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FASCINATING DIOPHANTINE 3-TUPLES FROM THE PAIR OF INTEGERS $\{u, v\}$

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ABSTRACT

This paper concerns with the construction of sequences of diophantine 3-tuples (a, b, c) ***from the pair of integers*** $\{u, v\}$ ***such that the product of any two elements of the set added by*** $D(\alpha^2k^2 + 2\alpha k(s-w) + s^2 - 2sw - uv + w^2)$ ***is a perfect square.***

KEYWORDS: ***Diophantine 3-tuples, sequences of triples***

INTRODUCTION

The problem of constructing the sets with property that product of any two of its distinct elements is one less than a square has a very long history and such sets have been studied by Diophantus. A set of m distinct positive integers $\{a_1, a_2, a_3, ..., a_m\}$ is said to have the property $D(n), n \in Z - \{0\}$ if $a_i a_j + n$ is a perfect square for all $1 \le i < j \le m$ or $1 \le j < i \le m$ and such a set is called a Diophantine m-tuple with property $D(n)$.

Many Mathematicians considered the construction of different formulations of diophantine triples with the property $D(n)$ for any arbitrary integer n [1] and also, for any linear polynomials in n. In this context, one may refer [2-13] for an extensive review of various problems on diophantine triples.

This paper concerns with the construction of sequences of diophantine 3-tuples (a,b,c) from the pair of integers $\{u,v\}$ such that the product of any two elements of the set added by $D(\alpha^2k^2+2\alpha k(s-w)+s^2-2sw-uv+w^2)$ is a perfect square. This paper is the generalization of [13].

METHOD OF ANALYSIS

Let u,v be any two given non-zero integers .For convenience and clear understanding ,take

$$a=u, c_0=v$$

It is observed that

$$ac_0+\alpha^2k^2+2\alpha k(s-w)+s^2-2sw-uv+w^2=(\alpha k+s-w)^2$$

Therefore, the pair (a, c_0) represents diophantine 2-tuple with the property

$$D(\alpha^2k^2+2\alpha k(s-w)+s^2-2sw-uv+w^2)$$

Let c_1 be any non-zero polynomial such that

$$ac_1+\alpha^2k^2+2\alpha k(s-w)+s^2-2sw-uv+w^2=p^2 \quad (1)$$

$$c_0c_1+\alpha^2k^2+2\alpha k(s-w)+s^2-2sw-uv+w^2=q^2 \quad (2)$$

Eliminating c_1 between (1) and (2), we have

$$c_0p^2-aq^2=(c_0-a)\ (\alpha^2k^2+2\alpha k(s-w)+s^2-2sw-uv+w^2) \quad (3)$$

Introducing the linear transformations

$$p=X+aT\ ,\ q=X+c_0T \quad (4)$$

in (3) and simplifying we get

$$X^2=ac_0T^2+(\alpha^2k^2+2\alpha k(s-w)+s^2-2sw-uv+w^2)$$

which is satisfied by $T=1\ ,\ X=\alpha k+s-w$

In view of (4) and (1), it is seen that

$$c_1=2(\alpha k+s)+u+v-2w$$

Note that (a, c_0, c_1) represents diophantine 3-tuple with property

$$D(\alpha^2k^2+2\alpha k(s-w)+s^2-2sw-uv+w^2)$$

Taking (a, c_1) and employing the above procedure, it is seen that the triple (a, c_1, c_2) where

$$c_2=4(\alpha k+s)+4u+v-4w$$

exhibits diophantine 3-tuple with property $D(\alpha^2k^2+2\alpha k(s-w)+s^2-2sw-uv+w^2)$

Taking (a, c_2) and employing the above procedure, it is seen that the triple (a, c_2, c_3) where

$$c_3=6(\alpha k+s)+9u+v-6w$$

exhibits diophantine 3-tuple with property $D(\alpha^2k^2+2\alpha k(s-w)+s^2-2sw-uv+w^2)$

Taking (a, c_3) and employing the above procedure, it is seen that the triple (a, c_3, c_4) where

$$c_4=8(\alpha k+s)+16u+v-8w$$

exhibits diophantine 3-tuple with property $D(\alpha^2k^2+2\alpha k(s-w)+s^2-2sw-uv+w^2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_{\partial-1}, c_{\partial})$where

$$c_{\partial-1}=2(\partial-1)(\alpha k+s)+(\partial-1)^2u+v-2(\partial-1)w\ ,\ \partial=1,2,3,...$$

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A few numerical examples are presented in Table below:

Table : Numerical Examples

α	k	s	w	u	v	(a, c_0, c_1)	(a, c_1, c_2)	(a, c_2, c_3)	Property
1	1	1	1	2	3	$(2, 3, 7)$	$(2, 7, 15)$	$(2, 15, 27)$	$D(-5)$
1	1	1	1	3	2	$(3, 2, 7)$	$(3, 7, 18)$	$(3, 18, 35)$	$D(-5)$
1	2	1	1	-2	5	$(-2, 5, 7)$	$(-2, 7, 5)$	$(-2, 5, -1)$	$D(14)$
1	1	1	1	2	2	$(2, 2, 6)$	$(2, 6, 14)$	$(2, 14, 26)$	$D(-3)$
0	0	0	$-2n-1$	$2n$	$2n+1$	$(2n, 2n+1, 8n+3)$	$(2n, 8n+3, 18n+5)$	$(2n, 18n+5, 32n+7)$	$D(2n+1)$
0	0	0	$-2n$	$2n$	$2n+1$	$(2n, 2n+1, 8n+1)$	$(2n, 8n+1, 18n+1)$	$(2n, 18n+1, 32n+1)$	$D(-2n)$
0	0	0	$-2n+1$	$2n$	$2n-1$	$(2n, 2n-1, 8n-3)$	$(2n, 8n-3, 18n-5)$	$(2n, 18n-5, 32n-7)$	$D(-2n+1)$
1	k	s	-1	k	k+3	$(k, k+3, 4k+2s+5)$	$(k, 4k+2s+5, 9k+4s+7)$	$(k, 9k+4s+7, 16k+6s+9)$	$D[(2s-1)k + (s+1)^2]$
1	k	s	$(1-\beta)s$	k	k+s	$(k, k+s, 4k+(2\beta+1)s)$	$(k, 4k+(2\beta+1)s, 9k+(4\beta+1)s)$	$(k, 9k+(4\beta+1)s, 16k+(6\beta+1)s)$	$D[(2\beta-1)ks + \beta^2 s^2]$
1	k	2	-2	6	3	$(6, 3, 2k+17)$	$(6, 4k+17, 4k+43)$	$(6, 4k+43, 6k+81)$	$D(k^2+8k-2)$

CONCLUSION

The researchers may attempt for the formulation of other sequences of diophantine 3-tuples such that the product of any two elements of the set added by a polynomial with integer coefficient is a perfect square.

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