ISSN (Online): 2455-3662
EPRA International Journal of Multidisciplinary Research (IJMR) - Peer Reviewed Journal
Volume: 6 | Issue: 9 | September 2020 || Journal DOI: 10.36713/epra2013 || SJIF Impact Factor: 7.032 ||ISI Value: 1.188

UNLOADING WAVES IN ELASTIC-PLASTIC FILTRATION OF A LIQUID IN UNSTABLE FORMATIONS

Kholiyarov Erkin Chorshanbiyevich
PhD in Physics and Mathematics,
Docent of the Department of Information Technology,
Termez State University

Toyirov Akbar Khasanovich
PhD Student
Department of Applied Mathematics and Informatics,
Termez State University

Kholliyev Fakhriddin Bokhodirovich
Teacher
Department of Applied Mathematics and Informatics,
Termez State University

ABSTRACT

Problems of unloading wave propagation in elastic-plastic filtration mode in unstable formations are considered. Unloading wave fronts in unstable formations in plane-radial formulations are determined. The numerical solution of the problem of elastic-plastic filtration in unstable formations is given and the features associated with the formation instability together with the irreversibility of deformation of the rock skeleton are determined. The influence of reservoir instability and the intensity of irreversible deformation on the propagation of the unloading wave has been studied.

KEYWORDS: ***Sustainable, deep-seated, deformations, pressure distribution, discharge wave front, Iterative process.***

INTRODUCTION

The issues of elastic-plastic filtration of liquids are becoming increasingly important in connection with the development of oil and gas fields at great depths. In deep-seated oil and gas fields, especially with abnormally high reservoir pressures (abnormally high reservoir pressures), as they are exploited, reservoir pressure significantly decreases, which leads to the occurrence of large effective stresses on the reservoir. This, in turn, causes significant deformations of the formation rock skeleton. One of the features of the formation deformation at great depths is the violation of its elasticity, i.e. transition from the elastic limit of rocks, which is accompanied by the appearance of plastic (irreversible) changes in reservoir characteristics. For the first time, attention was drawn to the irreversibility of changes in the permeability of oil reservoirs in [1, 2], although in oil practice such a decrease in reservoir properties of reservoirs as they are exploited (ie, decrease in reservoir pressure) have been known for a long time. The model schematization of the elastoplastic filtration regime was first proposed in [3, 4], which then formed the basis for the formation of ideas about the elastoplastic filtration in general and the assessment of its characteristics [5]. Note that in [3, 4], both in the mode of lowering and in the mode of recovery of reservoir pressure, the main relations of the dynamics of porosity of the reservoir depending on the current pressure are taken to be linear. Taking into account the nonlinearity of the formation deformation [6, 7], the equations of the elastic-plastic filtration mode are given in [8, 9], where the elastic-plastic change in the formation porosity and permeability is simultaneously taken into account. At great depths, oil reservoirs confined to poorly cemented terrigenous reservoirs, especially with abnormal pressures, lose their stability and integrity as effective stresses increase. This leads to the destruction of the skeleton of the formation rocks and the removal of rock particles formed due to destruction to the surface together with the produced oil. The latter can be the cause of major technological complications both in the process of oil production and its field treatment. Models of elastic-plastic fluid filtration in unstable reservoirs are proposed in [10, 11]. Analysis of the results of the implementation of the models shows that in unstable reservoirs the factors of irreversibility of deformation and reservoir instability have reciprocal effects on the reservoir properties of the reservoir.

The issues of unloading wave propagation in a porous medium are topical from both theoretical and practical points of view. The propagation of an unloading wave in a porous medium was first studied in [4]. In [9, 12], the problems of elastic-plastic filtration and propagation of the unloading wave are given taking into account the nonlinearity of deformation of the formation rocks using a number of simplifying assumptions. For a generalized model of elastic-plastic filtration in stable [8, 9] and unstable [11] formations, the problems of unloading wave propagation have not been studied.

In this work, on the basis of the well-known equation [8, 9], we first determine the unloading wave fronts for a nonlinearly deformable stable formation using the example of a plane-radial setting. Further, the same estimate is carried out for the one-dimensional plane-parallel case. Using both schematization of the reservoir (plane-parallel, plane-radial), the unloading wave fronts are numerically determined for an unstable reservoir. The influence of deformation plasticity and formation instability on the nature of the unloading wave propagation is estimated.

MAIN PART

Here the problems of elastic-plastic fluid filtration in an unstable radial-plane formation are considered. Some problems for a linear one-dimensional unstable reservoir were solved in [11, 14, 15]. The change in the coefficients of permeability and porosity depending on pressure will be written in the form [10, 11]

$$\downarrow k = k_0 \exp(-a_{k0}(p_0 - p)) + \theta(p_s - p) k_{s0}\left[1 - \exp(-a_{ks}(p_s - p))\right] \quad (1)$$

$$\downarrow m = m_0 \exp(-\beta_{m0}(p_0 - p)) + \theta(p_s - p) m_{s0}\left[1 - \exp(-\beta_{ms}(p_s - p))\right] \quad (2)$$

$$\uparrow k = k_0 \exp(-(a_{k0} - a_{k1})(p_0 - p_1)) \exp(-a_{k1}(p_0 - p)) + \\ + \theta(p_s - p_1) k_{s0}\left[1 - \exp(-a_{ks}(p_s - p_1))\right] \quad (3)$$

$$\uparrow m = m_0 \exp(-(\beta_{m0} - \beta_{m1})(p_0 - p_1)) \exp(-\beta_{m1}(p_0 - p)) + \\ + \theta(p_s - p_1) m_{s0}\left[1 - \exp(-\beta_{ms}(p_s - p_1))\right] \quad (4)$$

where p_s is the pressure at which the integrity of the formation is violated, $\theta(r)$ is the unit function of Heaviside, k_{s0}, m_{s0} is the maximum possible increase in k and m due to the removal of particles; a_{ks}, β_{ms} - coefficients of change k and m due to the removal of particles; $a_{k1} = a_{k0}\exp(-\eta_k(p_0 - p_1))$, $\beta_{m1} = \beta_{m0}\exp(-\eta_m(p_0 - p_1))$, $p_0 \geq p_1, \eta_m, \eta_k$ - coefficients of irreversible changes in porosity and permeability, p_1 - pressure distribution at the end of the pressure reduction phase.

Using (1) - (4), as well as the continuity equation and Darcy's law, we obtain the following filtration equations in the one-dimensional plane-radial case

$$\downarrow \frac{\partial}{\partial t}\left[\varphi + \frac{m_{s0}\theta(\sigma(\varphi))}{m_0}\left(1 - \delta_1 \varphi^{\beta_{ms}/\beta}\right)\varphi^{\beta_f/\beta}\right] = \\ = \chi_0 \frac{1}{r}\frac{\partial}{\partial r}\left\{\left[\varphi^{\gamma-1} + \frac{k_{s0}\theta(\sigma(\varphi))}{k_0}\left[1 - \delta_2 \varphi^{a_{ks}/\beta}\right]\varphi^{-(a_\mu + \beta_{m0})/\beta}\right] r \frac{\partial \varphi}{\partial r}\right\} \quad (5)$$

$$\uparrow \frac{\partial}{\partial t}\left\{\exp[-\varphi_2(r)]\exp[-\psi_2(r)(p_0 - p)] + \xi_2(r)\exp(-\beta_f(p_0 - p))\right\} = \chi_2 \frac{1}{r}\frac{\partial}{\partial r} \times \\ \times \left\{\left[\exp[-\varphi_1(r)]\exp[-\psi_1(r)(p_0 - p)] + \xi_1(r)\exp(-(\beta_f - a_\mu)(p_0 - p))\right] r \frac{\partial p}{\partial r}\right\} \quad (6)$$

where

$$\varphi = \exp(-\beta(p_0 - p)),\ \beta = \beta_{m0} + \beta_f,\ \varphi(t_0, r) = \exp(-\beta(p_0 - p_1)) = \psi(r),$$

$$\chi_2 = \frac{k_0}{m_0 \mu_0},\ \sigma(\varphi) = p_s - p_0 - (1/\beta)\ln\varphi,\ \delta_1 = \exp[-\beta_{ms}(p_0 - p_s)],$$

$$\delta_2 = \exp[-a_{ks}(p_0 - p_s)], \psi_1(r) = \beta_f - a_\mu + a_{k0}\psi(r)^{\eta_k/\beta},$$

$$\psi_2(r) = \beta_f + \beta_{m0}\psi(r)^{\eta_m/\beta}, \xi_1(r) = \frac{k_{s0}}{k_0}\left[1 - \delta_1(\psi(r))^{a_{ks}/\beta}\right]\theta(p_s - p_1),$$

$$\xi_2(r) = \frac{m_{s0}}{m_0}\left[1 - \delta_2(\psi(r))^{\beta_{ms}/\beta}\right]\theta(p_s - p_1),$$

$$\varphi_1(r) = a_{k0}\left[1 - \psi(r)^{\eta_k/\beta}\right]\left[-\frac{1}{\beta}\ln|\psi(r)|\right],$$

$$\varphi_2(r) = \beta_{m0}\left[1 - \psi(r)^{\eta_m/\beta}\right]\left[-\frac{1}{\beta}\ln|\psi(r)|\right]$$

To assess the change in pressure and other filtration characteristics in the modes of lowering and recovering reservoir pressure, it is necessary to solve equations (5) and (6) with the corresponding initial and boundary conditions. In view of their nonlinearity, numerical methods are the most appropriate.

To estimate the solutions of equations (5), (6), we formulate the following problem. Let in a semi-infinite planar-radial one-dimensional formation at $r = r_c$, starting from $t > 0$, pressure mode $p = p_c < p_0,\ p_c = const$ is set. Initially, the reservoir had a constant pressure distribution $p = p_0$.

In accordance with the formulation of the problem, the initial and boundary conditions in the pressure reduction mode have the form

$$p(0, r) = p_0, \quad p(t, r_c) = p_c, \quad p(t, \infty) = p_0 \tag{7}$$

or in notation relative to φ:

$$\varphi(0, r) = 1, \quad \varphi(t, r_c) = \varphi_c, \quad \varphi(t, \infty) = 1 \tag{8}$$

where $\varphi_c = \exp(-\beta(p_0 - p_c))$.

In the pressure recovery mode, the initial and boundary conditions are as follows:

$$p(t_0, r) = p_1, \quad \partial p(t, r_c)/\partial r = 0, \quad p(t, \infty) = p_0 \tag{9}$$

To solve the formulated problem, we use the finite difference method [16]. In area $D = \{0 \le r < \infty,\ 0 \le t \le t_0\}$, enter grid $\omega_{h\tau} = \{(r_i, t_j),\ i = 0, 1, \ldots,\ j = \overline{0, J},\ r_i = ih,\ t_j = j\tau,\ \tau = t_0/J\}$, where $h,\ \tau$ are grid steps of r and t. The grid solution corresponding to point (r_i, t_j) is denoted by φ_i^j.

Equation (5) is first written in the form

$$\downarrow \frac{\partial u(\varphi)}{\partial t} = \chi_0 \frac{1}{r}\frac{\partial}{\partial r}\left[r\, v(\varphi)\frac{\partial \varphi}{\partial r}\right] \tag{10}$$

where

$$u(\varphi) = \varphi + \frac{m_{s0}\theta(\sigma(\varphi))}{m_0}\left(1 - \delta_1\varphi^{\beta_{ms}/\beta}\right)\varphi^{\beta_f/\beta}$$

$$v(\varphi) = \varphi^{\gamma-1} + \frac{k_{s0}\theta(\sigma(\varphi))}{k_0}\left(1 - \delta_2\varphi^{a_{ks}/\beta}\right)\varphi^{-(a_\mu + \beta_{m0})/\beta}$$

Equation (10) on grid $\omega_{h\tau}$ is approximated with an accuracy of $O\left(\tau + h^2\right)$ by an explicit finite-difference scheme

$$\frac{u(\varphi_i^{j+1}) - u(\varphi_i^j)}{\tau} = \chi_1 \frac{1}{r_i h^2}\Big[r_{i-1/2} a_i(\varphi_i^j)\varphi_{i-1}^j - \left(r_{i-1/2} a_i(\varphi_i^j) + r_{i+1/2} a_{i+1}(\varphi_i^j)\right)\varphi_i^j + r_{i+1/2} a_{i+1}(\varphi_i^j)\varphi_{i+1}^j\Big] \qquad (11)$$

where $a_i\left(\varphi_i^j\right) = \frac{1}{2}\left[v\left(\varphi_{i-1}^j\right) + v\left(\varphi_i^j\right)\right]$.

From (11) we obtain the grid equations of the following form:

$$\varphi_i^{j+1} = V\left(\varphi_i^{j+1}\right) + F_i^j, \; i = \overline{1, I-1} \qquad (12)$$

where

$$V\left(\varphi_i^{j+1}\right) = -\frac{m_{s0}}{m_0}\left[1 - \delta_1\left(\varphi_i^{j+1}\right)^{\beta_{ms}/\beta}\right]\left(\varphi_i^{j+1}\right)^{\beta_f/\beta}\theta\left(\sigma\left(\varphi_i^{j+1}\right)\right),$$

$$F_i^j = \frac{\chi_1 \tau}{r_i h^2}\left[r_{i-1/2}\, a_i\left(\varphi_i^j\right)\varphi_{i-1}^j - \left(r_{i-1/2}\, a_i\left(\varphi_i^j\right) + r_{i+1/2}\, a_{i+1}\left(\varphi_i^j\right)\right)\varphi_i^j + r_{i+1/2}\, a_{i+1}\left(\varphi_i^j\right)\varphi_i^j\right] + \varphi_i^j + \frac{m_{s0}}{m_0}\left[1 - \delta_1\left(\varphi_i^j\right)^{\beta_{ms}/\beta}\right]\left(\varphi_i^j\right)^{\beta_{ж}/\beta}\theta\left(\sigma\left(\varphi_i^j\right)\right),$$

I is taken large enough so that $[r_c, r_I]$ contains the area of variation of other indicators of the problem.

Thus, in the pressure reduction mode, a nonlinear system of difference equations (12) was obtained, which was solved by a simple iteration method. The iterative process continues until conditions $\left|\left(\varphi_i^{j+1}\right)^{s+1} - \left(\varphi_i^{j+1}\right)^s\right| \le \varepsilon, \; i = \overline{0, I},$ are satisfied where ε is the accuracy of calculations, s is the iteration number.

Equation (6) after the finite-difference approximation is reduced to grid equations with respect to p_i^j, similar to (12):

$$p_i^{j+1} = K\left(p_i^{j+1}\right) + G_i^j, \quad i = \overline{1, I-1} \qquad (13)$$

where

$$K\left(p_i^{j+1}\right) = p_i^{j+1} - \exp\left[-\varphi_2(r_i)\right]exp\left[-\psi_2(r_i)\left(p_0 - p_i^{j+1}\right)\right] - \xi_2(r_i)\exp\left[-\beta_f\left(p_0 - p_i^{j+1}\right)\right]$$

$$G_i^j = \frac{\chi_2 \tau}{r_i h^2}\left[r_{i-1/2}\, c_i\left(p_i^j\right)p_{i-1}^j - \left(r_{i-1/2}\, c_i\left(p_i^j\right) + r_{i+1/2}\, c_{i+1}\left(p_i^j\right)\right)p_i^j + r_{i+1/2}\, c_{i+1}\left(p_i^j\right)p_{i+1}^j\right] + \exp\left[\varphi_2(r_i)\right]exp\left[-\psi_2(r_i)\left(p_0 - p_i^j\right)\right] + \xi_2(r_i)\exp\left[-\beta_f\left(p_0 - p_i^j\right)\right]$$

$$c_i\left(p_i^j\right) = \left[q\left(p_{i-1}^j\right) + q\left(p_i^j\right)\right]/2$$

$$q\left(p_i^j\right) = exp\left[-\varphi_1(r_i)\right]\exp\left[-\psi_1(r_i)\left(p_0 - p_i^j\right)\right] + \xi_1(r_i)\exp\left[-\left(\beta_f - a_\mu\right)\left(p_0 - p_i^j\right)\right]$$

The iterative process continues until conditions $\left|\left(p_i^{j+1}\right)^{s+1} - \left(p_i^{j+1}\right)^s\right| \le \varepsilon, \quad i = \overline{0, I}$. are met.

The initial and boundary conditions (8) are approximated as follows:

$$\varphi_i^0 = 1, \quad \varphi_0^{j+1} = \varphi_c, \quad \varphi_I^{j+1} = 1 \qquad (14)$$

at (9):

ISSN (Online): 2455-3662
EPRA International Journal of Multidisciplinary Research (IJMR) - Peer Reviewed Journal
Volume: 6 | Issue: 9 | September 2020 || Journal DOI: 10.36713/epra2013 || SJIF Impact Factor: 7.032 ||ISI Value: 1.188

$$p_i^0 = p_1(r_i), \quad p_0^{j+1} = \frac{4p_1^{j+1} - p_2^{j+1}}{3}, \quad p_I^{j+1} = p_0 \tag{15}$$

RESULTS AND DISCUSSION

The following initial data were used in the calculations: p_0=100 MPa, p_s=90 MPa, μ_0=2,0 Pa·s, k_0=10^{-12} m^2, m_0=0,15, a_μ=5·10^{-4} MPa^{-1}, β_f=10^{-3} MPa^{-1}, a_{k0}=0,02 MPa^{-1}, a_{ks}=0,015 MPa^{-1}, β_{m0}=0,015 MPa^{-1}, β_{ms}=0,01 MPa^{-1}, $t_0 = 2000$ s.

The filtration rate in the pressure reduction mode is determined as follows

$$w = -\frac{k_0}{\mu_0 \beta} \varphi^{\frac{\alpha-\beta}{\beta}-1} \frac{\partial \varphi}{\partial r} \tag{16}$$

At point $r = r_c$, finite-difference approximation (16) has the form

$$w_0^{j+1} = -\frac{k_0}{\mu_0 \beta} \left(\varphi_0^{j+1}\right)^{\frac{\alpha-\beta}{\beta}-1} \frac{4\varphi_1^{j+1} - \varphi_2^{j+1} - 3\varphi_0^{j+1}}{2h} \tag{17}$$

Some calculation results are shown in Fig. 1, 2. Analysis of the results shows that in this case, in comparison with the plane-parallel [11,14,15], the process of lowering the reservoir pressure occurs more slowly, the filtration rate is relatively higher in the destroyed zone, and the propagation of the boundary of this zone into the interior of the reservoir is slower. With increasing coefficients k_{s0}, m_{s0}, the intensity of destruction of the formation skeleton increases, which is reflected in profiles k, m, p (Fig. 1 and Fig. 2). In the destruction zone k and m are restored more significantly than at small k_{s0} and m_{s0}.

ISSN (Online): 2455-3662
EPRA International Journal of Multidisciplinary Research (IJMR) - Peer Reviewed Journal
Volume: 6 | Issue: 9 | September 2020 || Journal DOI: 10.36713/epra2013 || SJIF Impact Factor: 7.032 ||ISI Value: 1.188

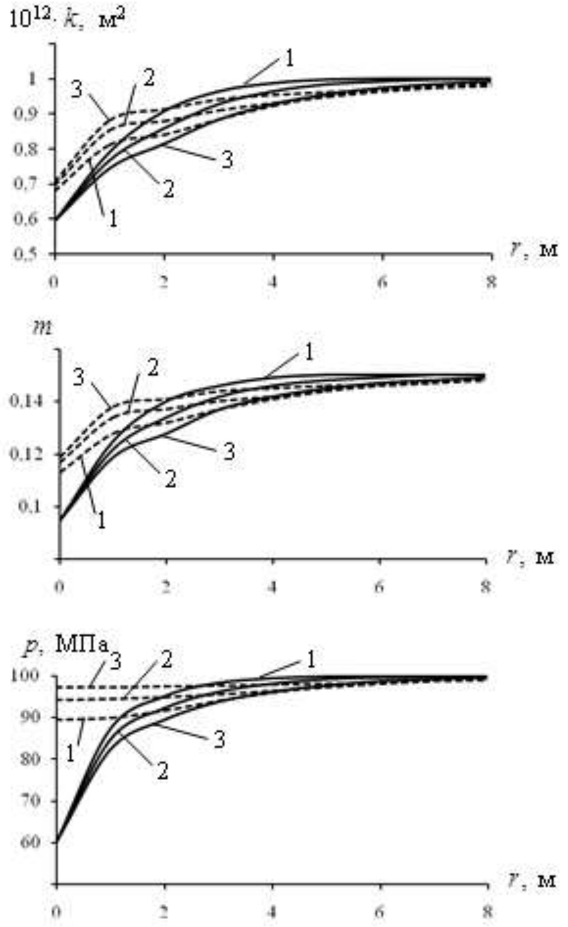

Figure: 1. Distribution k, m, p in the mode of pressure reduction (solid lines) at $t = 500$ s (1), 1000 (2), 2000 (3) and pressure recovery (dashed lines) at $t = 2300$ s (1), 3000 (2), 5000 (3)): $k_{s0} = 0{,}4 \cdot 10^{-12}$ m², $m_{s0} = 0{,}05$;

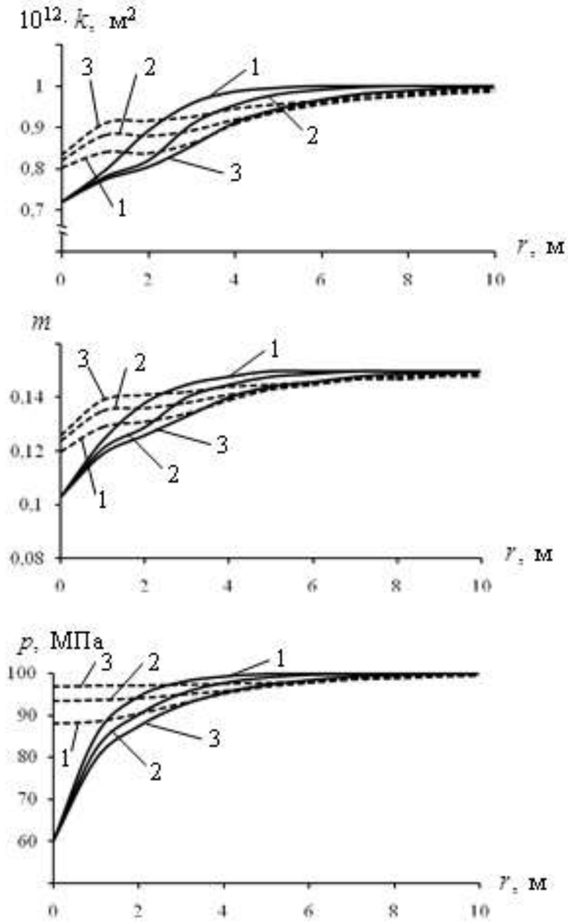

Figure: 2. Distribution k, m, p in the mode of pressure reduction (solid lines) at $t = 500$ s (1), 1000 (2), 2000 (3) and pressure recovery (dashed lines) at $t = 2300$ s (1), 3000 (2), 5000 (3)): $k_{s0} = 0{,}75 \cdot 10^{-12}$ m², $m_{s0} = 0{,}08$

Now, on the basis of the above solution to the problem of elastic-plastic filtration in unstable formations, we will estimate the unloading wave front. Determining the pressure field in the pressure recovery mode according to (13) on the pressure change graphs, we find points with $\Delta p = 0$, which is the front of the unloading wave. Typical results for one set of initial parameters are shown in Fig. 3. As can be seen from the presented results, an increase in the formation plastic properties (an increase in η_k and η_m) leads to an intensification of the propagation of unloading waves in the formation.

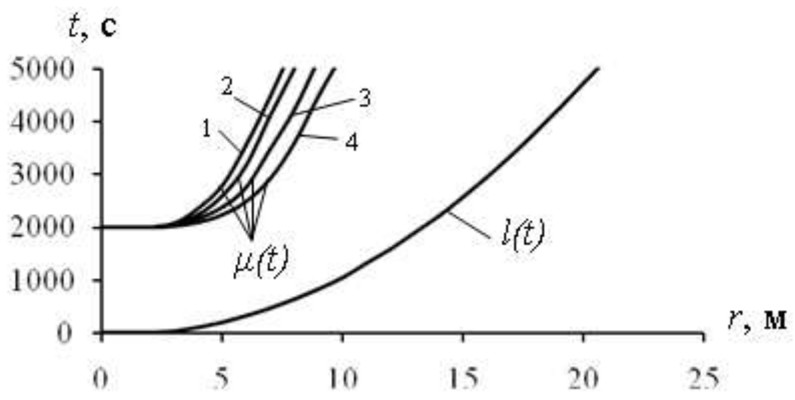

Figure: 3. Change in the front of the unloading wave $r = \mu(t)$, the boundaries of the area of influence $r = l(t)$ at a value of $k_{s0} = 0{,}75 \cdot 10^{-12}$ (m^2); $m_{s0} = 0{,}08$ and different values of parameters η_k (MPa^{-1}) and η_m (MPa^{-1}), respectively 1: 0.00021; 0.00014; 2: 0.03; 0.02; 3: 0.12; 0.08; 4: 0.21; 0.14

CONCLUSIONS

In unstable formations, the propagation of the unloading wave differs significantly from the propagation in stable formations. It is shown that with an increase in η_k, η_m, k_{s0} and m_{s0} the unloading wave front propagates deeper into the formation at the same times and other parameters of the problem. This means that if the effects of plasticity and fracture of the formation rocks are significant, then the unloading wave front propagates more intensively. In the region of the selected values of the initial parameters, it was found that the irreversibility factor of the reservoir characteristics more significantly affects the propagation of the unloading wave front than the factor of destruction of the reservoir rock.

REFERENCES

1. *Стрижов И. Н., Ходанович И. Е. Добыча газа. М.: Гостоптехиздат, 1946.*
2. *Исаков Г. В. О деформации нефтяных коллекторов // Нефтяное хозяйство. 1946. № 11. С. 17 – 24.*
3. *Баренблатт Г. И., Крылов А. П. Об упругопластическом режиме фильтрации // Изв. АН СССР. ОТН. 1955. № 2. С. 5 – 12.*
4. *Баренблатт Г. И. О некоторых задачах восстановления давления и распространения волн разгрузки при упруго-пластическом режиме фильтрации // Изв. АН СССР. ОТН. 1955. № 2. С. 14 – 26.*
5. *Barenblatt G. I., Entov V. M., Ryzhik V. M. Theory of fluid flows through natural rocks. Kluwer Academic Publishers. 1990. – 395 p.*
6. *Николаевский В. Н. К построению нелинейной теории упругого режима фильтрации жидкости и газа // ПМТФ. 1961. № 4. С. 67-76.*
7. *Николаевский В. Н., Басниев К. С., Горбунов А. Т., Зотов Г. А. Механика насыщенных горных пород. М.: Недра, 1970.*
8. *Горбунов А. Т. Упруго-пластический режим фильтрации жидкости в пористых средах // Изв. АН СССР. МЖГ. 1972. № 5. С. 84-90.*
9. *Горбунов А. Т. Разработка аномальных нефтяных месторождений. М.: Недра, 1981.*
10. *Хужаёров Б. Х. Об уравнениях фильтрации в упругопластическом режиме с выносом песка // Докл. АН РУз. 1997. № 4. С. 22 – 24.*
11. *Хужаёров Б. Х., Шодмонов И. Э., Холияров Э. Ч., Зокиров А. А. Упругопластическая фильтрация жидкости в неустойчивых пластах // ИФЖ. 2002. Т. 76. № 6. С. 123 – 128.*
12. *Авакян Э. А., Горбунов А. Т. К упруго-пластическому режиму фильтрации. – В кн.: Теория и практика добычи нефти. Ежегодник ВНИИ, 1968. М.: Недра. С. 174-185.*
13. *Баренблатт Г. И. О некоторых приближенных методах в теории одномерной нестационарной фильтрации при упругом режиме // Изв. АН СССР. ОТН. 1954. № 9. С. 36 – 49.*
14. *Хужаёров Б. Х., Давиденко М. А., Шодмонов И. Э. Фильтрации однородной жидкости в упруго-пластическом режиме с учетом разрушения пород // Узб. журн. "Проблемы механики". 1997. №2. С. 41-44.*
15. *Хужаёров Б. Х., Шодмонов И. Э. Упруго-пластическая фильтрация с учетом разрушения пласта // Узб. журн. "Проблемы механики". 1998. №2. С. 24-29.*
16. *Самарский А. А. Теория разностных схем. М.: Наука, 1982. – 616 с.*