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ON A GRACEFUL FAMILY OF 3-TUPLES

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ABSTRACT

This paper concerns with the study of formulating 3-tuples consisting of polygonal and pyramidal numbers such that, in each three tuple , the sum of any two members is a perfect square.

KEYWORDS: *3-tuples, polygonal numbers, pyramidal numbers.*

Notations:

- $SO_n = n(2n^2 - 1) =$ Stella Octangula number of rank n
- $CP_n^3 = \dfrac{n(n^2+1)}{2} =$ Centered triangular Pyramidal number of rank n
- $CS_n^4 = \dfrac{n(2n^2+1)}{3} =$ Centered square Pyramidal number of rank n
- $P_n^5 = \dfrac{n^2(n+1)^3}{2} =$ Pentagonal Pyramidal number of rank n
- $P_n^3 = \dfrac{n(n+1)(n+2)}{6} =$ Triangular Pyramidal number of rank n
- $CP_{9,n} = \dfrac{n(3n^2-1)}{2} =$ Centered nonagonal pyramidal number of rank n
- $CP_{24,n} = \dfrac{24n^3 - 18n}{6} =$ Centered icositetragonal pyramidal number of rank n

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- $t_{m,n} = n\left(1+\frac{(n-1)(m-2)}{2}\right)$ = Polygonal number of rank n with size m
- $PR_n = n(n+1)$ = Pronic number of rank n
- $S_n = 6n(n-1)+1$ = Star number of rank n
- $GNO_n = 2n-1$ = Gnomonic number of rank n

1. INTRODUCTION

Number patterns have occupied a unique position in the subject of Number Theory as they possess not only truth but also supreme beauty. Nearly every centuary has witnessed new and fascinating discoveries about the properties of numbers. For varieties of problems, one may refer [1-8]. The above problems motivated us for constructing three tuples. This paper concerns with the study of formulating 3-tuples consisting of polygonal and pyramidal numbers such that, in each three tuple , the sum of any two members is a perfect square.

2. METHOD OF ANALYSIS

Triple 1:

Let $a = 2t_{3,2k} = 4k^2 + 2k$ and $b = 2k+1$

$$a+b = (2k+1)^2$$

Let c be any non-zero integer such that

$$a+c=\alpha^2$$

$$b+c=\beta^2$$

Using some algebra
we have

$$c = 24k\,P_{k-1}^3 - 2k$$

Here $\left(2t_{3,2k}, 2k+1, 24k\,P_{k-1}^3 - 2k\right)$ is the required triple such that the sum of any two members is a perfect square.

Properties:

- $c - a + 2b + 2$ is a perfect square
- $c + 3a - 2b + 6$ is a perfect square
- $2a - b + c + 1 = 8k\,CP_k^3$

Triple 2:

Let $a = Ct_{10,2k} = 20k^2 + 10k + 1$ and $b = 5t_{10,2k} = 80k^2 - 30k$

$$a+b = (10k-1)^2$$

Let c be any non-zero integer such that

$$a+c=\alpha^2$$

$$b+c=\beta^2$$

Using some algebra
we have

$$c = 100(t_{8,k})^2 - 5t_{10,2k},\ k>1$$

Here $\left(Ct_{10,2k}, 5t_{10,2k}, 100(t_{8,k})^2 - 5t_{10,2k}\right)$ is the required triple such that the sum of any two members is a perfect square.

Properties:

- $4(a-1) - b \equiv 0 \pmod{70}$
- $3(a-1) + b \equiv 0 \pmod{140}$
- $c - 4b - 15a \equiv 0 \pmod{15}$

Triple 3:

Let $a = 8t_{3,k} = 4k^2 + 4k, k > 1$ and $b = 1$

$$a + b = (2k+1)^2$$

Let c be any non-zero integer such that

$$a + c = \alpha^2$$
$$b + c = \beta^2$$

Using some algebra
we have

$$c = 2k\,SO_k + 12CS_k^4 + 4t_{3,k-1} - 6k$$

Here $(8t_{3,k}, 1, 2k\,SO_k + 12CS_k^4 + 4t_{3,k-1} - 6k)$ is the required triple such that the sum of any two members is a perfect square.

Properties:

- $c - 2ka = 8k\,CP_k^3 - t_{25,k} - 15k$
- $2k^2a - c = 8k\,CP_k^3 - t_{10,k} + 5k$

For simplicity some more triples satisfying the required condition are given below:

Triple 4	$(t_{12,2n} + 2t_{3,2n} + 1, t_{8,2n} - 2n, 6n + 24nCP_{9,n})$
Triple 5	$(t_{34,n} + t_{42,n}, 11GNO_n - 10, 4(t_{20,n})^2 - 48nt_{20,n} - 68t_{3,n-1} + 142n^2)$
Triple 6	$(S_n, 6t_{12,n} + 18, 36n\,CP_{24,n} + 78n^2 + 6n)$
Triple 7	$(4PR_n, 1, 8nP_n^5 - 24CP_n^3 + 8n)$
Triple 8	$(7(6P_n^3 - 2P_n^5), 4t_{3,n} + 4, 36(t_{3,n})^2 - 28t_{3,n})$

3. CONCLUSION

In this paper we have presented triples involving polygonal and pyramidal numbers such that the sum of any two members of the triple is a perfect square. The readers of this paper may search for quadruples and higher order tuples with the sum of any two members as a perfect square.

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