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GENERATION OF DIOPHANTINE 3-TUPLES THROUGH MATRIX METHOD

A. Vijayasankar[1]
[1]Assistant Professor,
Department of Mathematics,
National College,
Affiliated to Bharathidasan University,
Trichy-620 001,
Tamil Nadu, India.

Sharadha Kumar[2]
[2*]Research Scholar,
Department of Mathematics,
National College,
Affiliated to Bharathidasan University ,
Trichy-620 001,
Tamil Nadu, India.

M.A. Gopalan[3]
[3]Professor,
Department of Mathematics,
Shrimati Indira Gandhi College,
Affiliated to Bharathidasan University,
Trichy-620 002,
Tamil Nadu, India.

ABSTRACT

This paper concerns with the formulation of sequences of Diophantine 3-tuples with property $D(k^2+10k-3)$ ***through matrix method.***

KEY WORDS: ***Diophantine 3-tuple, Sequence of Diophantine 3-tuples, Matrix application***

INTRODUCTION

A theory that can be explained in a regular and systematic way is a pattern. The essence of mathematical calculations is represented by numbers and the theory of numbers can be taught as a set of patterns [1-4]. It is to be noted here that the number pattern is a sequence of numbers establishing a same properties among them. Numbers exhibit fascinating, beautiful and curious varieties of patterns, namely, polygonal numbers, Fibonacci numbers, Lucas numbers, Ramanujan numbers, Kynea numbers, Jacobsthal numbers and so on. Let S be a set of m non-zero distinct integers $(a_1, a_2, a_3, ...a_m)$ have the property $D(n), n \in Z-\{0\}$ if $a_i a_j + n$ is a perfect square for all $1 \le i < j \le m$ or $1 \le j < i \le m$ and such a set S is called a Diophantine m-tuple with property $D(n)$. For illustration see [5-14]. This paper exhibits the construction of sequence of Diophantine 3-tuples with property $D(k^2+10k-3)$ through matrix method.

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METHOD OF ANALYSIS

Initially, construct a diophantine 2-tuple with property $D(k^2+10k-3)$ and then, extend it to diophantine 3-tuple.

Let $4,7$ be two distinct integers such that

$4*7+k^2+10k-3=(k+5)^2$, a perfect square

Therefore, the pair $(4,7)$ exhibits diophantine double having property $D(k^2+10k-3)$.

If c is the 3rd tuple, then it satisfies corresponding double equations

$$4c+k^2+10k-3=p^2 \tag{1}$$

$$7c+k^2+10k-3=q^2 \tag{2}$$

The eliminant of c in the above two equations leads to

$$7p^2-4q^2=3(k^2+10k-3) \tag{3}$$

Taking

$$p=X+4T\ ,\ q=X+7T \tag{4}$$

in (3) and simplifying, we get

$$X^2=28T^2+k^2+10k-3$$

which is satisfied by $T=1$, $X=k+5$

From (4) and (1), observe

$$c=2k+21$$

Note that $(4,7,2k+21)$ is a diophantine triple satisfying the property $D(k^2+10k-3)$

The process of obtaining other diophantine triples with property $D(k^2+10k-3)$ is illustrated below:

Let M be a 3×3 square matrix given by

$$M=\begin{pmatrix}1 & 0 & 2\\ 0 & 0 & -1\\ 0 & 1 & 2\end{pmatrix} \tag{5}$$

Now,

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$$(4,7,2k+21)M=(4,2k+21,4k+43)$$

Note that

$$4*(2k+21)+k^2+10k-3=(k+9)^2$$

$$4*(4k+43)+k^2+10k-3=(k+13)^2$$

$$(2k+21)*(4k+43)+k^2+10k-3=(3k+30)^2$$

$\therefore$ The triple $(4,2k+21,4k+43)$ is a diophantine triple having the property $D(k^2+10k-3)$.

Performing the above analysis, the general form of diophantine triple $(4,c_{s-1},c_s)$ is given by

$$(4,\ 4s^2+(2k+2)s-2k+1,\ 4(s+1)^2+(2k+2)s+3),s=1,2,3.......$$

A few numerical illustrations are given in table 1 below:

Table 1: Numerical Illustrations

k	$(4,c_0,c_1)$	$(4,c_1,c_2)$	$(4,c_2,c_3)$	$D(k^2+10k-3)$
0	$(4,7,21)$	$(4,21,43)$	$(4,43,73)$	$D(-3)$
-1	$(4,7,19)$	$(4,19,39)$	$(4,39,67)$	$D(-12)$
-2	$(4,7,17)$	$(4,17,35)$	$(4,35,61)$	$D(-19)$
-3	$(4,7,15)$	$(4,15,31)$	$(4,31,55)$	$D(-24)$
1	$(4,7,23)$	$(4,23,47)$	$(4,47,79)$	$D(8)$

It is noted that the triple (c_{s-1},c_s+4,c_{s+1}), $s=1,2,3.......$ forms an arithmetic progression.

Note 1:

It is obvious that $(7,4,2k+21)$ is a Diophantine 3-tuples with property $D(k^2+10k-3)$.

Following the procedure as above, the Diophantine triples obtained are $(7,4,2k+21)$, $(7,2k+21,4k+52)$, $(19,4k+52,6k+97)$, ... each with property $D(k^2+10k-3)$ whose general form $(7,c_{s-1},c_s)$ is $(7,7s^2+(2k-4)s-2k+1,7(s+1)^2+(2k-4)s-3)$,$s=1,2,3........$

Note that $\left(c_{s-1}, c_s + 7, c_{s+1}\right)$ forms an Arithmetic Progression.

Note 2:

In addition to (4), one may consider the linear transformation given by

$$p = X - 4T\,,\; q = X - 7T$$

For this case, employing the procedure as above one obtains two sets of sequences of Diophantine 3-tuples in which, each triple has the property $D\left(k^2 + 10k - 3\right)$. For simplicity and brevity, the general form of the triple in the sequence of Diophantine 3-tuples is presented:

Set 1: $\left(4, \alpha_{s-1}, \alpha_s\right)$ where $\alpha_{s-1} = 4s^2 + \left(-2k - 18\right)s + 2k + 21,\ s = 1,2,3.......$

Note that $\left(\alpha_{s-1}, \alpha_s + 4, \alpha_{s+1}\right)$ forms an Arithmetic Progression.

Set 2: $\left(7, \alpha_{s-1}, \alpha_s\right)$ where $\alpha_{s-1} = 7s^2 + \left(-2k - 24\right)s + 2k + 21,\ s = 1,2,3.......$

Note that $\left(\alpha_{s-1}, \alpha_s + 7, \alpha_{s+1}\right)$ forms an Arithmetic Progression.

Remark:

Instead of (5), suppose we have a third order square matrix N given by

$$N = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 2 \\ 0 & 1 & 2 \end{pmatrix}$$

Following the procedure presented above, one obtains 4 more sets of Diophantine triples , each with property $D\left(k^2 + 10k - 3\right)$.

To conclude, one may search for other choices of Matrices for the formulation of collections of Diophantine triples with suitable properties.

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