



MATHEMATICAL MODELING AND ASSESSMENT OF THE TENSION STATE IN THE THREAD CONNECTION OF WORKWEAR PARTS

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ABSTRACT

When connecting a dense garment fabric with a thread seam, considering it conditionally continuous, through holes are formed on the materials being fastened from the reciprocating movement of the sewing needle, i.e. the formation of stitches is associated with a violation of the continuity of the material. This article is devoted to the issue of assessing the stress state developed in a thread connection under the action of operational loads and the establishment of some regularities connecting the external load, material strength, hole diameter (thread), stitch pitch, etc.

KEY WORDS: *operating load, elastoplastic deformation, reciprocating motion, material continuity, stitches, line, sewing needle, tension, microcracks, isotropy, radial tensile force, intensity, outer contour.*

DISCUSSION

Under the action of repeatedly - variable operational loads that occur in the most loaded areas of workwear parts, elastic-plastic deformations of the material and corresponding tensions appear. These tensions are the main cause of destructive processes in the thread joints of clothing parts, in particular, materials connected by threads using various seam designs.

When connecting a dense garment fabric with a thread seam, considering it conditionally solid, through holes are formed on the materials being fastened from the reciprocating movement of the sewing needle, i.e. the formation of stitches is associated with a violation of the continuity of the material. Although these holes are filled with filament, the tensile strength of such a material will decrease due to the fact that the holes formed behave as tension concentrators with the greatest degree of uneven distribution of the acting loads. It is quite obvious that in the zone of the highest tension, destruction will begin in the form of an incipient microcrack that grows into a main crack.

It is of great practical interest to assess the tension state developed in a thread joint under the action of operational loads and to establish some regularities connecting the external load, the strength of the material (fabric), the diameter of the holes (thread), the stitch pitch, etc.

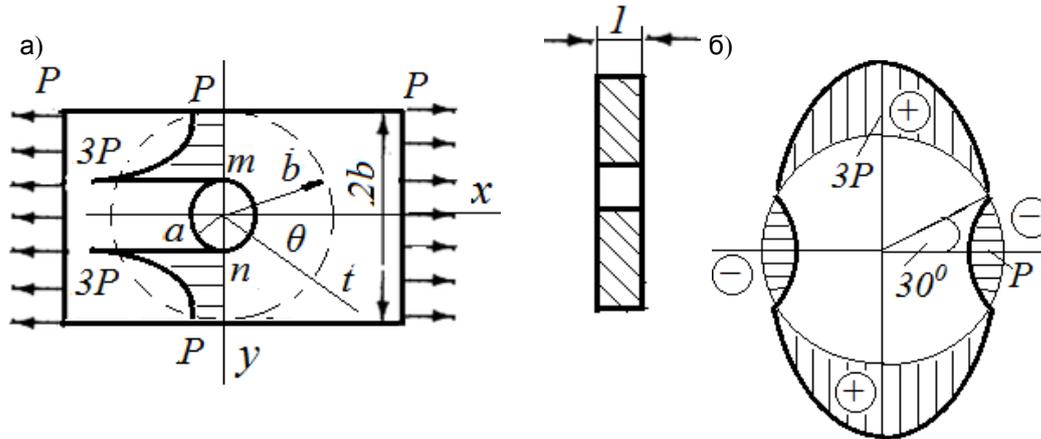


Fig. 1. A strip with a central circular hole evenly stretched by the force P per unit of width (a) and the diagrams of tangential (ring) stress changes along the border of the hole (b).

Within the width of the strip, there are two small circular holes with a radius. Consider one of these holes and place the origin of the coordinate axes in the center of the hole O. The axis is directed along the strip, the axis is perpendicular to it, the thickness of the strip is taken to be equal to unity (unit thickness).

The distribution of tensions at points located near a small hole changes sharply, however, in accordance with the Saint-Venant principle, the hole will not affect the tensions at points sufficiently distant from it (larger than the radius of the hole). Draw a circle with such a large radius from the center O, so that the tensions at the points of this circle can be considered unchanged due to the presence of a hole. Thus, annular plates with an inner radius a and an outer radius are distinguished. The tensions on a circle of radius will essentially be the same as in a plate without a hole.

Therefore, for an arbitrary point t , lying on the outer circle, the forces on the site tangent to the circle can be written using the same formulas as for simple tension [1]:

$$\left. \begin{aligned} \sigma_r &= p \cos^2 \theta = \frac{P}{2} (1 + \cos 2\theta) \\ \tau_r &= -\frac{P}{2} \sin 2\theta \end{aligned} \right\} \quad (1)$$

where is the θ angle, determined by the location of the section under consideration relative to the axis Ox . Expressions (1) are valid for $r=b$.

These forces inside the annular plate create such a stress state that can be considered as arising from two types of forces:

- 1) radial tensile force with intensity $P/2$ and constant along the entire outer contour;
- 2) efforts that vary depending on the angle θ :

$$\text{Normal } \frac{P}{2} \cos 2\theta \text{ and tangent } -\frac{P}{2} \sin 2\theta$$

The tensions arising in the annular plate from constant forces $P/2$ can be determined by the formulas obtained for a thick-walled pipe [2].

The stresses caused by varying forces can be determined from a stress function of the form

$$\varphi = f(r) \cos 2\theta \quad (2)$$

Substituting this expression into the continuity equation

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) \left(\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \varphi}{\partial \theta^2} \right) = 0, \quad (3)$$

we obtain an ordinary differential equation of the fourth order to determine:

$$\left(\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{4}{r^2} \right) \left(\frac{d^2 f}{dr^2} + \frac{1}{r} \frac{df}{dr} - \frac{4f}{r^2} \right) = 0 \quad (4)$$

The general solution will be presented as

$$f(r) = Ar^2 + Br^4 + C \frac{1}{r^2} + D \quad (5)$$

Taking into account (2), we obtain the tension function

$$\varphi = (Ar^2 + Br^4 + C \frac{1}{r^2} + D) \cos 2\theta \quad (6)$$

Having the tension function (6), normal σ_x , σ_y and shear tensions τ_{xy} are found as the second derivatives of it.

$$\sigma_x = \frac{\partial^2 \varphi}{\partial y^2}; \quad \sigma_y = \frac{\partial^2 \varphi}{\partial x^2}; \quad \tau_{xy} = -\frac{\partial^2 \varphi}{\partial x \partial y}$$

Converting these derivatives to polar coordinates, we obtained the following expressions for tensions

$$\left. \begin{aligned} \sigma_r &= \frac{1}{r} \frac{\partial \varphi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \varphi}{\partial \theta^2} \\ \sigma_\theta &= \frac{\partial^2 \varphi}{\partial r^2} \\ \tau_{r\theta} &= -\frac{1}{r} \frac{\partial^2 \varphi}{\partial r \partial \theta} + \frac{1}{r} \frac{\partial \varphi}{\partial \theta} = -\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \varphi}{\partial \theta} \right) \end{aligned} \right\} \quad (7)$$

Let us integrate expressions (7) and determine the arbitrary constants included in them from the conditions on the outer and inner contours of the annular plate. The conditions for the outer boundary correspond to dependencies (1), and the inner boundary is characterized by the fact that the hole edge is free from external forces.

$$\sigma_r = -(2A + \frac{6C}{r^4} + \frac{4D}{r^2}) \cos 2\theta; \quad (8)$$

$$\sigma_\theta = (2A + 12Br^2 + \frac{6C}{r^4}) \cos 2\theta;$$

$$\tau_{r\theta} = (2A + 6Br^2 - \frac{6C}{r^4} - \frac{2D}{r^2}) \sin 2\theta,$$

taking these conditions into account, we have a system of equations with constants A, B, C, and D

$$2A + \frac{6C}{b^4} + \frac{4D}{b^2} = -\frac{1}{2} P \quad (9)$$

$$2A + \frac{6C}{a^4} + \frac{4D}{a^2} = 0$$

$$2A + 6Bb^2 - \frac{6C}{b^4} - \frac{2D}{b^2} = -\frac{1}{2}p$$

$$2A + 6Ba^2 - \frac{6C}{a^4} - \frac{2D}{a^2} = 0$$

Having solved the system of equations (9) and sloping $a/b = 0$ (the plate is considered to be infinitely large and very small in comparison with, we discard the terms containing a/b). Then we obtain the following values of the integration constants:

$$A = -\frac{p}{4}; \quad B = 0 \quad C = -\frac{a^4}{4}p; \quad D = \frac{a^2}{2}p. \quad (10)$$

Substituting these values of the constants in Eqs. (8) and adding the tensions arising from uniform stretching of the intensity $p/2$ acting on the outer contour of the annular plate, as a result, we obtain the following expressions for the tensions.

$$\left. \begin{aligned} \sigma_r &= \frac{p}{2} \left(1 - \frac{a^2}{r^2}\right) + \frac{p}{2} \left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right) \cos 2\theta; \\ \sigma_\theta &= \frac{p}{2} \left(1 + \frac{a^2}{r^2}\right) - \frac{p}{2} \left(1 + \frac{3a^4}{r^4}\right) \cos 2\theta; \\ \tau_{r\theta} &= -\frac{p}{2} \left(1 - \frac{3a^4}{r^4} + \frac{2a^2}{r^2}\right) \sin 2\theta. \end{aligned} \right\} \quad (11)$$

With increasing radius r , i.e. with distance from the hole, the second and third terms in brackets in expressions (11) rapidly decrease. If we discard them, then for the distant points we obtain the same tension state as in simple tension, i.e. defined by formulas (1).

When approaching the edges of the hole, the tensions and decrease along the edges, when $r = a$, we get:

$$\sigma_r = \tau_{r\theta} = 0 \quad (12)$$

The voltages take on values

$$\sigma_\theta = p - 2p \cos \theta. \quad (13)$$

The results of the analysis of tension changes along the edge of the hole depending on the angle are shown in Fig. 1, c. This tension reaches its highest value at the nodes or, i.e., at the points m and n, lying at the ends of the diameter perpendicular to the direction of tension, and is equal to

$$\sigma_\theta^{\max} = 3p. \quad (14)$$

Thus, the maximum tensile stress at the edges of the hole is 3 times greater than the normal tensile tension in the unweakened section of the plate. At the ends of the longitudinal diameter, i.e. at $\theta = 0$ and $\theta = \pi$ the tension σ_θ is compressive and is

$$\sigma_\theta = -p. \quad (15)$$

At angles $\theta = \pm \frac{\pi}{6}$ and $\theta = 180 \pm \frac{\pi}{6}$, the tension σ_{θ} passes through zero and the tension field is distributed into zones with compressive and tensile tensions.

For the cross section of the plate passing through the center of the hole, i.e. for $\theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$, $\cos 2\theta = -1$ that is why

$$\sigma_{\theta} = \frac{p}{2} \left(2 + \frac{a^2}{r^2} + \frac{3a^4}{r^4} \right) \quad (16)$$

The diagram of these tensions is shown in Figure 1, b. From formula (16) and the graph it can be seen that large tensions arise only at the edges of the hole and as the radius r increases, they rapidly decrease, approaching the value $\sigma_{\theta} = p$. Thus, the effect of a small hole is of a purely local (local) nature. The local nature of tensions σ_{θ} substantiates the correctness of solution (11) obtained for an infinitely large plate to a plate of finite width. If the width of the plate is not less than four diameters of the hole, the error of solution (11) in the calculation σ_{θ}^{\max} does not exceed 6% [1].

A local increase in tensions in places of abrupt changes in the contours of a part is used to characterize the degree of tension concentration, assessed by the tension concentration factor for their elastic distribution or the theoretical tension concentration factor k [3]. The tension concentration factor can be calculated as the ratio to the tension σ_{θ}^{\max} in the unweakened section:

$$k = \frac{\sigma_{\theta}^{\max}}{p} \quad (17)$$

where the concentration coefficient k depends on the ratio of the hole diameter to the plate width (Fig. 2).

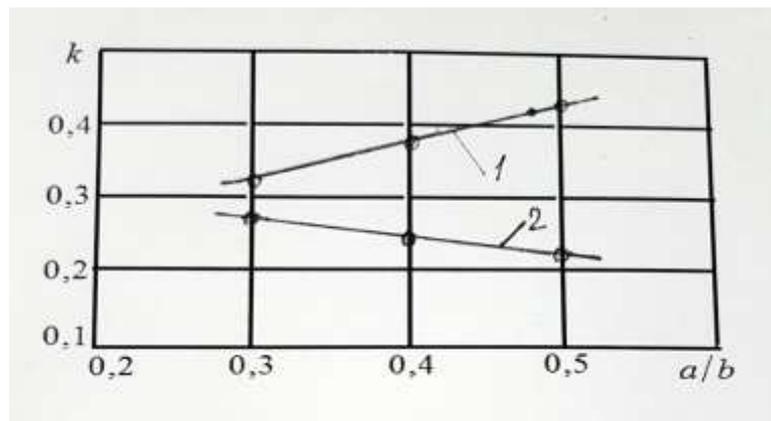


Fig. 2. Change in the concentration factor k depending on the ratio a / b , calculated as the ratio of the highest tension σ_{θ}^{\max} to tension in the unweakened section (1) and to the average tension in the weakened section (2)

In practical strength calculations, it is customary to compare the highest tension with the average uniform tension in a weakened section, determined by the formula:

$$\sigma_{cp} = \frac{p}{A_{HETTO}} \quad (18)$$

then the concentration factor should be calculated from the dependence

$$k = \frac{\sigma_{\theta}^{\max}}{\sigma_{cp}} \quad (19)$$

With this method of determining the concentration coefficient, it decreases as the hole diameter increases (Fig. 2).

Thus, it is obvious that the most dangerous holes of small diameters, causing the greatest concentration of tensions. All of the above is true under the assumption of an isotropic ideally elastic material. Taking into account the fact that during the operation of overalls in certain loaded areas, various types of tension-strain state may arise, it is also necessary to consider some special cases.

Having the solution (11) for tension or compression in one direction, using superposition, one can obtain a solution for tension or compression in two perpendicular directions. If we assume that the tensile tensions in the two perpendicular directions are equal p , then tensile tensions act on the boundary of the hole $\sigma_{\theta} = 2p$ [1].

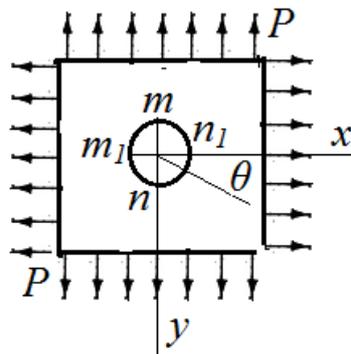


Fig. 3. Strip with a central circular hole under the action of tensile compressive stresses along the coordinate axes

Assuming that tensile tension acts in the direction x , and compressive tension p acts in the direction y tension $-p$ (Fig. 3), we get the case of pure shear. In accordance with dependence (11), the tangential (ring) tension at the boundary of the hole will be at $\theta = 0$ and $\theta = \pi$

$$\sigma_{\theta} = p - 2p \cos 2\theta - [s - 2s \cos(2\theta - \pi)] \quad (20)$$

At points m and n holes defined by angles $\theta = \frac{\pi}{2}$ or $\theta = \frac{3\pi}{2}$, tension $\sigma_{\theta} = 4p$ Thus, at pure shear at a sufficiently large plate size, the highest tangential tension is four times higher than the applied pure shear tension.

Normal and shear tension determined by Eqs. (11) are suitable for both the plane tension state and plane deformation. But for the case of plane deformation, axial tensions must act on the flat ends of the solid

$$\sigma_z = \nu(\sigma_T + \sigma_{\theta}) \quad (21)$$

where ν is Poisson's ratio.

These tension act perpendicular to the plane xy so as to make the strains equal ϵ_z to zero. If the hole diameter and the distance between the ends are of the same order of magnitude, then the problem becomes three-dimensional. In this case, tension σ_{θ} remains the largest component of the tension state and its value is very close to that given by the two-dimensional theory.

If in an infinite plate under the action of tensile tension P , there is an elliptical hole and one of the main axes of the ellipse is parallel to the direction of tension, then the tensions acting at points on the surface of the hole located on the other main axis are:

$$\sigma = p\left(1 + 2\frac{a}{b}\right) \quad (22)$$

where $2a$ is the axis of the ellipse, perpendicular to the direction of extension;
 $2b$ is the other axis of the ellipse.

Consideration of holes in the form of an ellipse in a plate can have a practical application when calculating the strength of thread joints, when under the influence of operational loads loosening of the seams occurs and the initial hole in the form of a circle is transformed into an ellipse.

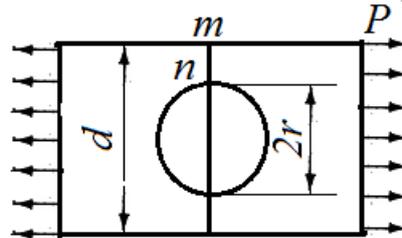


Fig. 4. A strip of finite width with a round hole on the axis of symmetry, evenly stretched by force P

In conclusion, consider the case of a plate of finite width with a round hole on the axis of symmetry (Fig. 4), when $2r = 0,5d$ [1]. The tangential tension at the boundary of the hole is n equal to $\sigma_{\theta} = 4,3p$ and $\sigma_{\theta} = 0,75p$ at a point m on the boundary surface of the plate.

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