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GENERALIZED ALPHA CLOSED SETS IN SPHERICAL FUZZY TOPOLOGICAL SPACES

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ABSTRACT

This paper is devoted to the study of spherical fuzzy topological spaces. In this paper Spherical fuzzy generalized alpha closed sets and Spherical fuzzy generalized open sets are introduced. Some of its properties are studied.

KEYWORDS: ***Spherical fuzzy topology, Spherical fuzzy generalized alpha closed sets, Spherical fuzzy generalized alpha open sets.***

1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [15] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [3] introduced intuitionistic fuzzy topological[IFS] spaces using the notion of intuitionistic fuzzy sets. IFSs has no ability to obtain any satisfactory result. To overcome this situation Yager [13] developed the idea of Pythagorean fuzzy set (PyFS) as a generalization of IFS, which satises that the value of square summation of its membership degrees is less then or equals to 1. Now the situation where the neutral membership degree calculate independently in real life problems, the IFS and PyFS fail to attain any satisfactory result. Based on these circumstances, to overcome this situation, Cuong and Kreinovich [4] initiated the idea of picture fuzzy set (PFS). He utilized three index (membership degree P(x), neutral-membership degree I(x), and non-membership degree N(x) in PFS with the condition that is $0 \leq P(x) + I(x) + N(x) \leq 1$. Obviously PFSs is more suitable that IFSs and PyFS to deal with fuzziness and vagueness. From last few decades, PFSs has been explored by many researchers and successfully applied to many practical fields like strategic decision making, attribute decision making and pattern recognition [4, 3, 5, 12].

Sometimes in real life, we face many problems which cannot be handled by PFS for example when $P(x) + I(x) + N(x) > 1$. In such condition, PFS has no ability to obtain any satisfactory result. Based on these circumstances, the idea of spherical fuzzy sets (SFSs) is introduced as a generalization of PFS. The spherical fuzzy sets are based on the fact that the hesitancy of a decision maker can be defined independently from membership and non-membership degrees, satisfying the following condition:

$$0 \leq {\mu_A}^2(x) + {\nu_A}^2(x) + {\pi_A}^2(x) \leq 1 \quad \forall\, x \in X.$$

In this paper we introduce spherical fuzzy topology and Spherical fuzzy generalized alpha closed sets. Also we study the Spherical fuzzy generalized alpha open sets and some of their properties.

2. PRELIMINARIES

Definition 2.1.[10] A Spherical Fuzzy Set (SFS) A_x of the universe of discourse X is given by,
$A_x = \{\mu_{A_x}, \ \nu_{A_x}, \pi_{A_x} |\ x \ \in X\}$ where $\mu_{A_x}: X \to [0,1]$, $\nu_{A_x}: X \to [0,1]$, $\pi_{A_x}: X \to [0,1]$ and
$0 \leq \ {\mu_A}^2(x) + \ {\nu_A}^2(x) + {\pi_A}^2(x) \leq 1.$
For each x, the numbers $\mu_{A_x}, \ \nu_{A_x}, \pi_{A_x}$ are the degree of membership, non-membership and hesitancy of x to A_x respectively.

Definition 2.2[10] Assuming that $A = \{\mu_A, \nu_A, \pi_A\}$ and $B = \{\mu_B, \nu_B, \pi_B\}$ be any two SFSs. Then their union, intersection and compliment are described as follows:

i) $A \subseteq B$ iff $\mu_A \leq \mu_B, \nu_A \geq \ \nu_B$ and $\pi_A \geq \pi_B$

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ii) $A = B$ iff $A \subseteq B$ and $B \subseteq A$
iii) $A \cup B = \{max(\mu_A, \mu_B), min(\nu_A, \nu_B), min(\pi_A, \pi_B)\}$
iv) $A \cap B = \{min(\mu_A, \mu_B), max(\nu_A, \nu_B), max(\pi_A, \pi_B)\}$
v) $A^c = \{\nu_A, \mu_A, 1 - \pi_A\}$

3. SPHERICAL FUZZY TOPOLOGY

Definition 3.1 A Spherical fuzzy topology (SFT in short) on X is a family τ of SFSs in X satisfying the following condition:

i) $0_\sim, 1_\sim \in \tau$
ii) $G_1 \cap G_2 \in \tau$ for any $G_{1,} G_2 \in \tau$
iii) $\cup G_i \in \tau$ for any family $\{G_i \mid i \in J\} \subseteq \tau$

In this case the pair (X, τ) is called a Spherical Fuzzy topological space (SFTS in short) and any SFS in τ is known as a spherical fuzzy open set (SFOS in short) in X.

The compliment A^c of a SFOS A in a SFTS in (X, τ) is called a Spherical fuzzy closed set (SFCS in short) in X.

Definition 3.2 Let (X, τ) be any SFS and $A = \{x, \mu_A, \nu_A, \pi_A\}$ be a SFS in X. Then the Spherical fuzzy interior and a spherical fuzzy closure are defined by

$int(A) = \cup \{G \mid G \text{ is a } SFOS \text{ in } X \text{ and } G \subseteq A\}$

$cl(A) = \cap \{K \mid K \text{ is a } SFCS \text{ in } X \text{ and } A \subseteq K\}$.

Note that for any SFS A in $(X, \tau)1$, we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 3.3 A SFS $A = \{x, \mu_A, \nu_A, \pi_A\}$ in a SFTS (X, τ) is said to be a spherical fuzzy semi closed set (SFSCS in short) if $int(cl(A)) \subseteq$ A.

Definition 3.4 A SFS $A = \{x, \mu_A, \nu_A, \pi_A\}$ in a SFTS (X, τ) is said to be a spherical fuzzy semi open set (SFSCS in short) if $A \subseteq cl(int(A))$.

Every SFOS in (X, τ) is a SFSOS in X.

Definition 3.5 A SFS A of a SFTS (X, τ) is a

i) Spherical fuzzy pre closed set (SFPCS in short) if $cl(int(A)) \subseteq$ A.
ii) Spherical fuzzy pre open set (SFPOS in short) if $A \subseteq int(cl(A))$.

Definition 3.6 A SFS A of a SFTS (X, τ) is a

i) Sphercial fuzzy α- open set (SFαOS in short) if $A \subseteq int\left(cl(int(A))\right)$.
ii) Sphercial fuzzy α- closed set (SFαCS in short) if $cl\left(int(cl(A))\right) \subseteq$ A.

The family of all SFαCSs (resp. SFαOSs) of a SFTS (X, τ) is denoted by SFαC(X) (resp. SFαO(X)).

Definition 3.7 A SFS A of a SFTS (X, τ) is a

i) Spherical fuzzy γ- open set (SF γOS in short) if $\subseteq int(cl(A)) \cup cl(int(A))$.
ii) Spherical fuzzy γ- closed set (SF γCS in short) if $cl(int(A)) \cap int(cl(A)) \subseteq A$.

Defintion 3.8 A SFS A of a SFTS (X, τ) is a spherical fuzzy semi pre open set (SFSPOS in short) if there exists a SFPOS B such that $B \subseteq A \subseteq cl(B)$.

Definition 3.9 A SFS A of a SFTS (X, τ) is a spherical fuzzy semi pre closed set (SFSPCS in short) if there exists a SFPCS B such that $int(B) \subseteq A \subseteq B$.

The family of all SFSPCSs (resp. SFSPOSs) of an SFTS (X, τ) is denoted by SFSPC(X) (resp. SFSPO(X)).

Definition 3.10 A SFS A of a SFTS (X, τ) is a

i) Spherical fuzzy regular open set (SFROS in short) if $A = int(cl(A))$.
ii) Spherical fuzzy regular closed set (SFRCS in short) if $A = cl(int(A))$.

Definition 3.11 A SFS A of a SFTS (X, τ) is a spherical fuzzy generalized closed set (SFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a SFOS in X.

Note that every SFCS in (X, τ) is a SFGCS in X.

Definition 3.12 A SFS A of a SFTS (X, τ) is a spherical fuzzy generalized open set (SFGOS in short) if A^c is a SFGCS in X.

Definition 3.13 Let A be a SFS of a SFTS (X, τ). Then the semi closure of A (scl(A) in short) is defined as scl(A) $= \cap \{K \mid K \text{ is a } SFSCS \text{ in } X \text{ and } A \subseteq K\}$.

Definition 3.14 Let A be a SFS of a SFTS (X, τ). Then the semi interior of A (sint(A) in short) is defined as sint(A) $= \cup \{K \mid K \text{ is a } SFSOS \text{ in } X \text{ and } K \subseteq A\}$.

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Definition 3.15 A SFS A of a SFTS (X,τ) is a spherical fuzzy generalized semi closed set (SFGSCS in short) if scl(A) $\subseteq U$ whenever $A \subseteq U$ and U is a SFOS in X.
Note that every SFCS in (X,τ) is a SFGSCS in X.
Definition 3.16 A SFS A of a SFTS (X,τ) is a spherical fuzzy generalized semi open set (SFGSOS in short) if A^c is a SFGSCS in X.
The family of all SFGSCSs (resp. SFGSOSs) of a SFTS (X,τ) is denoted by SFGCS(X) (resp. SFGSO(X).
Definition 3.17 Let A be a SFS of a SFTS (X,τ). Then

i) $scl(A) = A \cup int(cl(A))$.

ii) $sint(A) = A \cap cl(int(A))$.

If A is a SFS of X then $scl(A^c) = (sint(A))^c$.
Definition 3.18 Let A be a SFS of a SFTS (X,τ). Then the pre closure of A (pcl(A) in short) is defined as pcl(A) $= \cap \{K \mid K\ is\ a\ SFPCS\ in\ X\ and\ A \subseteq K\}$.
Definition 3.19 Let A be a SFS of a SFTS (X,τ). Then the pre interior of A (pint(A) in short) is defined as pint(A) = $\cup \{K \mid K\ is\ a\ SFPOS\ in\ X\ and\ K \subseteq A\}$.
Definition 3.20 A SFS A of a SFTS (X,τ) is a spherical fuzzy generalized pre closed set (SFGPCS in short) if pcl(A) $\subseteq U$ whenever $A \subseteq U$ and U is a SFOS in X.
Definition 3.21 A SFS A of a SFTS (X,τ) is a spherical fuzzy generalized pre open set (SFGPOS in short) if A^c is a SFGPCS in X.
Definition 3.22 Let A be a SFS of a SFTS (X,τ). Then the α closure of A (α cl(A) in short) is defined as α cl(A) $= \cap \{K \mid K\ is\ a\ SF\alpha CS\ in\ X\ and\ A \subseteq K\}$.
Definition 3.19 Let A be a SFS of a SFTS (X,τ). Then the α interior of A (α int(A) in short) is defined as α int(A) = $\cup \{K \mid K\ is\ a\ SF\alpha OS\ in\ X\ and\ K \subseteq A\}$.

4. SPHERICAL FUZZY GENERALIZED ALPHA CLOSED SETS

In this section we introduce spherical fuzzy generalized alpha closed sets and study some of their properties.
Definition 4.1 A SFS A of a SFTS (X,τ) is a spherical fuzzy generalized alpha closed set (SFGαCS in short) if α cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is a SFαOS in (X,τ).
The family of all SFGαCSs of a SFTS (X,τ) is denoted by SFG$\alpha C(X)$.
Example 4.2 Let $X = \{a,b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x, (0.2, 0.3, 0.7), (0.4, 0.3, 0.5)\rangle$. Here the only α open sets are $0_\sim, 1_\sim\ and\ G$. Then the SFS $A = \langle x, (0.2, 0.4, 0.6), (0.3, 0.5, 0.7)\rangle$ is a SFGαCS in (X,τ).
Theorem 4.3 Every SFCS in (X,τ) is a SFGαCS, but not conversely.
Proof. Let $A \subseteq U$ and U is a SFαOS in (X,τ). Since $\alpha cl(A) \subseteq cl(A)$ and A is a SFCS, $\alpha cl(A) \subseteq cl(A) = A \subseteq U$. Therefore A is a SFGαCS in X.
Example 4.4 Let $X = \{a,b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x, (0.2, 0.3, 0.6), (0.3, 0.3, 0.5)\rangle$. Let $A = \langle x, (0.1, 0.2, 0.7), (0.2, 0.4, 0.7)\rangle$ be any SFS in X. Here $\alpha cl(A) \subseteq G$ whenever $A \subseteq G$ for all SFαOS G in X. A is a SFαCS, but not a SFCS in X, since $cl(A) = G^c \neq A$.
Theorem 4.5 Every SFαCS is a SF$G\alpha$CS but not conversely.
Proof. Let $A \subseteq U$ and U is a SFαOS in (X,τ). By hypothesis $\alpha cl(A) = A$. Hence $\alpha cl(A) \subseteq U$. Therefore A is a SF$G\alpha$CS in X.
Example 4.6 Let $X = \{a,b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x, (0.3, 0.3, 0.5), (0.4, 0.2, 0.5)\rangle$. Let $A = \langle x, (0.1, 0.5, 0.5), (0.5, 0.5, 0.7)\rangle$ be any SFS in X. Clearly $\alpha cl(A) \subseteq G$ whenever $A \subseteq G$ for all SFαOS G in X. Therefore A is a SFGαCS, but not a SFαCS in X, since $cl\left(int(cl(A))\right) = G^c \nsubseteq A$.
Theorem 4.7 Every SFRCS is a SFGαCS, but not conbersely.
Proof. Let A be a SFRCS in (X,τ). By definition $A = cl(int(A))$. This implies $cl(A) = cl(int(A))$. Therefore $cl(A) = A$. That is A is a SFCS in X. By theorem 4.3, A is a SFGαCS in X.
Example 4.8 Let $X = \{a,b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x, (0.2, 0.3, 0.6), (0.3, 0.2, 0.5)\rangle$. Let $A = \langle x, (0.2, 0.4, 0.7), (0.4, 0.2, 0.7)\rangle$ be any SFS in X. A is a SFGαCS, but not SFRCS in (X,τ) since $cl(int(A)) = G^c \neq A$.
Example 4.9 Every SFGCS need not be SFGαCS in X.
Let $X = \{a,b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where

$G = \langle x, (0.3, 0.4, 0.5), (0.3, 0.4, 0.7)\rangle$. Let $A = \langle x, (0.2, 0.5, 0.5), (0.4, 0.5, 0.8)\rangle$ be any SFS in X. Here A is a SFGCS in X. Consider the SFαOS $G_1 = \langle x, (0.5, 0.2, 0.1), (0.5, 0.2, 0.6)\rangle$. Here $A \subseteq G_1$ but $\alpha cl(A) \nsubseteq G_1$. Hence A is not a SFGαCS in (X, τ).

Theorem 4.10 Every SFGαCS is a SFαGCS in X. But the converse is not true in general.

Proof. Let $A \subseteq U$ and U is a SFαOS in (X, τ). Since every α open set is an open set, we have $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a SFOS in (X, τ). Hence A is a SFαGCS in X.

Example 4.11 Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x, (0.4, 0.3, 0.7), (0.3, 0.4, 0.8)\rangle$. Let $A = \langle x, (0.3, 0.4, 0.6), (0.4, 0.5, 0.5)\rangle$ be any SFS in X. Here A is a SFαGCS in X. Consider the SFαOS $G_1 = \langle x, (0.3, 0.3, 0.5), (0.4, 0.5, 0.5)\rangle$. Here $A \subseteq G_1$ but $\alpha cl(A) \nsubseteq G_1$. Hence A is not a SFGαCS in (X, τ).

Theorem 4.12 Every SFGαCS is a SFGSCS but its converse may not be true.

Proof. Let $A \subseteq U$ and U is a SFαOS in (X, τ). By hypothesis, $\alpha cl(A) \subseteq U$, which implies $cl\left(int\big(cl(A)\big)\right) \subseteq U$. That is, $int\big(cl(A)\big) \subseteq U$, which implies $A \cup int\big(cl(A)\big) \subseteq U$. Therefore $scl(A) \subseteq U$, U is a SFOS. Therefore A is a SFGSCS in (X, τ).

Example 4.13 Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x, (0.2, 0.5, 0.7), (0.4, 0.5, 0.6)\rangle$ be any SFS in X. Then $scl(A) = G$. Clearly $scl(A) \subseteq G$, whenever $A \subseteq G$, for all SFOS G in X. A is a SFGSCS in (X, τ), but not SFGαCS, since $\alpha cl(A) = G^c \nsubseteq G$.

Theorem 4.14 Every SFGαCS is a SFGPCS but its converse may not be true.

Proof. Let $A \subseteq U$ and U is a SFαOS in (X, τ). By hypothesis, $\alpha cl(A) \subseteq U$, which implies $cl\left(int\big(cl(A)\big)\right) \subseteq U$. That is, $cl\big(int(A)\big) \subseteq U$, which implies $A \cup cl\big(int(A)\big) \subseteq U$. Therefore $pcl(A) \subseteq U$, U is a SFOS. Therefore A is a SFGPCS in (X, τ).

Example 4.15 Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x, (0.3, 0.4, 0.7), (0.4, 0.3, 0.6)\rangle$. Here the only α open sets are $0_\sim, 1_\sim$ *and* G. Let $A = \langle x, (0.2, 0.4, 0.8), (0.3, 0.5, 0.6)\rangle$ be any SFS in X. $pcl(A) \subseteq G$. Therefore A is a SFGPCS in (X, τ), but not a SFGαCS, since $\alpha cl(A) = G^c \nsubseteq G$.

Remark 4.16 A SFP closedness is independent of a SFGα closedness.

Example 4.17 Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x, (0.2, 0.5, 0.7), (0.3, 0.4, 0.5)\rangle$. Here the only α open sets are $0_\sim, 1_\sim$ *and* G. Let $A = \langle x, (0.3, 0.4, 0.7), (0.3, 0.4, 0.6)\rangle$ be a SFGαCS(X) but not a SFPCS(X), since $cl\big(int(A)\big) = G^c \nsubseteq G$.

Example 4.18 Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x, (0.4, 0.5, 0.5), (0.4, 0.4, 0.3)\rangle$. Let $A = \langle x, (0.4, 0.5, 0.5), (0.3, 0.4, 0.7)\rangle$ be a SFPCS(X) but not a SFGαCS(X), since $\alpha cl(A) = G^c \nsubseteq G$.

Remark 4.19 A SFS closedness is independent of a SFGα closedness.

Example 4.20 Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x, (0.3, 0.4, 0.5), (0.4, 0.3, 0.3)\rangle$. Let $A = \langle x, (0.2, 0.4, 0.4), (0.3, 0.4, 0.7)\rangle$ be a SFGαCS(X) but not a SFGSCS(X), since $int\big(cl(A)\big) = G \nsubseteq A$.

Example 4.21 Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x, (0.3, 0.5, 0.6), (0.4, 0.5, 0.6)\rangle$. Let $A = \langle x, (0.3, 0.5, 0.6), (0.4, 0.5, 0.6)\rangle$ be a SFSCS(X) but not a SFGαCS(X), since $\alpha cl(A) = G^c \nsubseteq G$.

Remark 4.22 The intersection of any two SFGαCS is not a SFGαCS in general as seen from the following example.

Example 4.23 Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x, (0, 0.2, 0.6), (0.1, 0.2, 0.5)\rangle$. Then the SFSs $A = \langle x, (0, 0.3, 0.7), (0.3, 0.3, 0.5)\rangle$, $B = \langle x, (0, 0.4, 0.5), (0.1, 0.1, 0.4)\rangle$ are SFGαCSs but $A \cap B$ is not a SFGαCS in X.

Theorem 4.24 If A is a SFOS and a SFGαCS in (X, τ), then A is a SFαCS in X.

Proof. Let A be a SFOS in X. Since $A \subseteq A$, by hypothesis $\alpha cl(A) \subseteq A$. But from the definition $A \subseteq \alpha cl(A)$. Therefore $\alpha cl(A) = A$. Hence A is a SFαCS of X.

Theorem 4.25 Let (X, τ) be a SFTS. Then $SF\alpha O(X) = SF\alpha C(X)$ if and only if every SFS in (X, τ) is a SFGαCS.

Proof. **Necessity:**

Suppose that $SF\alpha O(X) = SF\alpha C(X)$. Let $A \subseteq U$ and U is a SFOS in X. This implies $\alpha cl(A) \subseteq \alpha cl(U)$ and U is a $SF\alpha OS$ in X. Since by hypothesis U is a $SF\alpha CS$ in X, $\alpha cl(U) = U$. This implies $\alpha cl(A) \subseteq U$. Therefore A is a $SFG\alpha CS$ of X.

Sufficiency:

Suppose that every SFS in (X,τ) is a $SFG\alpha CS$. Let $U \in SFO(X)$, then $U \in SF\alpha O(X)$ and by hypothesis $\alpha cl(U) \subseteq U \subseteq \alpha cl(U)$. Therefore $U \in SF\alpha C(X)$. Hence $SF\alpha O(X) \subseteq SF\alpha C(X)$. Let $A \in SF\alpha C(X)$, then A^c is a $SF\alpha OS$ in X. But $SF\alpha O(X) \subseteq SF\alpha C(X)$. Therefore $A \in SF\alpha O(X)$. Hence $SF\alpha C(X) \subseteq SF\alpha O(X)$. Thus $SF\alpha O(X) = SF\alpha C(X)$.

5. SPHERICAL FUZZY GENERALIZED ALPHA OPEN SETS

In this section we introduce Spherical Fuzzy generalized alpha open sets and studied some of its properties.

Definition 5.1. A SFS A is said to be a Spherical fuzzy generalized alpha open set ($SFG\alpha OS\ in\ short$) in (X,τ) if the compliment A^c is a $SFG\alpha CS$ in X.
The family of all $SFG\alpha OSs$ of a $SFTS\ (X,\tau)$ is denoted by $SFG\alpha O(X)$.

Theorem 5.2. For any $SFTS\ (X,\tau)$, we have the following:

1. Every $SFOS$ is a $SFG\alpha OS$.
2. Every $SF\alpha OS$ is a $SFG\alpha OS$.
3. Every $SFROS$ is a $SFG\alpha OS$. But the converse is not true in general.

Proof. Straight forward.

Example 5.3. Let $X = \{a,b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x,(0.2,0.3,0.7),(0.3,0.2,0.6)\rangle$. Here the only α open sets are $0_\sim, 1_\sim\ and\ G$. Let $A = \langle x,(0.2,0.4,0.5),(0.3,0.3,0.6)\rangle$ be any SFS in X. A is a $SFG\alpha OS$, but not a $SFOS$ in X.

Example 5.4. Let $X = \{a,b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x,(0.2,0.3,0.7),(0.3,0.2,0.6)\rangle$. Here the only α open sets are $0_\sim, 1_\sim\ and\ G$. Let $A = \langle x,(0.2,0.4,0.5),(0.3,0.3,0.6)\rangle$ be any SFS in X. A is a $SFG\alpha OS$, but not a $SF\alpha OS$ in X.

Example 5.5. Let $X = \{a,b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x,(0.3,0.4,0.6),(0.3,0.4,0.7)\rangle$. Here the only α open sets are $0_\sim, 1_\sim\ and\ G$. Let $A = \langle x,(0.4,0.3,0.5),(0.3,0.3,0.6)\rangle$ be any SFS in X. A is a $SFG\alpha OS$, but not a $SFROS$ in X.

Theorem 5.6. For any $SFTS\ (X,\tau)$, we have the following:

1. Every $SFG\alpha OS$ is a $SFGSOS$.
2. Every $SFG\alpha OS$ is a $SFSGOS$.
3. Every $SFG\alpha OS$ is a $SFGPOS$. But the converses are not true in general.

Proof. Straight Forward.

Example 5.7. Let $X = \{a,b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x,(0.4,0.5,0.5),(0.3,0.5,0.6)\rangle$. Here the only α open sets are $0_\sim, 1_\sim\ and\ G$. Let $A = \langle x,(0.6,0.3,0.4),(0.7,0.2,0.2)\rangle$ be any SFS in X. A is a $SFGSOS$, but not a $SFG\alpha OS$ in X.

Example 5.8. Let $X = \{a,b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x,(0.3,0.4,0.5),(0.2,0.5,0.6)\rangle$. Here the only α open sets are $0_\sim, 1_\sim\ and\ G$. Let $A = \langle x,(0.5,0.2,0.5),(0.5,0.1,0.4)\rangle$ be any SFS in X. A is a $SFSGOS$, but not a $SFG\alpha OS$ in X.

Example 5.9. Let $X = \{a,b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x,(0.3,0.4,0.5),(0.2,0.5,0.6)\rangle$. Here the only α open sets are $0_\sim, 1_\sim\ and\ G$. Let $A = \langle x,(0.5,0.2,0.5),(0.5,0.1,0.4)\rangle$ be any SFS in X. A is a $SFGPOS$, but not a $SFG\alpha OS$ in X.

Remark 5.10. The union of any two $SFG\alpha OS$ is not a $SFG\alpha OS$ in general as seen from the following example.

Example 5.11. Let $X = \{a,b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is a spherical fuzzy topology on X, where $G = \langle x,(0.4,0.3,0.5),(0.2,0.5,0.6)\rangle$. Here the only α open sets are $0_\sim, 1_\sim\ and\ G$. Let $A = \langle x,(0.1,0.3,0.4),(0.3,0.4,0.4)\rangle$, $B = \langle x,(0.3,0.2,0.3),(0.4,0.5,0.5)\rangle$ are SFGαOSs but $A \cup B$ is not a SFGαOS in X.

Theorem 5.12. A $SFS\ A$ of a $SFTS\ (X,\tau)$ is a SFGαOS if and only if $G \subseteq \alpha int(A)$, whenever G is a SFαCS(X) and $G \subseteq A$.

Proof. **Necessity:**

Assume that A is a SFGαOS in X. Also let G be a SFαCS in X such that $G \subseteq A$. Then G^c is a SFαOS in X such that $A^c \subseteq G^c$. Since A^c is a SFGαCS, $\alpha cl(A^c) \subseteq G^c$. But $\alpha cl(A^c) = \left(\alpha int(A)\right)^c$. Hence $\left(\alpha int(A)\right)^c \subseteq G^c$. This implies $G \subseteq \alpha int(A)$.

Sufficiency:

Assume that $G \subseteq \alpha int(A)$, whenever G is a SFαCS and $G \subseteq A$. Then $\left(\alpha int(A)\right)^c \subseteq G^c$, whenever G^c is a SFαOS and $\alpha cl(A^c) \subseteq G^c$. Therefore A^c is a SFGαCS. This implies A is a SFGαOS.

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