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STRONG CYLINDRICAL MAGNETOGASDYNAMIC SHOCK UNDER INFLUENCE OF RADIATION FLUX

Prof. (Dr.)Shailesh Nath Pandey
Department of Mathematics (Applied Science)
B.N.College of Engineering & Technology, Lucknow

Abstract

This paper deals with the strong cylindrical magnetogasdynamics shock under influence of radiation flux. The radiation pressure and radiation energy are considered to be small in comparison with gas pressure and gas energy respectively and therefore only radiation flux is taken into account in case of cylindrical symmetry. Runge- Kutta method has been employed to obtain the solution. The variations of pressure and radiation flux with distance have been illustrated through graphs for different values of radiation parameter. From figures it is very clear that the region of disturbance near the shock surface decreases with the increasing value of radiative parameters.

1-INTRODUCTION:

The effect of radiation transfer on high speed flow in optically thick medium has been studied extensively by several authors, e.g. Wang [1], Koch [2] , Helliwell [3] and others. Elliot [4] discussed the self-similar solution for spherical blast wave using Rosseland's diffusion approximation under the assumption that there is no effect of heat flux at the center of symmetry. Propagation of spherical blast waves in a transparent medium was carried out by Erickson and Olfe [5]. Koch and Gross [6]. and Oppenhein [7] studied propagation of strong shock wave in an optically thin atmosphere taking different radiation models.

Wang and Helliwell studied strong plane shocks in a thick atmosphere of uniform density using Plank's diffusion approximation. Unlike Wang and Helliwell, self-similar solutions for the propagation of plane shock waves have been obtained in transparent grey atmosphere of variable density under the influence of magnetic fields. The radiation pressure and radiation energy are considered to be small in comparison with gas pressure and energy respectively and therefore only radiation flux is taken into account in case of the cylindrical symmetry. Runge Kutta method has been employed to obtain the solution.

Ahead of the shock, the density distribution ρ_0 and magnetic field distribution h_0 is taken to vary as

$\rho_0 = \rho_0 r^{\alpha}$

$h_0 = h_c r^{\delta}$

where, ρc, hc, α and δ are constants.

2. EQUATIONS OF MOTION AND BOUNDARY CONDITIONS :

The equations of motion of a fluid having infinite electrical conductivity when expressed in cylindrical symmetry, symmetric Eulerian form with zero viscosity and thermal conductivity are,

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}+\rho\frac{\partial u}{\partial r}+\frac{\rho u}{r}=0 \quad (2.1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{h}{\rho}\frac{\partial h}{\partial r}+\frac{h^2}{\rho r}=0 \quad (2.2)$$

$$\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial r}+h\frac{\partial u}{\partial r}+\frac{hu}{r}=0 \quad (2.3)$$

$$\frac{1}{\gamma-1}\left[\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}\right]-\frac{\gamma p}{\rho(\gamma-1)}\left[\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}\right]+\frac{\partial F}{\partial r}=0 \quad (2.4)$$

where u is the material velocity, ρ the density, p the material pressure, h the component of magnetic field and F the radiative flux, all at a distance r from the plan of explosion at time t;, γ stands for the ratio of specific heats.

Also, assuming local thermodynamic equilibrium and taking Plank's diffusion approximation we have,

$$\frac{\partial F}{\partial r}=4\bar{k}_p\sigma T^4 \quad (2.5)$$

where $\overline{k_p}$ is Plank's mean absorption coefficient, σ the Stefan-Boltzmann constant, and T the absolute temperature.

As the gas is taken as ideal, the equation of state is given by

$$p = \rho R T$$

R being the gas constant.

We next take $\overline{k}$ p as a power-law function of the density and temperature as,

$$\overline{k_p} = \mu o \, \rho^{\alpha'} \, T^{\beta'} \qquad (2.6)$$

where μo, α' and β' are constants.

The jump conditions for a strong shock wave propagating in a perfectly conducting gas are,

$$\frac{u_1}{v} = \frac{2}{\gamma+1} \qquad (2.7)$$

$$\frac{\rho_1}{\rho_0} = \frac{\gamma+1}{\gamma-1} \qquad (2.8)$$

$$\frac{p_1}{\rho_0 v^2} = \frac{2}{\gamma+1} \qquad (2.9)$$

$$\frac{h_1}{h_0} = \frac{\gamma+1}{\gamma-1} \qquad (2.10)$$

where the suffices 0 and 1 denote the regions just out side and just inside the shock plane, respectively and v is the shock velocity.

3 SOLUTION OF THE EQUATIONS :

We seek solutions of the equations in the form,

$$u = \frac{r}{t} U(\eta)$$

$$\rho = r^{k} t^{\lambda} \Omega(\eta)$$

$$p = r^{k+2} t^{\lambda-2} p(\eta) \qquad (3.1)$$

$$h = r^{\frac{k+2}{2}} t^{\frac{\lambda-2}{2}} H(\eta)$$

$$F = r^{k+3} t^{\lambda-3} Y(\eta)$$

where $\eta = r^{a} \; t^{b}$ (3.2)

.

Where K, λ, a and b are constants and η is called as similarity constant .

We choose the shock fronts to be given by η_o = constant, its value depending on the constant amount of explosion energy.

By direct substitution of the form (3.1) in the equations of motion and shock conditions, we find that similarity conditions are compatible when $K = \alpha$, $\lambda = 0$, $a = -(\alpha+3)$, $b = 2$, $\delta = -½$, and $\beta' = 3/2$; α'-remaining arbitrary.

For our study however, we choose the particular value $\alpha' = 1$. This unable us to consider the mean free path of radiation to be adequately large when the density is small and temperature is reasonable high.

Therefore, the shock velocity is

$$v = \frac{2}{3+\alpha}\frac{d}{t} \qquad (3.3)$$

where d is the distance of shock front at time t from the location of the initial impulse. Next combining the equations (2.1), (2.2.) (2.3) & (2.4) we have,

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial r}[(E+p^*)u] + \frac{\partial F}{\partial r} = 0 \qquad (3.4)$$

where, $$E = \frac{p}{\gamma - 1} + \frac{1}{2}\rho u^2 + \frac{h^2}{2}$$

$$p^* = p + \frac{h^2}{2}$$

AS E is of the form $r^{\alpha+2} t^{-2} \phi(\eta)$, equation (3.4) can be written as,

$$-\frac{\partial}{\partial r}\left[\left(\frac{2}{3+\alpha}\right)\frac{r}{t}E\right] + \frac{\partial}{\partial r}[(E+p^*)u] + \frac{\partial F}{\partial r} = 0$$

which on integration, we get,

$$\frac{F}{\rho v_{3}} = \frac{1}{2} u'^{2}(r' - u') - \frac{2}{(\gamma + 1)_{2}} w^{2}(\gamma u' - r')$$

$$+\frac{1}{2} M_A^{-2}(r' - 2u') \qquad (3.5)$$

where, we have taken

$$w^2 = \frac{p}{p_1}\frac{\rho_1}{\rho}$$

$$u' = \frac{u}{v}$$

$$M_A^2 = \frac{\rho v^2}{h^2}$$

and, $$r' = \frac{r}{d}$$

M_A being the Alfven mach number.

Equation (3.5) helps us finally to calculate radiative flux F when the other flow variables have been determined.

After some manipulation, the following set of equations are obtained from (2.1) to (2.5)

$$\frac{dw^2}{dr'} = \frac{w^2}{r'-u'}[(\gamma-1)\frac{du'}{dr'} - (\alpha+1) + L(w^2)^{3/2}] \quad (3.6)$$

$$\frac{(\gamma+1)^2}{4(\gamma-1)}[(\alpha+1)u' + 2(r'-u')\frac{du'}{dr'}] + \frac{w^2}{(r'-u')} - \frac{\gamma w^2}{(r'-u')}\frac{du'}{dr'}$$

$$+\frac{(\gamma+1)^2}{4(\gamma-1)}\frac{M_A^{-2}}{(r'-u')}[1-2\frac{du'}{dr'}] - \frac{L(w^2)^{5/2}}{(r'-u')} = 0 \quad (3.7)$$

$$\frac{dB^2}{dr'} = M_A^{-2}[2(r'-u')]\frac{du'}{dr'} + (\alpha+1)u']$$

$$+\frac{4(\gamma-1)}{(\gamma+1)^2}\frac{M_A^{-2}W^2}{(r'-u')}[1-\frac{\gamma du'}{dr'} - L(w^2)^{3/2}] \quad (3.8)$$

$$(r'-u')\frac{ds}{dr'} + 1 - \frac{\gamma du'}{dr'} - L(w^2)^{3/2} = 0 \quad (3.9)$$

where we have putting,

$S = \log p / p_1$

$B = \log h / h_1$

and

$$L = \frac{32\sqrt{2}}{(3+\alpha)^2} \frac{\mu_0 \sigma (\gamma - 1)^{5/2}}{\eta_0 R^{5/2} d^{\alpha} (\gamma + 1)^3}$$

The symbol L represents the radiation parameter depending on the nature and optical properties of the medium, as well as the explosion energy.

4.RESULTS AND DISCUSSION:

Similarity solutions of the problem of propagation of strong cylindrical magnetogasdynamics shock wave in an optically thin atmosphere have been obtained. The numerical integration has been done by Runge-Kutta program. The variations of pressure, density, magnetic field, velocity, temperature and radiation flux with distance have been illustrated through graph for different values of radiation parameter L and for $\gamma = 4/3$, $M^{-2}_{A} = 10$.

Radiation parameter L has significant effect on pressure, represented by figure 1. Near the shock surface, pressure decreases rapidly to a maximum and then increases asymptotically in a narrow region; when L taken maximum value 20. But in case when L = 1, near the shock velocity, temperature and radiation flux is represented by figs. 4, 5, 6. It is very clear that the region of disturbance near the shock surface decreases with the increasing value of radiative parameters.

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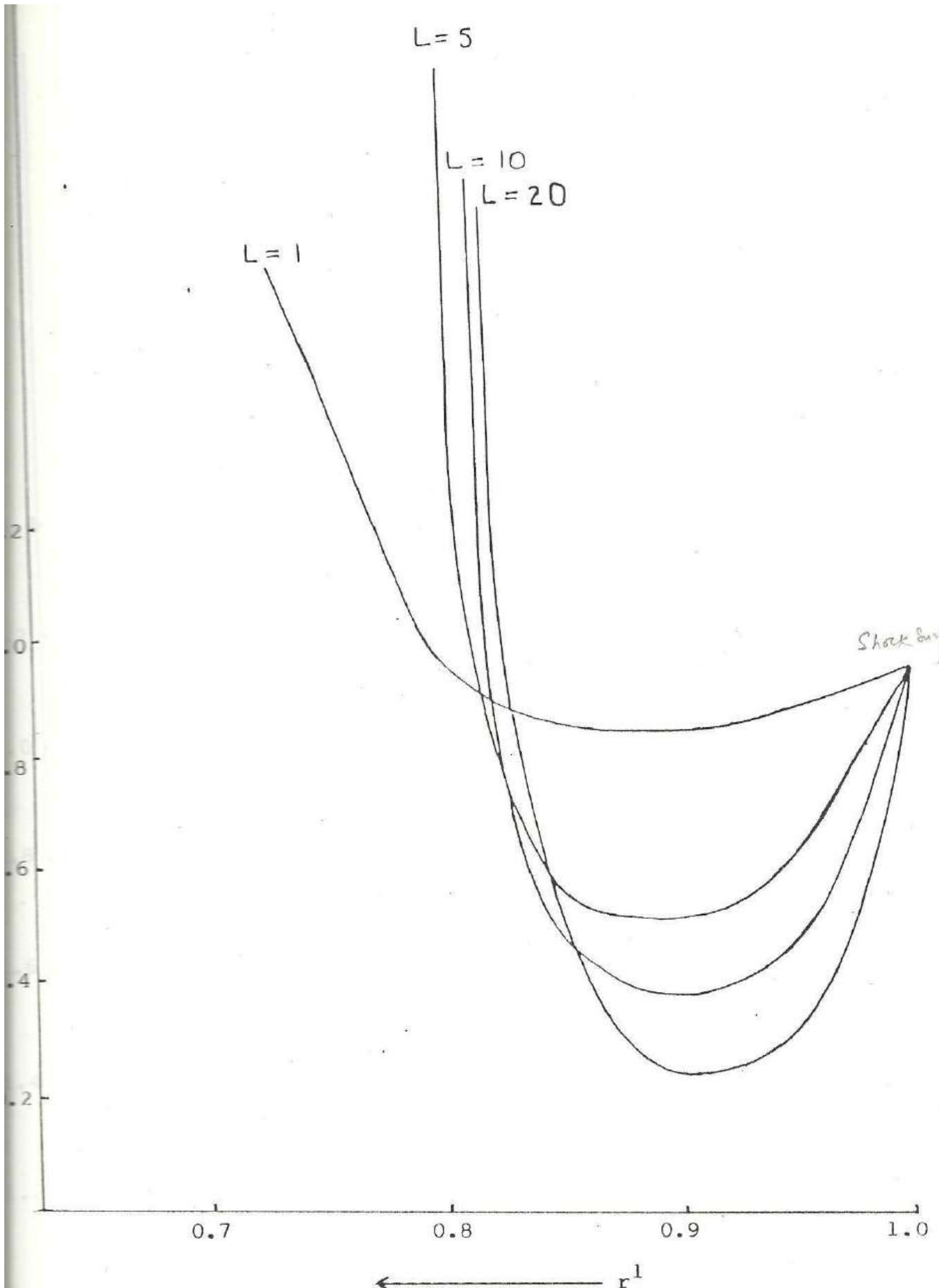

Fig. 1 Pressure distribution

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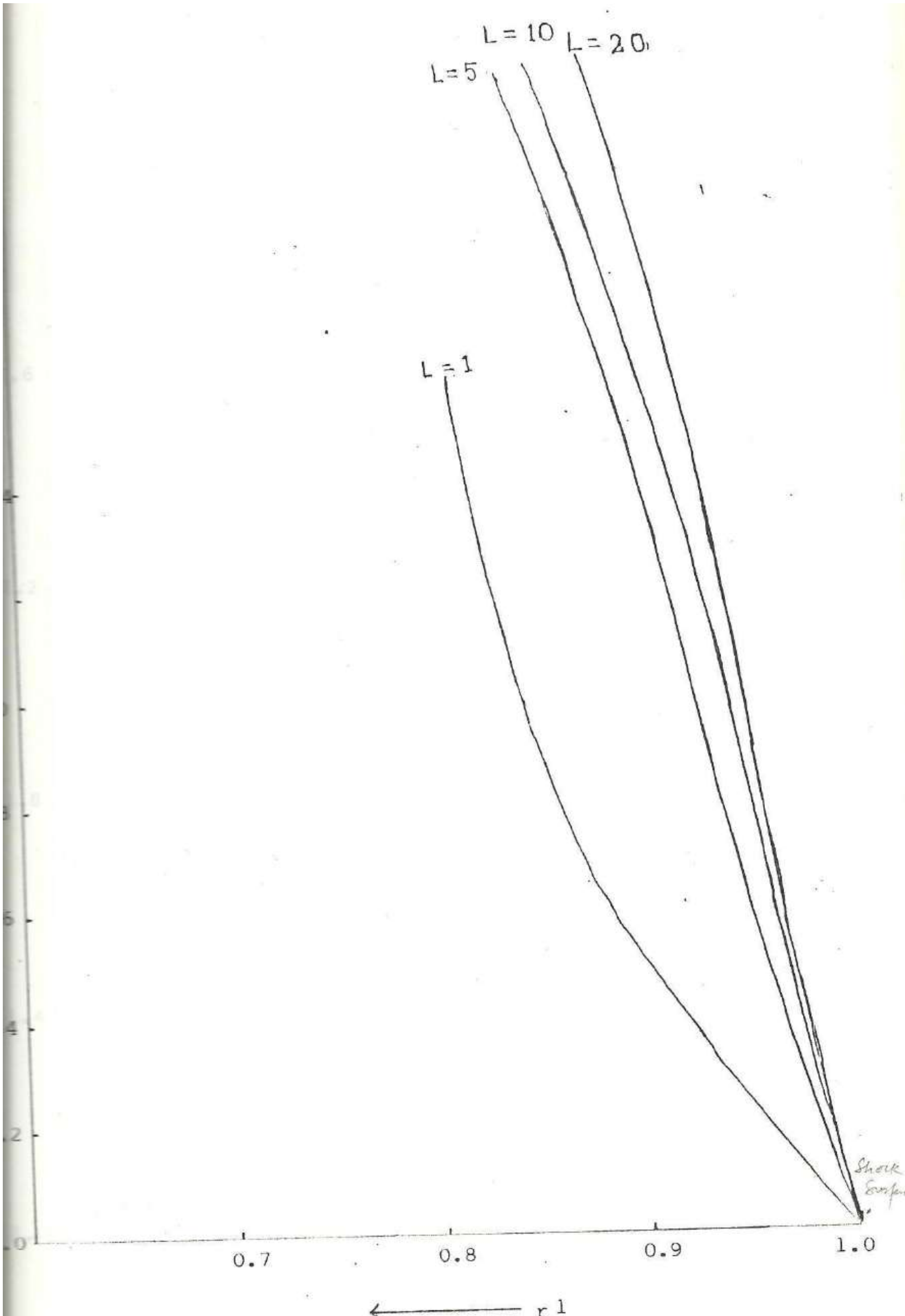

Fig. 2 Density distribution

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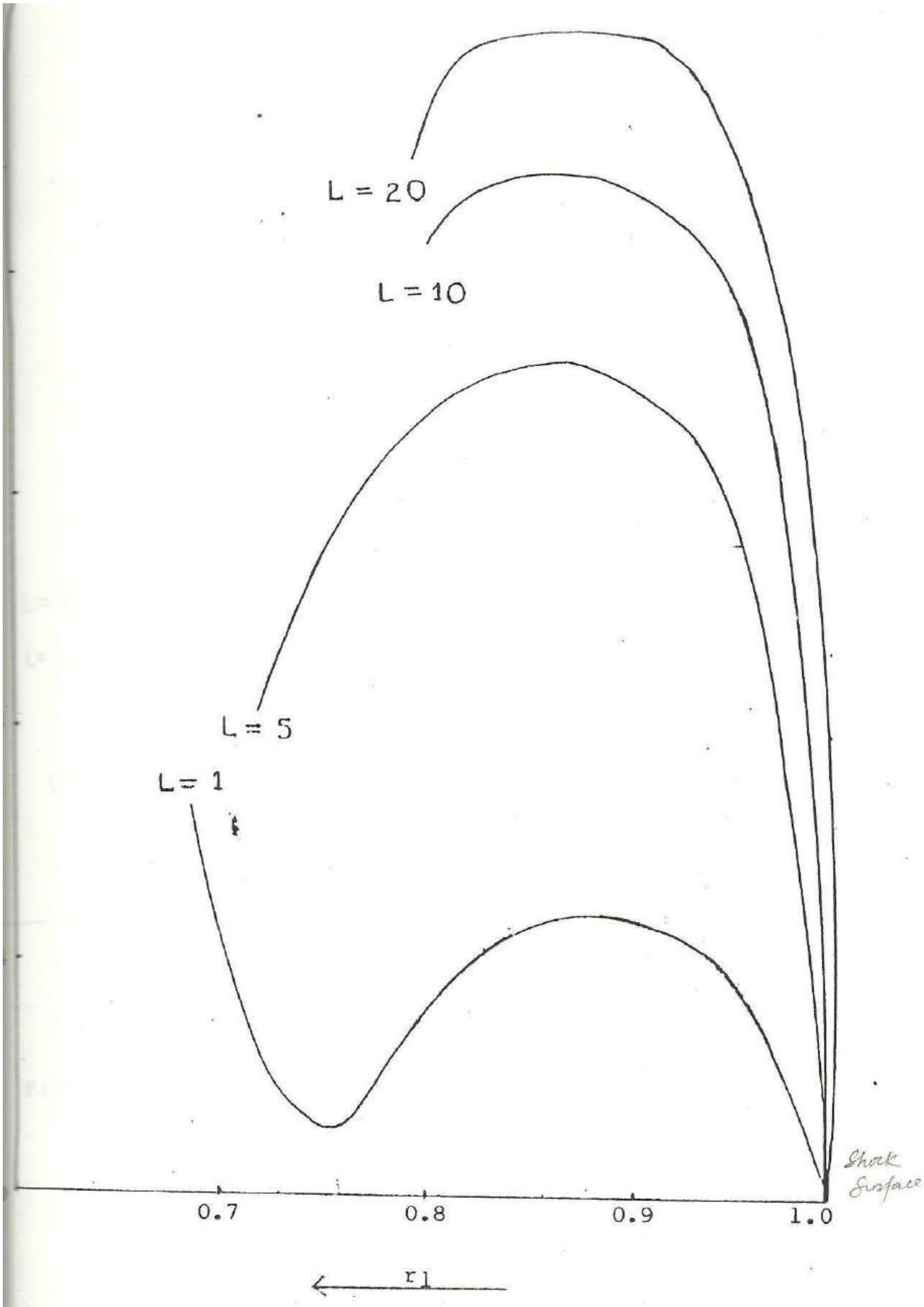

Fig. 3 Magnetic field distribution

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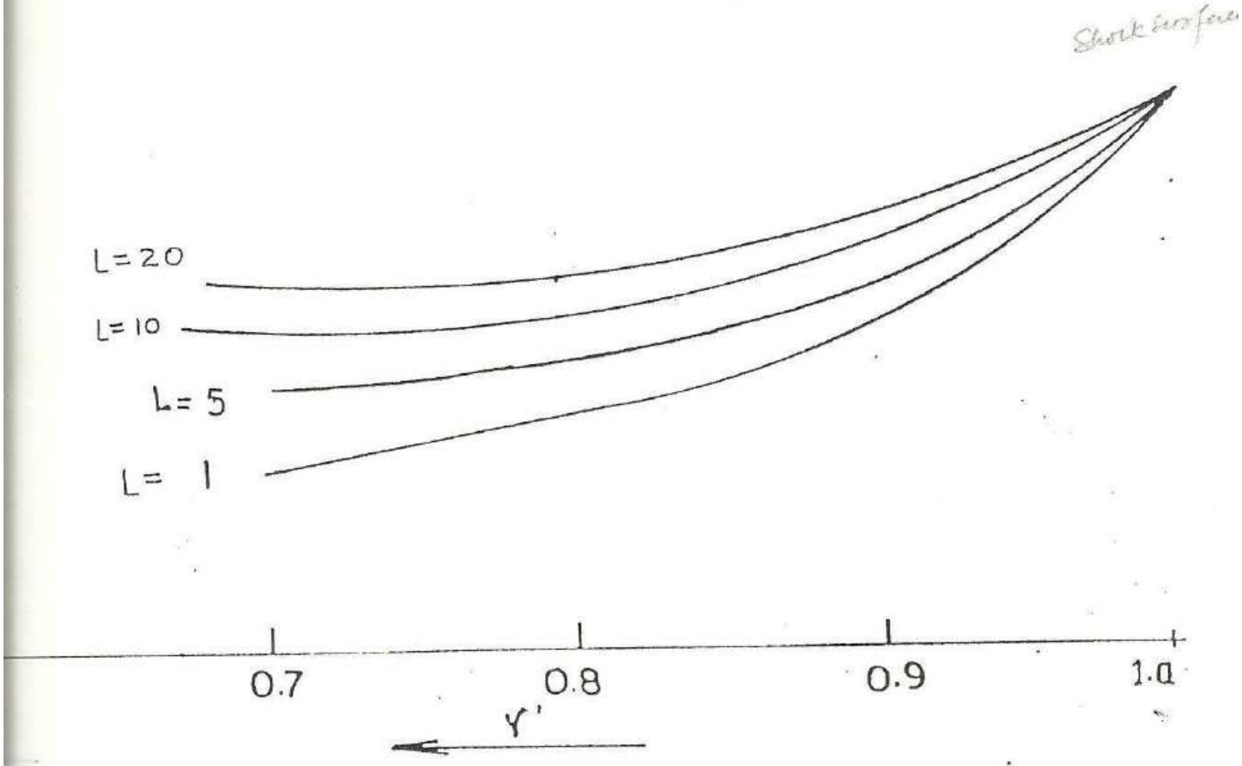

Fig. 4 Velocity distribution

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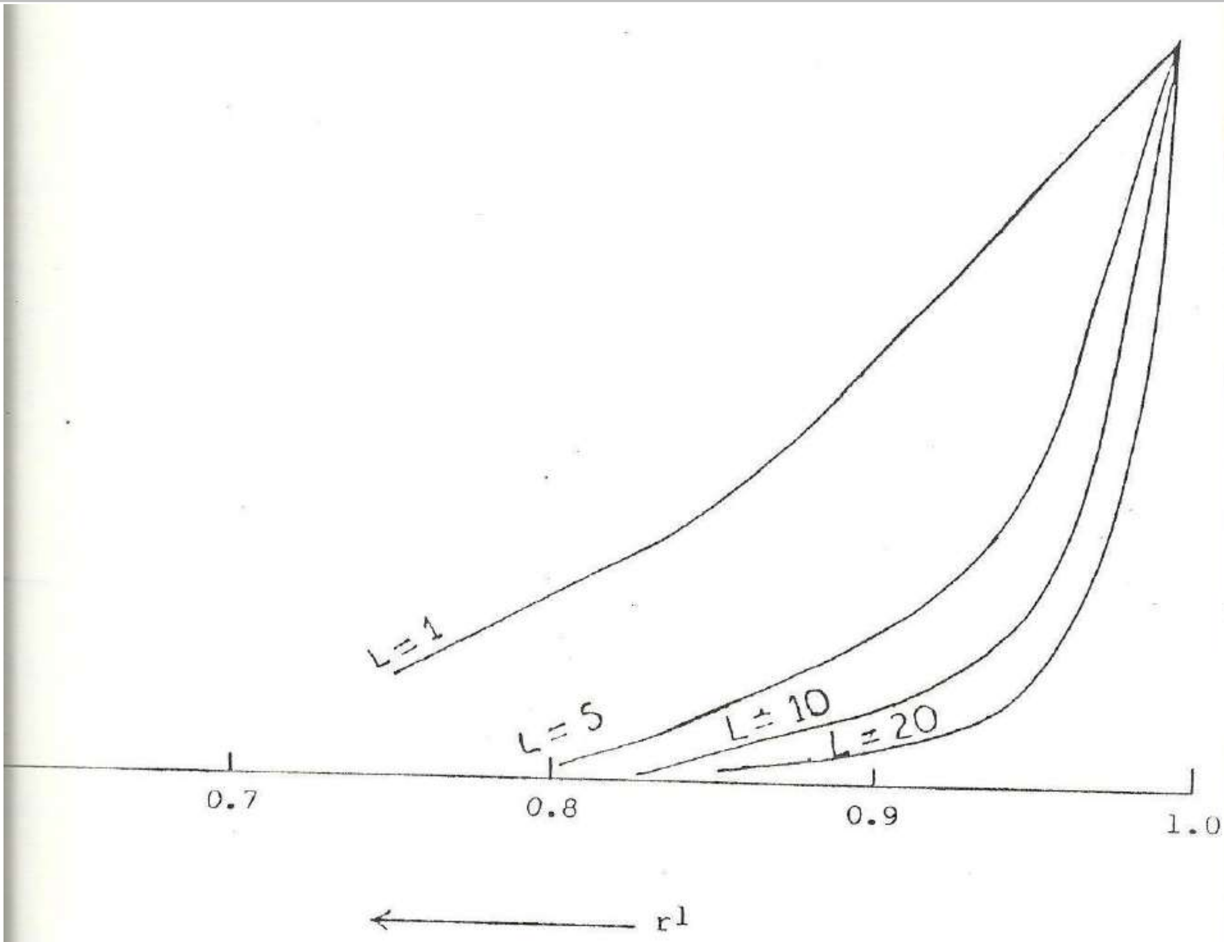

Fig. 5 Temperature distribution

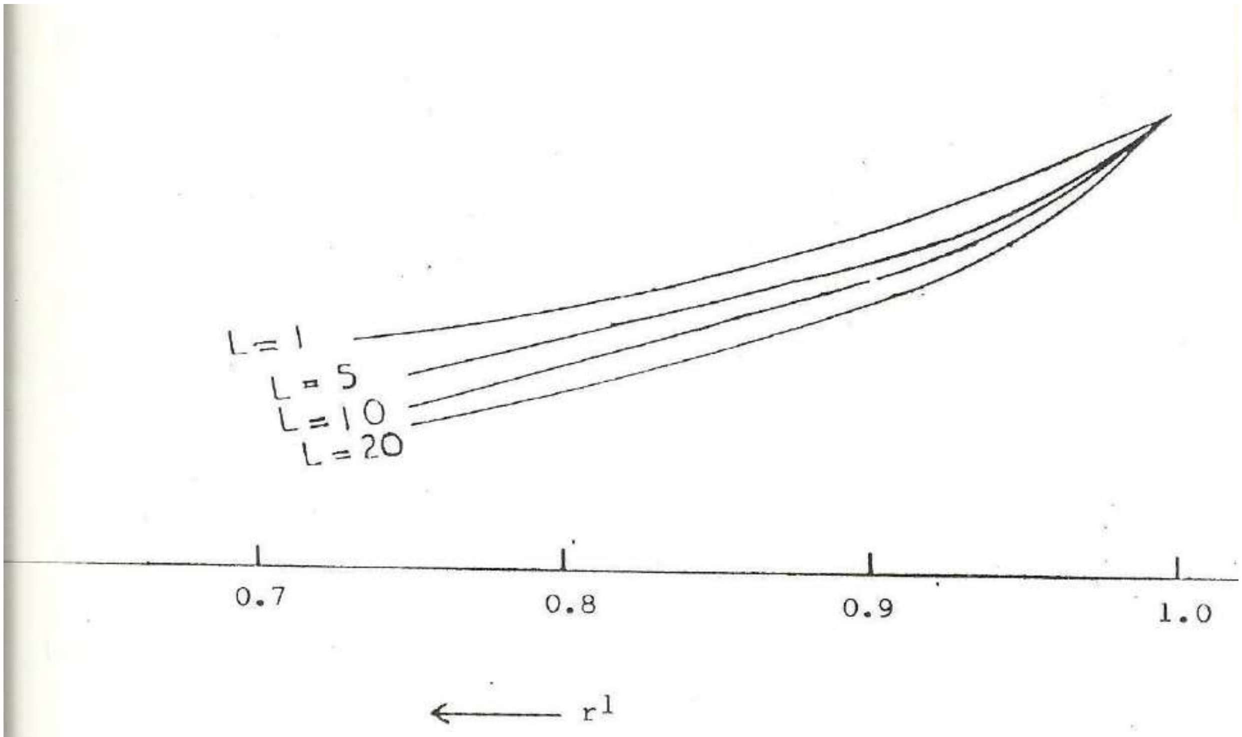

Fig. 6 Radiation flux distribution

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