt and his collaborators discovered a migration of particles to the pipe's axis of symmetry when the liquid speed is very high. In this way, a liquid ring is formed which flows over the inner walls of the pipe and, therefore, prevents

direct contact of the particles with the pipe wall. As a result, frictional pressure losses between solids and internal walls of the line are negligible from the total drop in frictional pressure.

The actual speeds of the liquid and solid phases are calculated using the following equations:

$$v_L = \frac{v_{SL}}{\varepsilon} \quad \left[\frac{m}{s}\right] \qquad (26)$$

$$v_P = \frac{v_{SP}}{1-\varepsilon} \quad \left[\frac{m}{s}\right] \qquad (27)$$

The fraction of gaps or holdup of the liquid phase can be determined by using equation 4:

$$\varepsilon = \frac{Q_L}{Q_L + Q_P} = \frac{\dfrac{W_L}{\rho_L}}{\dfrac{W_L}{\rho_L} + \dfrac{W_P}{\rho_P}} \qquad (4)$$

Where: Q_L and Q_P - volumetric flows of the liquid and solid phases in m^3/h.
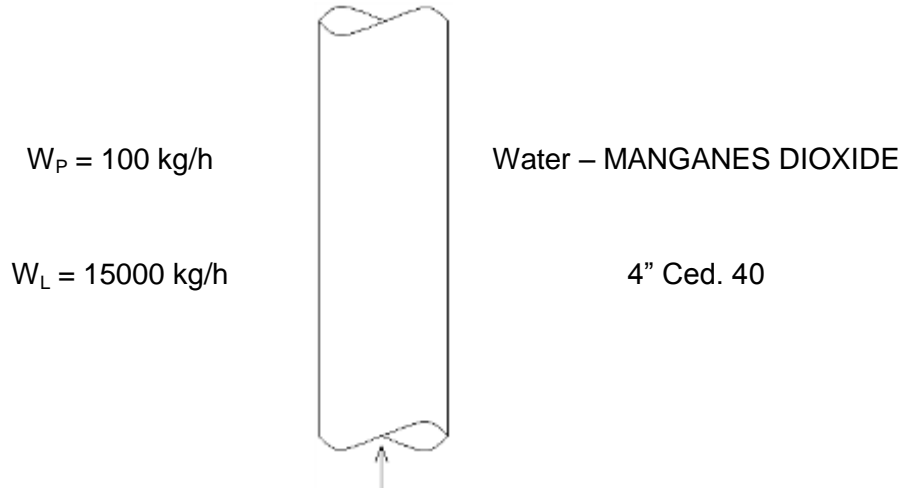
6.1.- <u>Kopko-Barton-McCormick method</u>

1.- Determine the flow pattern using Grace's map (Figure 9). If the flow type determined corresponds to hydraulic transport, the method must be continued.

2.- Calculate the fraction of gaps or liquid holdup with equation 4.

3.- Calculate the actual speeds of both phases with equations 26 and 27.

4.- Calculate the Darcy friction factor of the liquid using the Moody graph or with the Hagen-Poiseuille or Chen equations, depending on the flow rate (laminar or turbulent, respectively).

5.- Determine the total pressure drop with equation 25.

Example 5

By a vertical pipe of 4 inches 40 ascend 15000 kg/h of water at 30oC, and 100 kg/h of manganese dioxide particles, which have an average diameter of 1.57 mm and a density of 4100 kg/m3. Calculate the total pressure drop if the pipe length is 25 m.

1.-TRANSLATION

W_P = 100 kg/h   Water – MANGANES DIOXIDE

W_L = 15000 kg/h   4" Ced. 40

2. Planning

2.1.-Discussion

To determine the flow pattern, Grace's map for liquid-solid systems (Figure 9) and to calculate the pressure drop on the line will be used, the Kopko-Barton-McCormick method shall be used, provided that the flow pattern corresponds to hydraulic transport.

3.-CALCULATIONS

3.1.-Flow pattern

The properties of water at 30°C are:

$\square \rho_L$ = 995.68 kg/m^3

$\square \mu_L$ = 0.8007 cp = 8.007x10^{-4} kg/(m s)

For a pipe of 4" nominal diameter card 40, its internal diameter is:

D = 4.026 in = 0.1023 m

A = 0.008213 m^2

v_{SL} = 0.51 m/s

d_P^* = 56.78

u* = 17.54

On Grace's map in Figure 9, these coordinates indicate the presence of a hydraulic transport inside the pipe.

3.2.-Actual velocity of each phase.

$$v_L = \frac{0.51\frac{m}{s}}{0.9984} = 0.511\frac{m}{s}$$

$$v_{SP} = \frac{100\frac{kg}{h}}{3600\frac{s}{h}\left(4100\frac{kg}{m^3}\right)\left(0.008213\,m^2\right)} = 8.249\times10^{-4}\frac{m}{s}$$

$$v_P = \frac{8.249\times10^{-4}\frac{m}{s}}{1-0.9984} = 0.516\frac{m}{s}$$

3.3.- Total pressure drop.

$$Re_L = \frac{0.1023\,m\left(0.511\frac{m}{s}\right)\left(995.68\frac{kg}{m^3}\right)}{8.007\times10^{-4}\frac{kg}{m\,s}} = 65005 \qquad \text{turbulent flow}$$

From Moody:

ε/D = 0.00045

$f_L = f_D = 0.021$

$$\Delta P_{2F} = \frac{0.9984\left(995.68\frac{kg}{m^3}\right)\left(0.511\frac{m}{s}\right)^2}{2\left(9.81\frac{kg\,m}{kgf\,s^2}\right)} + \frac{(1-0.9984)\left(4100\frac{kg}{m^3}\right)\left(0.516\frac{m}{s}\right)^2}{2\left(9.81\frac{kg\,m}{kgf\,s^2}\right)}$$

$$+0.9984\left(995.68\frac{kg}{m^3}\right)(25\,m)sen(90°)+(1-0.9984)\left(4100\frac{kg}{m^3}\right)(25\,m)sen(90°)$$

$$+ \frac{0.021\left(0.511\frac{m}{s}\right)^2\left(995.68\frac{kg}{m^3}\right)}{2\left(0.1023\,m\right)\left(9.81\frac{kg\,m}{kgf\,s^2}\right)}$$

$$\Delta P_{2F} = 25032.2\frac{kgf}{m^2}$$

4.-RESULT

The total pressure drop is 25032.2 kgf/m^2.

Although fixed and fluidized beds are beyond the scope of this article, as they do not constitute a true two-phase flow, they will be briefly exposed for the sole purpose of providing greater elements for the design of liquid-solid systems. To delve deeper into the subject of particle beds, the reader is advised to consult the works of Davidson and Harrison [27], of Kunii and Levenspiel [24], by Leva [23], by Valiente Barderas[22], and the two of Rhodes [28,29]

7.- Fluidization

Similar to gas-solid systems, the total pressure drop for this regime has no acceleration contributions, and is then given by the equation proposed by Foscolo and Gilabaro [3]

$$\Delta P_{2F} = \frac{\left[\varepsilon\rho_L + (1-\varepsilon)\rho_P\right]g\,L}{g_C} \quad \left[\frac{kgf}{m^2}\right] \qquad (28)$$

The fraction of voids is calculated using the Richardson-Zaki equation [16], as the correlation obtained by these authors is only applicable for fixed and fluidized beds:

$$\varepsilon = \left(\frac{v_{SL}}{v_i}\right)^{1/n} \qquad (29)$$

Where: n =exponent of Richardson-Zaki. ; (Vi) terminal speed set to m/s:

$$v_i = v_t\,10^{-\frac{d_P}{D}} \quad \left[\frac{m}{s}\right] \qquad (30)$$

Rowe [17] developed an explicit equation to determine the value of this exponent, which is based on the Reynolds terminal and is given by the equation:

$$n = 2.35\left[\frac{2 + 0.175\,(Re_P)_t^{3/4}}{1 + 0.175\,(Re_P)_t^{3/4}}\right] \qquad (31)$$

Where: $(Re_P)t = $ terminal particle Reynolds:

$$(Re_P)_t = \frac{d_P\, v_t\, \rho_L}{\mu_L} \qquad (32)$$

These equations are applicable to particulate and aggregative fluidizations, for upstream flow. In the case of downflow, the fraction of gaps or liquid holdup for bed flows with particulate fluidization and aggregative fluidization, is calculated using equation 4, since they are not actually fluidized beds in which case the equations of Richardson-Zaki and Rowe would be valid:

$$\varepsilon = \frac{Q_L}{Q_L + Q_P} = \frac{\dfrac{W_L}{\rho_L}}{\dfrac{W_L}{\rho_L} + \dfrac{W_P}{\rho_P}} \qquad (33)$$

Example 6

Determine the pressure drop in an 8-inch vertical line 40 by which 100 kg/h of water rises through a bed consisting of 300 kg of uranium dioxide. The water temperature is 25°C, the depth of the bed is 2.5 m, the diameter of the particles is 152 μm and its density is 3520 kg/m^3.

1.-TRANSLATION

M_P = 300 kg

Water – Uranium dioxide

W_L = 100 kg/h
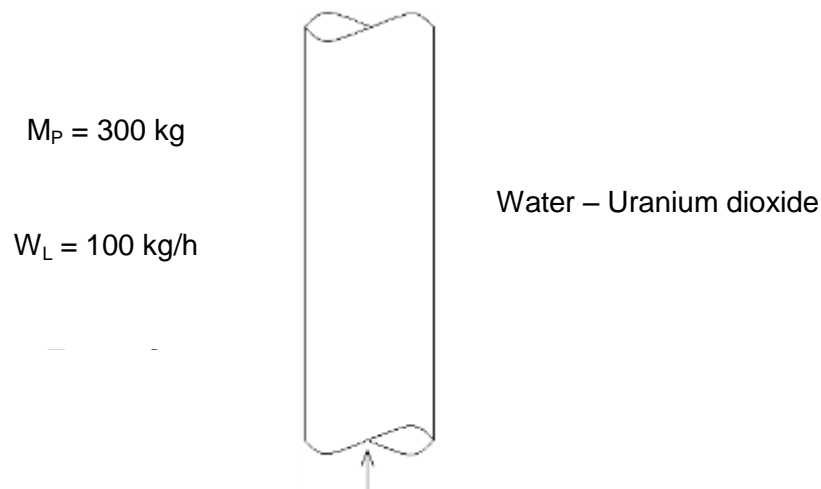
2.-Planning.

 2.1.-Discussion

The flow pattern is determined by the Grace chart in Figure 9, and the pressure drop is obtained using equation 28, as long as the pattern corresponds to that of fluidized bed.

 3.-CALCULATIONS

3.1.-Flow pattern

The properties of water at 25°C are:

ρ_L = 997.08 kg/m^3

μ_L = 0.8937 cp = 8.937x10^{-4} kg/(m s)

For a pipe of 8" nominal diameter card 40, its internal diameter is:

D = 7.981 in = 0.2027 m

A = 0.032275 m^2

v_{SL} = 8.632x10^{-4} m/s

d_P* = 4.77

u* = 0.031

On Grace's map, these coordinates intersect in the region corresponding to the fluidization. In order to determine the type of fluidization present in the bed of uranium dioxide particles, the particle classification Di Felice map must be used (Figure 2):

Ar = 108.5

De = 0.28

According to the latter map, the fluidization of the bed is particulate type.

3.2.-Total pressure drop

The terminal speed of the particles is obtained using Figure 5. With d_P* = 4.77, the following is observed:

u_t* = 1

v_t = 0.02813 m/s

$$(Re_P)_t = \frac{\left(152 \times 10^{-6}\,m\right)\left(0.02813\,\frac{m}{s}\right)\left(997.08\,\frac{kg}{m^3}\right)}{8.937 \times 10^{-4}\,\frac{kg}{ms}} = 4.77$$

$$v_i = \left(0.02813\,\frac{m}{s}\right)10^{-\frac{152\times10^{-6}\,m}{0.2027m}} = 0.02808\,\frac{m}{s}$$

$$n = 2.35\left[\frac{2+0.175\left(4.77\right)^{\frac{3}{4}}}{1+0.175\left(4.77\right)^{\frac{3}{4}}}\right] = 3.85$$

$$\varepsilon = \left(\frac{8.632 \times 10^{-4} \frac{m}{s}}{0.02808 \frac{m}{s}} \right)^{1/3.85} = 0.4048$$

$$\Delta P_{2F} = \left[0.4048 \left(997.08 \frac{kg}{m^3} \right) + (1 - 0.4048) \left(3520 \frac{kg}{m^3} \right) \right] (2.5\,m) = 6246.81 \frac{kgf}{m^2}$$

4.-RESULT

The total pressure drop along the fluidized bed is 6246.81 kgf/m^2.

8.- Fixed bed

Based on theoretical considerations, Gibilaro, Di Felice, Waldram and Foscolo [18] modified the Ergun [19] equation, to be applicable to liquid-solid systems, remaining as follows:

$$\frac{\Delta P}{L} = \left(\frac{17.3}{Re_{SP}} + 0.336 \right) \frac{\rho_L v_{SL}^2}{d_P g_C} (1 - \varepsilon) \varepsilon^{-4.8} \qquad \left[\frac{kgf/m^2}{m} \right] \qquad (33)$$

Where: e = void fraction. ; Re $_{SP}$ = Superficial Particle Reynolds:

$$Re_{SP} = \frac{d_P v_{SL} \rho_L}{\mu_L} \qquad (5)$$

The void fraction is determined by the equations of Richardson-Zaki (equations 29 and 30) and Rowe (equation 31), for fixed bed with gas upflow. The Gibilaro equation and collaborators can be used for downflow (packed bed flow) by replacing the surface velocity of the liquid with the relative or sliding speed between phases:

$$v_{rel} = \left| \frac{v_{SL}}{\varepsilon} - \frac{v_{SP}}{1 - \varepsilon} \right| \qquad \left[\frac{m}{s} \right] \qquad (34)$$

The void fraction for downstream vertical flow is obtained by equation 4, as it is not a true fixed bed:
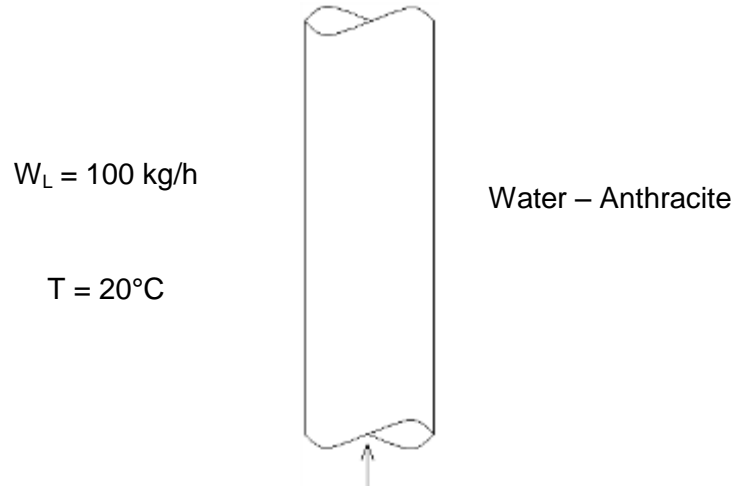
$$\varepsilon = \frac{Q_L}{Q_L + Q_P} = \frac{\frac{W_L}{\rho_L}}{\frac{W_L}{\rho_L} + \frac{W_P}{\rho_P}} \qquad (4)$$

Example 7

Get pressure drop in a 6-inch pipe 40, which is a packed bed of anthracite particles, whose depth is 2 m. The average particle diameter is 1.32 mm and its density is 1400 kg/m3. For the interstitials of the bed, 100 kg/h of water rises at 20°C.

1.-TRANSLATION

$W_L = 100$ kg/h

$T = 20°C$

Water – Anthracite

2.-Planning.

2.1.-Discussion

To confirm the existence of a packed bed, the Grace map will be used (Figure 9). The fraction of gaps will be obtained by the equations of Richardson-Zaki and Rowe. The pressure drop will be determined using the Gibilaro-Di Felice-Waldram-Foscolo equation.

3.-CALCULATIONS

3.1.-Flow pattern

The properties of water at 20°C are:

$\rho_L = 998.23$ kg/m^3

$\mu_L = 1.005$ cp $= 1.005 \times 10^{-3}$ kg/(m s)

For a 6" nominal diameter pipe 40, its internal diameter is:

$D = 6.065$ in $= 0.1541$ m

$A = 0.0186388$ m^2

$v_{SL} = 0.001493$ m/s

$d_P^* = 20.77$

$u^* = 0.0942$

On Grace's map in Figure 9, these coordinates check for the existence of the fixed bed inside the pipe.

3.2.-Fraction of gaps

The terminal speed of the particles is obtained using Figure 5. With $d_P^* = 20.77$, the following is observed:

$u_t^* = 5.7$

$v_t = 0.0903$ m/s

$$(Re_P)_t = \frac{\left(1.32 \times 10^{-3}\, m\right)\left(0.0903\, \frac{m}{s}\right)\left(998.23\, \frac{kg}{m^3}\right)}{1.005 \times 10^{-3}\, \frac{kg}{ms}} = 118.39$$

$$v_i = \left(0.0903\, \frac{m}{s}\right)10^{-\frac{1.32 \times 10^{-3}\, m}{0.1541m}} = 0.0885\, \frac{m}{s}$$

$$n = 2.35 \left[\frac{2 + 0.175\,(118.39)^{3\!/\!4}}{1 + 0.175\,(118.39)^{3\!/\!4}}\right] = 2.67$$

$$\varepsilon = \left(\frac{0.001493\, \frac{m}{s}}{0.0885\, \frac{m}{s}}\right)^{1\!/\!2.67} = 0.2168$$

3.3.- Total pressure drop.

$$Re_{SP} = \frac{\left(1.32 \times 10^{-3}\, m\right)\left(0.001493\, \frac{m}{s}\right)\left(998.23\, \frac{kg}{m^3}\right)}{1.005 \times 10^{-3}\, \frac{kg}{ms}} = 1.96$$

$$\Delta P = \left(\frac{17.3}{1.96} + 0.336\right)\frac{998.23\, \frac{kg}{m^3}\left(0.001493\, \frac{m}{s}\right)^2}{\left(1.32 \times 10^{-3}\, m\right)\left(9.81\, \frac{kg\,m}{kgf\,s^2}\right)}(1 - 0.2168)(0.2168)^{-4.8}(2m) = 3792.63\, \frac{kgf}{m^2}$$

4.-RESULT

The total pressure drop along the bed is 3792.63 kgf/m2.

The Kopko-Barton-McCormick method predicts pressure drops to two liquid-solid phases in vertical pipes with an error of no more than 20%. Gibilaro and collaborators' equation accurately predicts pressure losses in packaged beds.

There are other semi-empirical correlations for the calculation of pressure drops, among which it is recommended to review those of Newitt-Richardson-Gliddon [15], Condolios-Chapus [9], Aude et al. [20], among others.

GENERAL CONSIDERATIONS

In the design of liquid-solid flow systems, it is important to consider the erosion caused by the relentless collision of solid particles against the internal walls of equipment and lines. This phenomenon represents a great annual cost due to frequent replacement of equipment parts and fittings, such as pumps and valves, and pipe runs damaged by particle flow. Erosion in turn causes the phenomenon of corrosion to appear, due to the contact between the liquid and the metal exposed by erosion.

Erosion can be decreased by properly selecting the liquid velocity, which must be sufficiently higher than the sedimentation rate of the particles, corresponding to the transition rate between heterogeneous horizontal flow patterns and dunes, or at the terminal velocity for vertical pipes. In this way, corrosion on the eroded surface can be avoided. In addition, the flow pattern present in the pipe can be selected in order to set the surface speed of the liquid phase. Generally, a speed between 1 m/s and 2 m/s (4 ft/s and 7 ft/s) per economy is recommended. Erosion occurs at liquid speeds between 2.4 m/s and 3 m/s (8 and 10 ft/s), aggravating at higher speeds. For heterogeneous flow, surface rates of liquid greater than 2 m/s are suggested, but care should be taken with erosion. It is also advisable to transport the solid particles at a concentration of between 10% and 40% by volume.

For the sizing of horizontal pipes homogeneous flows with transverse dunes are recommended, because of their relative stability. Heterogeneous flows with longitudinal dunes should be avoided due to their high tendency to sedimentation, and therefore, because they are very erosive flow patterns. Also, mobile bed flow is not recommended, as it tends to block the lines. For vertical lines, the upstream hydraulic transport pattern and the three downflow patterns do not have major drawbacks.

As for the flow system [20],[21], long and extra-long radius elbows should be used to minimize erosion caused by changes in particle direction. It is recommended to use ball valves, which must be installed together with the necessary connections for draining and cleaning, thus avoiding the accumulation of solid particles in the valve. Pipes and transport lines are usually made of ordinary steel or some special alloy of steel, cast iron, rubber, plastic or rubber-coated steel or some other polymer. The material of the lines where the liquid flows at speeds greater than 4.5 m/s (15 ft/s) must be concrete or plastic, which better withstand erosion.

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