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# FORMULATION OF SEQUENCES OF DIOPHANTINE 3-TUPLES THROUGH THE PAIR $(3,6)$

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## ABSTRACT

***This paper aims at formulating sequences of Diophantine 3-tuples through the pair*** $(3,6)$

**KEY WORDS:** ***Diophantine 3-tuple,sequence of Diophantine 3-tuples***

## INTRODUCTION

The problem of constructing the sets with property that product of any two of its distinct elements is one less than a square has a very long history and such sets have been studied by Diophantus. A set of m distinct positive integers $\{a_1, a_2, a_3, ....., a_m\}$ is said to have the property $D(n), n \in Z - \{0\}$ if $a_i a_j + n$ is a perfect square for all $1 \le i < j \le m$ or $1 \le j < i \le m$ and such a set is called a Diophantine m-tuple with property $D(n)$.

Many Mathematicians considered the construction of different formulations of diophantine triples with the property $D(n)$ for any arbitrary integer n [1] and also, for any linear polynomials in n. In this context, one may refer [2-13] for an extensive review of various problems on diophantine triples.

This paper concerns with the construction of sequences of diophantine 3-tuples $(a, b, c)$ such that the product of any two elements of the set added by $(-2), (-9), (-14), (-17), D(k^2 + 8k - 2), D(k^2 - 8k - 2)$ in turn is a perfect square.

**Sequence: 1**

Let $a = 6, \ c_0 = 3$

It is observed that

$ac_0 - 2 = 16$, a perfect square

Therefore, the pair $(a, c_0)$ represents diophantine 2-tuple with the property $D(-2)$ .

Let $c_1$ be any non-zero polynomial such that

$$ac_1 - 2 = p^2 \tag{1}$$

$$c_0 c_1 - 2 = q^2 \tag{2}$$

Eliminating $c_1$ between (1) and (2), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(-2) \tag{3}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0 T \tag{4}$$

in (3) and simplifying, we get

$$X^2 = ac_0 T^2 - 2$$

which is satisfied by $T = 1$ , $X = 4$

In view of (4) and (1), it is seen that

$$c_1 = 17$$

Note that $(\mathbf{a}, \mathbf{c_0}, \mathbf{c_1})$ represents diophantine 3-tuple with property $D(-2)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$$c_2 = 43$$

exhibits diophantine 3-tuple with property $D(-2)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$$c_3 = 81$$

exhibits diophantine 3-tuple with property $D(-2)$

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

$$c_4 = 131$$

exhibits diophantine 3-tuple with property $D(-2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_{s-1} = 6s^2 - 4s + 1,\ \ s = 1, 2, 3, \ldots$$

**Sequence: 2**

Let $a = 6,\ \ c_0 = 3$

It is observed that
$ac_0 - 9 = 9$, a perfect square

Therefore, the pair $(a, c_0)$ represents diophantine 2-tuple with the property $D(-9)$ .

Let $c_1$ be any non-zero polynomial such that

$$ac_1 - 9 = p^2 \tag{5}$$

$$c_0 c_1 - 9 = q^2 \tag{6}$$

Eliminating $c_1$ between (5) and (6), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(-9) \tag{7}$$

Introducing the linear transformations

$$p = X + aT\ ,\ \ q = X + c_0 T \tag{8}$$

in (7) and simplifying we get

$$X^2 = ac_0 T^2 - 9$$

which is satisfied by $T = 1\ ,\ X = 3$

In view of (8) and (5), it is seen that

$$c_1 = 15$$

Note that $(a, c_0, c_1)$ represents diophantine 3-tuple with property $D(-9)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$$c_2 = 39$$

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exhibits diophantine 3-tuple with property $D(-9)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$$c_3 = 75$$

exhibits diophantine 3-tuple with property $D(-9)$

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

$$c_4 = 123$$

exhibits diophantine 3-tuple with property $D(-9)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_{s-1} = 6s^2 - 6s + 3, \quad s = 1, 2, 3, \ldots$$

**Sequence: 3**

Let $a = 6, \quad c_0 = 3$

It is observed that
$ac_0 - 14 = 4$, a perfect square

Therefore, the pair $(a, c_0)$ represents diophantine 2-tuple with the property $D(-14)$ .

Let $c_1$ be any non-zero polynomial such that

$$ac_1 - 14 = p^2 \tag{9}$$

$$c_0 c_1 - 14 = q^2 \tag{10}$$

Eliminating $c_1$ between (9) and (10), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(-14) \tag{11}$$

Introducing the linear transformations

$$p = X + aT, \quad q = X + c_0 T \tag{12}$$

in (11) and simplifying we get

$$X^2 = ac_0 T^2 - 14$$

which is satisfied by $T=1$ , $X=2$

In view of (12) and (9), it is seen that

$$c_1 = 13$$

Note that $(a, c_0, c_1)$ represents diophantine 3-tuple with property $D(-14)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$$c_2 = 35$$

exhibits diophantine 3-tuple with property $D(-14)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$$c_3 = 69$$

exhibits diophantine 3-tuple with property $D(-14)$

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

$$c_4 = 115$$

exhibits diophantine 3-tuple with property $D(-14)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_{s-1} = 6s^2 - 8s + 5 \ , \ s = 1, 2, 3, ...$$

**Sequence: 4**

Let $a = 6, \quad c_0 = 3$

It is observed that
$ac_0 - 17 = 1$, a perfect square

Therefore, the pair $(a, c_0)$ represents diophantine 2-tuple with the property $D(-17)$ .

Let $c_1$ be any non-zero polynomial such that

$$ac_1 - 17 = p^2 \tag{13}$$

$$c_0 c_1 - 17 = q^2 \tag{14}$$

Eliminating $c_1$ between (13) and (14), we have

$$c_0p^2 - aq^2 = (c_0 - a)(-17) \tag{15}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0T \tag{16}$$

in (15) and simplifying we get

$$X^2 = ac_0T^2 - 17$$

which is satisfied by $T = 1 \ , X = 1$

In view of (16) and (13), it is seen that

$$c_1 = 11$$

Note that $(a, c_0, c_1)$ represents diophantine 3-tuple with property $D(-17)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$$c_2 = 31$$

exhibits diophantine 3-tuple with property $D(-17)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$$c_3 = 63$$

exhibits diophantine 3-tuple with property **D(–17)**

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

$$c_4 = 107$$

exhibits diophantine 3-tuple with property $D(-17)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_{s-1} = 6s^2 - 10s + 7 \ , \ s = 1, 2, 3, \ldots$$

**Sequence: 5**

Let $a = 6, \ \ c_0 = 3$

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It is observed that
$ac_0 + k^2 + 8k - 2 = (k+4)^2$, a perfect square

Therefore, the pair $(a, c_0)$ represents diophantine 2-tuple with the property $D(k^2 + 8k - 2)$ .

Let $c_1$ be any non-zero polynomial such that

$$ac_1 + k^2 + 8k - 2 = p^2 \tag{17}$$

$$c_0 c_1 + k^2 + 8k - 2 = q^2 \tag{18}$$

Eliminating $c_1$ between (17) and (18), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(k^2 + 8k - 2) \tag{19}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0 T \tag{20}$$

in (19) and simplifying we get

$$X^2 = ac_0 T^2 + k^2 + 8k - 2$$

which is satisfied by $T = 1$ , $X = k + 4$

In view of (20) and (17), it is seen that

$$c_1 = 2k + 17$$

Note that $(a, c_0, c_1)$ represents diophantine 3-tuple with property $D(k^2 + 8k - 2)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$$c_2 = 4k + 43$$

exhibits diophantine 3-tuple with property $D(k^2 + 8k - 2)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$$c_3 = 6k + 81$$

exhibits diophantine 3-tuple with property $D(k^2 + 8k - 2)$

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

$$c_4 = 8k + 131$$

exhibits diophantine 3-tuple with property $D(k^2 + 8k - 2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_{s-1} = 2(s-1)k + 6s^2 - 4s + 1 \ , \ s = 1, 2, 3, ...$$

**Sequence: 6**

Let $a = 6, \ \ c_0 = 3$

It is observed that
$ac_0 + k^2 - 8k - 2 = (k-4)^2$, a perfect square

Therefore, the pair $(a, c_0)$ represents diophantine 2-tuple with the property $D(k^2 - 8k - 2)$ .

Let $c_1$ be any non-zero polynomial such that

$$ac_1 + k^2 - 8k - 2 = p^2 \tag{21}$$

$$c_0 c_1 + k^2 - 8k - 2 = q^2 \tag{22}$$

Eliminating $c_1$ between (21) and (22), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(k^2 - 8k - 2) \tag{23}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0 T \tag{24}$$

in (23) and simplifying we get

$$X^2 = ac_0 T^2 + k^2 - 8k - 2$$

which is satisfied by $T = 1 \ , \ X = k - 4$

In view of (24) and (21), it is seen that

$$c_1 = 2k + 1$$

Note that $(a, c_0, c_1)$ represents diophantine 3-tuple with property $D(k^2 - 8k - 2)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$$c_2 = 4k + 11$$

exhibits diophantine 3-tuple with property $D(k^2 - 8k - 2)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$$c_3 = 6k + 33$$

exhibits diophantine 3-tuple with property $D(k^2 - 8k - 2)$

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

$$c_4 = 8k + 67$$

exhibits diophantine 3-tuple with property $D(k^2 - 8k - 2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_{s-1} = 2(s-1)k + 6s^2 - 20s + 17 \;,\; s = 1, 2, 3, \ldots$$

It is noted that, in each of the above sequences,the following relations are observed:

- The triple $(c_s, c_{s+1} + 6, c_{s+2})$ forms an arithmetic progression.

**Sequence: 7**

Let $a = 3, \;\; c_0 = 6$

It is observed that

$ac_0 - 2 = 16$, a perfect square

Therefore, the pair $(a, c_0)$ represents diophantine 2-tuple with the property $D(-2)$ .

Let $c_1$ be any non-zero polynomial such that

$$ac_1 - 2 = p^2 \tag{25}$$

$$c_0 c_1 - 2 = q^2 \tag{26}$$

Eliminating $c_1$ between (25) and (26), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(-2) \tag{27}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0 T \qquad (28)$$

in (27) and simplifying, we get

$$X^2 = ac_0 T^2 - 2$$

which is satisfied by $T = 1 \ , \ X = 4$

In view of (28) and (25), it is seen that

$$c_1 = 17$$

Note that $(\mathbf{a}, \mathbf{c_0}, \mathbf{c_1})$ represents diophantine 3-tuple with property $D(-2)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$$c_2 = 34$$

exhibits diophantine 3-tuple with property $D(-2)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$$c_3 = 57$$

exhibits diophantine 3-tuple with property $D(-2)$

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

$$c_4 = 86$$

exhibits diophantine 3-tuple with property $D(-2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_{s-1} = 3s^2 + 2s + 1, \ s = 1, 2, 3, \ldots$$

**Sequence: 8**

Let $a = 3, \ \ c_0 = 6$

It is observed that
$ac_0 - 9 = 9$, a perfect square

Therefore, the pair $(a, c_0)$ represents diophantine 2-tuple with the property $D(-9)$ .

Let $c_1$ be any non-zero polynomial such that

$$ac_1 - 9 = p^2 \tag{29}$$

$$c_0 c_1 - 9 = q^2 \tag{30}$$

Eliminating $c_1$ between (29) and (30), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(-9) \tag{31}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0 T \tag{32}$$

in (7) and simplifying we get

$$X^2 = ac_0 T^2 - 9$$

which is satisfied by $T = 1 \ , X = 3$

In view of (32) and (29), it is seen that

$$c_1 = 15$$

Note that $(a, c_0, c_1)$ represents diophantine 3-tuple with property $D(-9)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$$c_2 = 30$$

exhibits diophantine 3-tuple with property $D(-9)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$$c_3 = 51$$

exhibits diophantine 3-tuple with property $D(-9)$

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

$$c_4 = 78$$

exhibits diophantine 3-tuple with property $D(-9)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

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$$c_{s-1} = 3s^2 + 3, \; s = 1, 2, 3, ...$$

**Sequence: 9**

Let $a = 3, \; c_0 = 6$

It is observed that
$ac_0 - 14 = 4$, a perfect square

Therefore, the pair $(a, c_0)$ represents diophantine 2-tuple with the property $D(-14)$ .

Let $c_1$ be any non-zero polynomial such that

$$ac_1 - 14 = p^2 \tag{33}$$

$$c_0c_1 - 14 = q^2 \tag{34}$$

Eliminating $c_1$ between (33) and (34), we have

$$c_0p^2 - aq^2 = (c_0 - a)(-14) \tag{35}$$

Introducing the linear transformations

$$p = X + aT \;,\; q = X + c_0T \tag{36}$$

in (35) and simplifying we get

$$X^2 = ac_0T^2 - 14$$

which is satisfied by $T = 1 \;, X = 2$

In view of (36) and (33), it is seen that

$$c_1 = 13$$

Note that $(a, c_0, c_1)$ represents diophantine 3-tuple with property $D(-14)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$$c_2 = 26$$

exhibits diophantine 3-tuple with property $D(-14)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$$c_3 = 45$$

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exhibits diophantine 3-tuple with property $D(-14)$

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

$$c_4 = 70$$

exhibits diophantine 3-tuple with property $D(-14)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_{s-1} = 3s^2 - 2s + 5, \quad s = 1, 2, 3, ...$$

**Sequence: 10**

Let $a = 3, \quad c_0 = 6$

It is observed that
$ac_0 - 17 = 1$, a perfect square

Therefore, the pair $(a, c_0)$ represents diophantine 2-tuple with the property $D(-17)$ .

Let $c_1$ be any non-zero polynomial such that

$$ac_1 - 17 = p^2 \qquad (37)$$

$$c_0 c_1 - 17 = q^2 \qquad (38)$$

Eliminating $c_1$ between (37) and (38), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(-17) \qquad (39)$$

Introducing the linear transformations

$$p = X + aT \;,\; q = X + c_0 T \qquad (40)$$

in (39) and simplifying we get

$$X^2 = ac_0 T^2 - 17$$

which is satisfied by $T = 1 \;,\; X = 1$

In view of (40) and (37), it is seen that

$$c_1 = 11$$

Note that $(a, c_0, c_1)$ represents diophantine 3-tuple with property $D(-17)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$$c_2 = 22$$

exhibits diophantine 3-tuple with property $D(-17)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$$c_3 = 39$$

exhibits diophantine 3-tuple with property $\mathbf{D(-17)}$

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

$$c_4 = 62$$

exhibits diophantine 3-tuple with property $D(-17)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_{s-1} = 3s^2 - 4s + 7\ ,\ \ s = 1, 2, 3, \ldots$$

**Sequence: 11**

Let $a = 3,\ \ c_0 = 6$

It is observed that
$ac_0 + k^2 + 8k - 2 = (k+4)^2$, a perfect square

Therefore, the pair $(a, c_0)$ represents diophantine 2-tuple with the property $D(k^2 + 8k - 2)$ .

Let $c_1$ be any non-zero polynomial such that

$$ac_1 + k^2 + 8k - 2 = p^2 \tag{41}$$

$$c_0c_1 + k^2 + 8k - 2 = q^2 \tag{42}$$

Eliminating $c_1$ between (41) and (42), we have

$$c_0p^2 - aq^2 = (c_0 - a)(k^2 + 8k - 2) \tag{43}$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0 T \tag{44}$$

in (43) and simplifying we get

$$X^2 = ac_0 T^2 + k^2 + 8k - 2$$

which is satisfied by $T = 1$ , $X = k + 4$

In view of (44) and (41), it is seen that

$$c_1 = 2k + 17$$

Note that $(a, c_0, c_1)$ represents diophantine 3-tuple with property $D(k^2 + 8k - 2)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$$c_2 = 4k + 34$$

exhibits diophantine 3-tuple with property $D(k^2 + 8k - 2)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$$c_3 = 6k + 57$$

exhibits diophantine 3-tuple with property $D(k^2 + 8k - 2)$

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

$$c_4 = 8k + 86$$

exhibits diophantine 3-tuple with property $D(k^2 + 8k - 2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_{s-1} = 2(s-1)k + 3s^2 + 2s + 1, \ s = 1, 2, 3, ...$$

**Sequence: 12**

Let $a = 3, \ c_0 = 6$

It is observed that
$ac_0 + k^2 - 8k - 2 = (k-4)^2$ , a perfect square

Therefore, the pair $(a, c_0)$ represents diophantine 2-tuple with the property $D(k^2 - 8k - 2)$ .

Let $c_1$ be any non-zero polynomial such that

$$ac_1 + k^2 - 8k - 2 = p^2 \quad (45)$$

$$c_0c_1 + k^2 - 8k - 2 = q^2 \quad (46)$$

Eliminating $c_1$ between (45) and (46), we have

$$c_0p^2 - aq^2 = (c_0 - a)(k^2 - 8k - 2) \quad (47)$$

Introducing the linear transformations

$$p = X + aT \ , \ q = X + c_0T \quad (48)$$

in (47) and simplifying we get

$$X^2 = ac_0T^2 + k^2 - 8k - 2$$

which is satisfied by $T = 1 \ , \ X = k - 4$

In view of (48) and (45), it is seen that

$$c_1 = 2k + 1$$

Note that $(a, c_0, c_1)$ represents diophantine 3-tuple with property $D(k^2 - 8k - 2)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$$c_2 = 4k + 2$$

exhibits diophantine 3-tuple with property $D(k^2 - 8k - 2)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$$c_3 = 6k + 9$$

exhibits diophantine 3-tuple with property $D(k^2 - 8k - 2)$

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

$$c_4 = 8k + 22$$

exhibits diophantine 3-tuple with property $D(k^2 - 8k - 2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_{s-1} = 2(s-1)k + 3s^2 - 14s + 17, \ s = 1, 2, 3, ...$$

It is noted that, in each of the above sequences 7-12,

the triple $(c_s, c_{s+1} + 3, c_{s+2})$ forms an arithmetic progression.

In conclusion, one may attempt for obtaining sequences of higher order Diophantine tuples with suitable properties.

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