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FORMULATION OF SEQUENCES OF DIOPHANTINE 3-TUPLES THROUGH THE PAIR (3,6)

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ABSTRACT

This paper aims at formulating sequences of Diophantine 3-tuples through the pair (3,6)

KEY WORDS: Diophantine 3-tuple, sequence of Diophantine 3-tuples

INTRODUCTION

The problem of constructing the sets with property that product of any two of its distinct elements is one less than a square has a very long history and such sets have been studied by Diophantus. A set of m distinct positive integers $\{a_1, a_2, a_3, \dots, a_m\}$ is said to have the property $D(n), n \in \mathbb{Z} - \{0\}$ if $a_i a_j + n$ is a perfect square for all $1 \le i < j \le m$ or $1 \le j < i \le m$ and such a set is called a Diophantine m-tuple with property D(n).

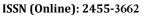
Many Mathematicians considered the construction of different formulations of diophantine triples with the property D(n) for any arbitrary integer n [1] and also, for any linear polynomials in n. In this context, one may refer [2-13] for an extensive review of various problems on diophantine triples.

This paper concerns with the construction of sequences of diophantine 3-tuples (a,b,c) such that the elements added product by $(-2), (-9), (-14), (-17), D(k^2 + 8k - 2), D(k^2 - 8k - 2)$ in turn is a perfect square.

Sequence: 1

Let
$$a = 6$$
, $c_0 = 3$

It is observed that





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 $ac_0 - 2 = 16$, a perfect square

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property D(-2).

Let $\, \mathbf{c}_1 \,$ be any non-zero polynomial such that

$$ac_1 - 2 = p^2 \tag{1}$$

$$c_0 c_1 - 2 = q^2 (2)$$

Eliminating c_1 between (1) and (2), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(-2)$$
 (3)

Introducing the linear transformations

$$p = X + aT$$
, $q = X + c_0T$ (4)

in (3) and simplifying, we get

$$X^2 = ac_0 T^2 - 2$$

which is satisfied by T=1, X=4

In view of (4) and (1), it is seen that

$$c_1 = 17$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property D(-2)

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 43$$

exhibits diophantine 3-tuple with property D(-2)

Taking (a,c_2) and employing the above procedure, it is seen that the triple (a,c_2,c_3) where

$$c_3 = 81$$

exhibits diophantine 3-tuple with property D(-2)

Taking (a,c_3) and employing the above procedure, it is seen that the triple (a,c_3,c_4) where

$$c_4 = 131$$



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exhibits diophantine 3-tuple with property D(-2)

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (a, c_s, c_{s+1}) where

$$c_{s-1} = 6s^2 - 4s + 1$$
, $s = 1, 2, 3, ...$

Sequence: 2

Let
$$a = 6$$
, $c_0 = 3$

It is observed that

 $ac_0 - 9 = 9$, a perfect square

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property D(-9).

Let c_1 be any non-zero polynomial such that

$$ac_1 - 9 = p^2 \tag{5}$$

$$c_0 c_1 - 9 = q^2 \tag{6}$$

Eliminating c_1 between (5) and (6), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(-9)$$
 (7)

Introducing the linear transformations

$$p = X + aT$$
, $q = X + c_0T$ (8)

in (7) and simplifying we get

$$X^2 = ac_0 T^2 - 9$$

which is satisfied by T = 1, X = 3

In view of (8) and (5), it is seen that

$$c_1 = 15$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property D(-9)

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 39$$



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exhibits diophantine 3-tuple with property D(-9)

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 75$$

exhibits diophantine 3-tuple with property D(-9)

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 123$$

exhibits diophantine 3-tuple with property D(-9)

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (a, c_s, c_{s+1}) where

$$c_{s-1} = 6s^2 - 6s + 3$$
, $s = 1, 2, 3,...$

Sequence: 3

Let
$$a = 6$$
, $c_0 = 3$

It is observed that

 $ac_0 - 14 = 4$, a perfect square

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property D(-14).

Let c_1 be any non-zero polynomial such that

$$ac_1 - 14 = p^2 (9)$$

$$c_0 c_1 - 14 = q^2 \tag{10}$$

Eliminating C_1 between (9) and (10), we have

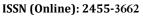
$$c_0 p^2 - aq^2 = (c_0 - a)(-14)$$
(11)

Introducing the linear transformations

$$p = X + aT$$
, $q = X + c_0T$ (12)

in (11) and simplifying we get

$$X^2 = ac_0T^2 - 14$$





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which is satisfied by T = 1, X = 2

In view of (12) and (9), it is seen that

$$c_1 = 13$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property D(-14)

Taking (a,c_1) and employing the above procedure, it is seen that the triple (a,c_1,c_2) where

$$c_2 = 35$$

exhibits diophantine 3-tuple with property D(-14)

Taking $\left(a\,,c_{_2}\right)$ and employing the above procedure, it is seen that the triple $\left(a\,,c_{_2}\,,c_{_3}\,\right)$ where

$$c_3 = 69$$

exhibits diophantine 3-tuple with property D(-14)

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 115$$

exhibits diophantine 3-tuple with property D(-14)

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (a, c_s, c_{s+1}) where

$$c_{s-1} = 6s^2 - 8s + 5$$
, $s = 1, 2, 3, ...$

Sequence: 4

Let
$$a = 6$$
, $c_0 = 3$

It is observed that

$$ac_0 - 17 = 1$$
, a perfect square

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property D(-17).

Let c_1 be any non-zero polynomial such that

$$ac_1 - 17 = p^2 (13)$$

$$c_0 c_1 - 17 = q^2 \tag{14}$$



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Eliminating c_1 between (13) and (14), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(-17)$$
(15)

Introducing the linear transformations

$$p = X + aT$$
, $q = X + c_0T$ (16)

in (15) and simplifying we get

$$X^2 = ac_0T^2 - 17$$

which is satisfied by T = 1, X = 1

In view of (16) and (13), it is seen that

$$c_1 = 11$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property D(-17)

Taking (a,c_1) and employing the above procedure, it is seen that the triple (a,c_1,c_2) where

$$c_2 = 31$$

exhibits diophantine 3-tuple with property D(-17)

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 63$$

exhibits diophantine 3-tuple with property D(-17)

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 107$$

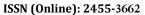
exhibits diophantine 3-tuple with property D(-17)

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (a, c_s, c_{s+1}) where

$$c_{s-1} = 6s^2 - 10s + 7$$
, $s = 1, 2, 3,...$

Sequence: 5

Let
$$a = 6$$
, $c_0 = 3$





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It is observed that

$$ac_0 + k^2 + 8k - 2 = (k + 4)^2$$
, a perfect square

Therefore, the pair $\left(a\,,c_{_0}\right)$ represents diophantine 2-tuple with the property $D\left(k^2+8k-2\right)$.

Let C_1 be any non-zero polynomial such that

$$ac_1 + k^2 + 8k - 2 = p^2 (17)$$

$$c_0 c_1 + k^2 + 8k - 2 = q^2 (18)$$

Eliminating c_1 between (17) and (18), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(k^2 + 8k - 2)$$
(19)

Introducing the linear transformations

$$p = X + aT$$
, $q = X + c_0T$ (20)

in (19) and simplifying we get

$$X^2 = ac_0T^2 + k^2 + 8k - 2$$

which is satisfied by T = 1, X = k + 4

In view of (20) and (17), it is seen that

$$c_1 = 2k + 17$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property $D(k^2 + 8k - 2)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 4k + 43$$

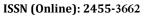
exhibits diophantine 3-tuple with property $D(k^2 + 8k - 2)$

Taking (a,c_2) and employing the above procedure, it is seen that the triple (a,c_2,c_3) where

$$c_2 = 6k + 81$$

exhibits diophantine 3-tuple with property $D(k^2 + 8k - 2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where





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$$c_4 = 8k + 131$$

exhibits diophantine 3-tuple with property $D(k^2 + 8k - 2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (a, c_s, c_{s+1}) where

$$c_{s-1} = 2(s-1)k + 6s^2 - 4s + 1$$
, $s = 1, 2, 3, ...$

Sequence: 6

Let
$$a = 6$$
, $c_0 = 3$

It is observed that

$$ac_0 + k^2 - 8k - 2 = (k - 4)^2$$
, a perfect square

Therefore, the pair $\left(a\,,c_0^{}\right)$ represents diophantine 2-tuple with the property $D\left(k^2-8k-2\right)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 + k^2 - 8k - 2 = p^2 (21)$$

$$c_0 c_1 + k^2 - 8k - 2 = q^2 (22)$$

Eliminating c_1 between (21) and (22), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(k^2 - 8k - 2)$$
(23)

Introducing the linear transformations

$$p = X + aT$$
, $q = X + c_0T$ (24)

in (23) and simplifying we get

$$X^2 = ac_0T^2 + k^2 - 8k - 2$$

which is satisfied by T=1, X=k-4

In view of (24) and (21), it is seen that

$$c_1 = 2k + 1$$

Note that $\left(a,c_{0},c_{1}\right)$ represents diophantine 3-tuple with property $\left.D(k^{2}-8k-2)\right.$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where



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$$c_2 = 4k + 11$$

exhibits diophantine 3-tuple with property $D(k^2 - 8k - 2)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 6k + 33$$

exhibits diophantine 3-tuple with property $D(k^2 - 8k - 2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 8k + 67$$

exhibits diophantine 3-tuple with property $D(k^2 - 8k - 2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (a, c_s, c_{s+1}) where

$$c_{s-1} = 2(s-1)k + 6s^2 - 20s + 17$$
, $s = 1, 2, 3,...$

It is noted that, in each of the above sequences, the following relations are observed:

• The triple $(c_s, c_{s+1} + 6, c_{s+2})$ forms an arithmetic progression.

Sequence: 7

Let
$$a = 3$$
, $c_0 = 6$

It is observed that

$$ac_0 - 2 = 16$$
, a perfect square

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property D(-2).

Let C_1 be any non-zero polynomial such that

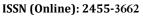
$$ac_1 - 2 = p^2 \tag{25}$$

$$c_0 c_1 - 2 = q^2 (26)$$

Eliminating c_1 between (25) and (26), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(-2)$$
(27)

Introducing the linear transformations





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$$p = X + aT$$
, $q = X + c_0T$ (28)

in (27) and simplifying, we get

$$X^2 = ac_0 T^2 - 2$$

which is satisfied by T = 1, X = 4

In view of (28) and (25), it is seen that

$$c_1 = 17$$

Note that $(\mathbf{a}, \mathbf{c}_0, \mathbf{c}_1)$ represents diophantine 3-tuple with property D(-2)

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 34$$

exhibits diophantine 3-tuple with property D(-2)

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 57$$

exhibits diophantine 3-tuple with property D(-2)

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 86$$

exhibits diophantine 3-tuple with property D(-2)

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (a, c_s, c_{s+1}) where

$$c_{s-1} = 3s^2 + 2s + 1, \ s = 1, 2, 3, ...$$

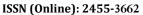
Sequence: 8

Let
$$a = 3$$
, $c_0 = 6$

It is observed that

$$ac_0 - 9 = 9$$
, a perfect square

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property D(-9).





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Let c_1 be any non-zero polynomial such that

$$ac_1 - 9 = p^2 \tag{29}$$

$$c_0 c_1 - 9 = q^2 (30)$$

Eliminating c_1 between (29) and (30), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(-9)$$
(31)

Introducing the linear transformations

$$p = X + aT$$
, $q = X + c_0T$ (32)

in (7) and simplifying we get

$$X^2 = ac_0 T^2 - 9$$

which is satisfied by T = 1, X = 3

In view of (32) and (29), it is seen that

$$c_1 = 15$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property D(-9)

Taking (a,c_1) and employing the above procedure, it is seen that the triple (a,c_1,c_2) where

$$c_2 = 30$$

exhibits diophantine 3-tuple with property D(-9)

Taking (a,c_2) and employing the above procedure, it is seen that the triple (a,c_2,c_3) where

$$c_3 = 51$$

exhibits diophantine 3-tuple with property D(-9)

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 78$$

exhibits diophantine 3-tuple with property D(-9)

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (a, c_s, c_{s+1}) where



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$$c_{s-1} = 3s^2 + 3$$
, $s = 1, 2, 3, ...$

Sequence: 9

Let
$$a = 3$$
, $c_0 = 6$

It is observed that

 $ac_0 - 14 = 4$, a perfect square

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property D(-14).

Let c_1 be any non-zero polynomial such that

$$ac_1 - 14 = p^2$$
 (33)

$$c_0 c_1 - 14 = q^2 \tag{34}$$

Eliminating c_1 between (33) and (34), we have

$$c_0 p^2 - a q^2 = (c_0 - a)(-14)$$
(35)

Introducing the linear transformations

$$p = X + aT$$
, $q = X + c_0T$ (36)

in (35) and simplifying we get

$$X^2 = ac_0T^2 - 14$$

which is satisfied by T = 1, X = 2

In view of (36) and (33), it is seen that

$$c_1 = 13$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property D(-14)

Taking (a,c_1) and employing the above procedure, it is seen that the triple (a,c_1,c_2) where

$$c_2 = 26$$

exhibits diophantine 3-tuple with property D(-14)

Taking $\left(a\,,c_{_2}\right)$ and employing the above procedure, it is seen that the triple $\left(a\,,c_{_2}\,,c_{_3}\,\right)$ where

$$c_3 = 45$$



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exhibits diophantine 3-tuple with property D(-14)

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 70$$

exhibits diophantine 3-tuple with property D(-14)

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (a, c_s, c_{s+1}) where

$$c_{s-1} = 3s^2 - 2s + 5$$
, $s = 1, 2, 3, ...$

Sequence: 10

Let
$$a = 3$$
, $c_0 = 6$

It is observed that

 $ac_0 - 17 = 1$, a perfect square

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property D(-17).

Let c_1 be any non-zero polynomial such that

$$ac_1 - 17 = p^2 (37)$$

$$c_0 c_1 - 17 = q^2 (38)$$

Eliminating c_1 between (37) and (38), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(-17)$$
(39)

Introducing the linear transformations

$$p = X + aT$$
, $q = X + c_0T$ (40)

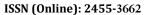
in (39) and simplifying we get

$$X^2 = ac_0 T^2 - 17$$

which is satisfied by T = 1, X = 1

In view of (40) and (37), it is seen that

$$c_1 = 11$$





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Note that $(\mathbf{a}, \mathbf{c}_0, \mathbf{c}_1)$ represents diophantine 3-tuple with property D(-17)

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 22$$

exhibits diophantine 3-tuple with property D(-17)

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 39$$

exhibits diophantine 3-tuple with property D(-17)

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 62$$

exhibits diophantine 3-tuple with property D(-17)

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (a, c_s, c_{s+1}) where

$$c_{s-1} = 3s^2 - 4s + 7$$
, $s = 1, 2, 3, ...$

Sequence: 11

Let
$$a = 3$$
, $c_0 = 6$

It is observed that

$$ac_0 + k^2 + 8k - 2 = (k+4)^2$$
, a perfect square

Therefore, the pair $\left(a,c_{0}\right)$ represents diophantine 2-tuple with the property $D\left(k^{2}+8k-2\right)$.

Let c_1 be any non-zero polynomial such that

$$ac_1 + k^2 + 8k - 2 = p^2 (41)$$

$$c_0 c_1 + k^2 + 8k - 2 = q^2 (42)$$

Eliminating C_1 between (41) and (42), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(k^2 + 8k - 2)$$
(43)

Introducing the linear transformations



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$$p = X + aT$$
, $q = X + c_0T$ (44)

in (43) and simplifying we get

$$X^2 = ac_0T^2 + k^2 + 8k - 2$$

which is satisfied by T=1 , X=k+4

In view of (44) and (41), it is seen that

$$c_1 = 2k + 17$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property $D(k^2 + 8k - 2)$

Taking (a,c_1) and employing the above procedure, it is seen that the triple (a,c_1,c_2) where

$$c_2 = 4k + 34$$

exhibits diophantine 3-tuple with property $D(k^2 + 8k - 2)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3 = 6k + 57$$

exhibits diophantine 3-tuple with property $D(k^2 + 8k - 2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 8k + 86$$

exhibits diophantine 3-tuple with property $D(k^2 + 8k - 2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (a, c_s, c_{s+1}) where

$$c_{s-1} = 2(s-1)k + 3s^2 + 2s + 1, \ s = 1, 2, 3, ...$$

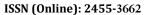
Sequence: 12

Let
$$a = 3$$
, $c_0 = 6$

It is observed that

$$ac_0 + k^2 - 8k - 2 = (k - 4)^2$$
, a perfect square

Therefore, the pair $\left(a\,,c_{_0}\right)$ represents diophantine 2-tuple with the property $D\left(k^2-8k-2\right)$.





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Let C_1 be any non-zero polynomial such that

$$ac_1 + k^2 - 8k - 2 = p^2 (45)$$

$$c_0 c_1 + k^2 - 8k - 2 = q^2 (46)$$

Eliminating C_1 between (45) and (46), we have

$$c_0 p^2 - aq^2 = (c_0 - a)(k^2 - 8k - 2)$$
(47)

Introducing the linear transformations

$$p = X + aT$$
, $q = X + c_0T$ (48)

in (47) and simplifying we get

$$X^2 = ac_0T^2 + k^2 - 8k - 2$$

which is satisfied by T=1, X=k-4

In view of (48) and (45), it is seen that

$$c_1 = 2k + 1$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property $D(k^2 - 8k - 2)$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2 = 4k + 2$$

exhibits diophantine 3-tuple with property $D(k^2 - 8k - 2)$

Taking (a,c_2) and employing the above procedure, it is seen that the triple (a,c_2,c_3) where

$$c_3 = 6k + 9$$

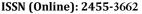
exhibits diophantine 3-tuple with property $D(k^2 - 8k - 2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4 = 8k + 22$$

exhibits diophantine 3-tuple with property $D(k^2 - 8k - 2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by (a, c_s, c_{s+1}) where





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$$c_{s-1} = 2(s-1)k + 3s^2 - 14s + 17, \ s = 1, 2, 3, ...$$

It is noted that, in each of the above sequences 7-12,

the triple $(c_s, c_{s+1} + 3, c_{s+2})$ forms an arithmetic progression.

In conclusion, one may attempt for obtaining sequences of higher order Diophantine tuples with suitable properties.

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