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ASSOCIATED GRAPH BASED ON QUASI-IDEALS BCIK-ALGEBRAS

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ABSTRACT

In this paper, we use the notions graph based nodal ideals of BCIK – algebra and its properties. Then we study relationships between associated graph based notion of (l-prime) quasi ideals and zero divisors are first introduced and related properties are investigated. The concept of associative graph of a BCIK-algebra is introduced, and several example are displayed.

KEYWORDS: ***BCIK-algebra, l-prime(quasi-ideals), nodal ideals.***

1. INTRODUCTION

In 2021 [3,4], S Rethina Kumar introduce combination BCK–algebra and BCI–algebra to define BCIK–algebra and its properties and also using Lattices theory to derived the some basic definitions, and they also the idea introduced a regular f-derivation in BCIK-algebras. We give the Characterizations f-derivation p-semi simple algebra and its properties. After the work in 2021[3],S Rehina Kumar have given the notion of t-derivation of BCIK-algebras and studied p-semi simple BCIK—algebras by using the idea of regular t-derivation in BCIK-algebras have extended the results of BCIK-algebra in the same paper they defined and studied the notion of left derivation of BCIK-algebra and investigated some properties of left derivation in p-semi simple BCIK-algebras. In 2021 [5], S Rethina Kumar discuss about the notions of a node and nodal ideals of BCIK-algebras. Then relationships between nodal ideals and some other types of ideals, like prime ideal and maximal ideal in BCIK-algebras.

Many authors studied the graph theory in connection with (commutative) semi group and (commutative and non-commutative) rings as we can refer to references. For example, Beck[1] associated to any commutative rings R its zero-divisor graph G(R) whose vertices are the zero-divisors of R(including 0), with two vertices a,b joined by an edge in case ab=0. Also, DeMeyer et al. [2] defined the zero-divisor graph of a commutative semi group S with zero (0x=0 for all x϶S).

In this paper, motivated by these works, we study the associated graphs of BCIK-algebras. We first introduce the notions of (l-prime) quasi-ideals and zero divisors and investigated related properties. We introduce the concept of associative graph of a BCIK-algebra and provide several examples. We give conditions for a proper (quasi)ideals of a BCIK-algebra to be l-prime. We show that associated graph of a BCIK-algebra is a connected graph in which every nonzero vertex is adjacent to 0, but the associative graph of a BCIK-algebra is not connected by providing an example.

2. PRELIMINARIES

Definition 2.1.[3,4] BCIK algebra

Let X be a non-empty set with a binary operation * and a constant 0. Then (X, * ,0) is called a BCIK Algebra, if it satisfies the following axioms for all x, y, z Є X:

(BCIK-1) x*y = 0, y*x = 0, z*x = 0 this imply that x = y = z.

(BCIK-2) ((x*y) * (y*z)) * (z*x) = 0.

(BCIK-3) (x*(x*y)) * y = 0.

(BCIK-4) x*x = 0, y*y = 0, z*z = 0.

(BCIK-5) 0*x =0, 0*y = 0, 0*z = 0.

For all x, y, z Є X. An inequality ≤ is a partially ordered set on X can be defined x ≤ y if and only if

(x*y) * (y*z) = 0.

Properties 2.2. [3,4] I any BCIK – Algebra X, the following properties hold for all x, y, z Є X:

(1) 0 Є X.
(2) x*0 = x.
(3) x*0 = 0 implies x = 0.
(4) 0*(x*y) = (0*x) * (0*y).
(5) X*y = 0 implies x = y.
(6) X*(0*y) = y*(0*x).
(7) 0*(0*x) = x.
(8) x*y Є X and x Є X imply y Є X.
(9) (x*y) * z = (x*z) * y
(10) x*(x*(x*y)) = x*y.
(11) (x*y) *(y*z) = x*y.
(12) 0 ≤ x ≤ y for all x, y Є X.
(13) x ≤ y implies x*z ≤ y*z and z*y ≤ z*x.
(14) x*y ≤ x.
(15) x*y ≤ z ⇔ x*z ≤ y for all x, y, z Є X
(16) x*a = x*b implies a = b where a and b are any natural numbers (i. e)., a, b Є N
(17) a*x = b*x implies a = b.
(18) a*(a*x) = x.

Definition 2.3. [3, 4], Let X be a BCIK – algebra. Then, for all x, y, z Є X:

(1) X is called a positive implicative BCIK – algebra if (x*y) * z = (x*z) * (y*z).
(2) X is called an implicative BCIK – algebra if x*(y*x) = x.
(3) X is called a commutative BCIK – algebra if x*(x*y) = y*(y*x).
(4) X is called bounded BCIK – algebra, if there exists the greatest element 1 of X, and for any x Є X, 1*x is denoted by GG_x,
(5) X is called involutory BCIK – algebra, if for all x Є X, GG_x = x.

Definition 2.4. [5,6] Let X be a bounded BCIK-algebra. Then for all x, y Є X:

(1) $G1 = 0$ and $G0 = 1$,
(2) $GG_x \leq x$ that $GG_x = G(G_x)$,
(3) $G_x * G_y \leq y*x$,
(4) $y \leq x$ implies $G_x \leq G_y$,
(5) $G_{x*y} = G_{y*x}$
(6) $GGG_x = G_x$.

Theorem 2.5.[4] Let X be a bounded BCIK-algebra. Then for any x, y Є X, the following hold:

(1) X is involutory,
(2) $x*y = G_y * G_x$,
(3) $x*G_y = y * G_x$,
(4) $x \leq G_y$ implies $y \leq G_x$.

Theorem 2.6.[3,4] Every implicative BCIK-algebra is a commutative and positive implicative BCIK-algebra.

Definition 2.7. [3,4] Let X be a BCIK-algebra. Then:

(1) X is said to have bounded commutative, if for any x, y Є X, the set A(x,y) = {t Є X : $t*x \leq y$} has the greatest element which is denoted by x o y,
(2) (X, *, ≤) is called a BCIK-lattices, if (X,≤) is a lattice, where ≤ is the partial BCIK-order on X, which has been introduced in Definition 2.1.

Definition 2.8. [3,4] Let X be a BCIK-algebra with bounded commutative. Then for all x, y, z Є X:

(1) $y \leq x \text{ o } (y*x)$,
(2) $(x \text{ o } z) * (y \text{ o } z) \leq x*y$,
(3) $(x*y) * z = x*(y \text{ o } z)$,
(4) If $x \leq y$, then $x \text{ o } z \leq y \text{ o } z$,
(5) $z*x \leq y \Leftrightarrow z \leq x \text{ o } y$.

Theorem 2.9. [3,4] Let X be a BCIK-algebra with condition bounded commutative. Then, for all

x, y, z Є X, the following are equivalent:

(1) X is a positive implicative,
(2) $x \leq y$ implies $x \text{ o } y = y$,
(3) $x \text{ o } x = x$,
(4) $(x \text{ o } y) * z = (x*z) \text{ o } (y*z)$,
(5) $x \text{ o } y = x \text{ o } (y*x)$.

Theorem 2.10. [3,4] Let X be a BCIK-algebra.

(1) If X is a finite positive implicative BCIK-algebra with bounded and commutative the (X, ≤) is a distributive lattice,
(2) If X is a BCIK-algebra with bounded and commutative, then X is positive implicative if and only if (X, ≤) is an upper semi lattice with $x \vee y = x \text{ o } y$, for any x, y Є X,
(3) If X is bounded commutative BCIK-algebra, then BCIK-lattice (X, ≤) is a distributive lattice, where $x \wedge y = y*(y*x)$ and $x \vee y = G(G_x \wedge G_y)$.

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Theorem 2.11.[3,4] Let X be an involutory BCIK-algebra, Then the following are equivalent:

(1) (X, ≤) is a lower semi lattice,
(2) (X, ≤) is an upper semi lattice,
(3) (X, ≤) is a lattice.

Theorem 2.12. [4] Let X be a bounded BCIK-algebra. Then:

(1) every commutative BCIK-algebra is an involutory BCIK-algebra.
(2) Any implicative BCIK-algebra is a Boolean lattice (a complemented distributive lattice).

Theorem 2.13. [3, 4] Let X be a BCK-algebra, Then, for all x, y, z Є X, the following are equivalent:

(1) X is commutative,
(2) x*y = x*(y*(y*x)),
(3) x*(x*y) = y*(y*(x*(x*y))),
(4) x≤ y implies x = y*(y*x).

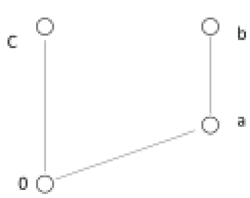
3.[5] NODAL IDEAL OF A BCIK-ALGEBRA

We denote BCIK-algebra (X,*,0) by X.

Definition 3.1.[5] A node of X is an element of X is comparable with every element of X. It is clear that 0 is a node in every BCIK-algebra
.

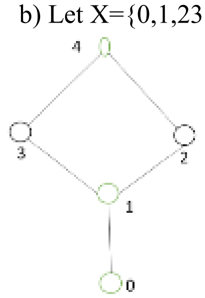
Proposition 3.1.[5] An element a∈X is a node if and only if for every x∈X either a*x=0 or a=0.

Example 3.2. a) Let X={0,a,b,c}. For all x,y∈X, we define, * as follows:



*	0	a	b	c
0	0	0	0	0
A	A	0	0	a
B	B	a	0	b
C	C	c	c	0

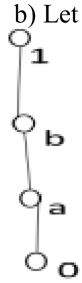
Then (X,*0) is a BCIK-algebra. 0 is only node of X and atom(X)={a,c}.
b) Let X={0,1,23,4}. For all x,y∈X, we define, * as follows:



*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	0	0	0
2	2	1	0	1	0
3	3	3	3	0	0
4	4	4	4	4	0

Then (X,*,0) is a BCIK-algebra. {0,1,4} is the set of all nodes of X and S(X) = {0,4} and atom(X) = {1}.

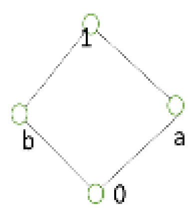
b) Let X={0,a,b,1}. For all x,y∈ X, we define, * as follows:



*	0	a	B	1
0	0	0	0	0
a	a	0	0	0
b	b	b	0	0
1	1	b	A	0

Then (X,*0) is a BCIK-algebra . All elements of X are node S(X)={0,a,b,1} and atom(X)={a}.

d) Let X={0,a,b,1}. For all x,y∈ X, we define, * as follows:



*	0	A	b	1
0	0	0	0	0
a	a	0	a	0
b	b	B	0	0
1	1	1	1	0

Then(X,*,0) is a BCIK-algebra. {0,1} is the set of all nodes of X, S(C) ={0,1} and atom(X)={a,b}.

Proposition 3.3. If X is bounded, then {0,1} $\subseteq$ node(X) $\bigcap$ S(X).

Definition 3.2. An ideal I of X will be called a nodal ideal of X, if I is a node of I(X). we denote all nodal ideals of X by N(X).

Example 3.4.[5]a) X and {0} are trivial nodal ideals of ever X.

b) In Example 3.3, (a), we have I(X)={{0},{0,c},{0,a.b},X}. But only {0} and X are nodal ideal of X.

c) In Example 3.3, (b), {0},{0,1,2},{0,1,2,3}, X are all of nodal ideals of X.

d) In Example 3.3, (c), {0},{0,a}, X are all of nodal ideal of X.

e) In Example 3.3, (d), we have I(X)={{0},{0,a},{0,b},{0,a,b},X}. And {0},{0,a,b}, X are all of nodal ideals of X.

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Theorem 3.5.[5] Let I be an ideal of X. If for all x,y∈ X such that x∈ I and y ∉ I, the relation x<y is satisfied, then I is a nodal ideal of X.
Proof. If I is not a nodal ideal of X. So there x,y∈ X such that x∈ I –J,yx∈ J-I and x≮y. Thus it is contrary, so every ideal J of X is comparable with I, that is, I is a nodal ideal of X.

Example 3.6. In Example 3.3,(d), Let I={0,b}. we have a,b∈ X, b∈ I and a∉I, but b≮a, so I is not a nodal ideal of X.

Theorem 3.7. Let I be a nodal ideal of positive implicative BCIK-algebra X. Then for every x,y∈ X, such that x∈ I and y ∉ I, the relation x<y is satisfied.
Proof. Since I is a nodal ideal of X, so for all x,y∈ X, such that x∈ I and y ∉ I, we have (x]⊆I and I⊆(y]. Thus (x]⊆I⊆(y], so x∈ (y], then (…(x*y)*…)*y=0, since X is a positive implicative BCIK-algebra, we have (x*y)*=x*y, therefore x*y=0, thus x<y.

Example 3.8. In Example3.3,(b), X is a BCIK-algebra but is not positive implicative BCIK-algebra. Let I={0,1,2}. Then I is a nodal ideal of X, 2∈ I and 3 ∉ I, but 2≮ 3.

Corollary 3.9. Let X be a BCIK-chain. If I is a (positive implicative) ideal, then I is a nodal ideal.

Proposition 3.10. x is a node in X if and only if principle ideal (x] is a nodal ideal of X.
Proof. Let x be a node in X and I be a ideal of X. If x∈ I, then (x]⊆I. Let x ∉ I. Now, if I⊈(x], then there exists a y∈ I such that y ∉ (x], so for all n∈ N, y≰x^n, so y≰x and since x is a node, then x<y. Thus (x] ⊆ (y] ⊆ I. That is x∈ I, its contrary, so if x ∉ I, then I⊆(x].

Theorem 3.11. Let be a non-principal nodal ideal of lower BCIK- semi lattice X. Then I is a prime ideal.
Proof. Let I be a non-principal nodal ideal of X and x ∧ y∈ I and x ∉ I and y ∉ I. Thus (x ∧ y] ⊆ I. On the other hand, since x ∉ I and y ∉ I then (x]⊈I and (y]⊈I, so I ⊂ (x], and I ⊂ (y], thus I ⊂ (x] ⋂ (y]=(x ∧ y], it is contrary, thus x∈ I or y∈ I, so I is a prime ideal.

Example 3.12. In example 3.3,(d), I={0,a} is prime ideal but is not a nodal ideal.

Corollary 3.13. Let I be a principal nodal ideal of implicative BCIK-algebra X. Then I is a prime (maximal, irreducible, obstinate) ideal.

Theorem 3.14. Let X be a lower BCIK-semi lattice. Then the annihilator of node(X) is a nodal ideal of X.
Proof. If node(x)={0}, then (node(X))*=X. Now, let 0≠ a∈ node(X). We have (node(X))*=$\bigcap_{s\in node(X)}$ {s}* and {a}*={x ∈ X:x ∧ a=0}. Since 0≠a∈ node(X), sox ∧ a=x or x ∧ a=a, thus x ∧ a=0 if and only if x=0, then {a}*={0}. So (node(X))*=$\bigcap_{s\in node(X)}${s}*={0}. Therefore (node(X))* is a nodal ideal of X.

Theorem 3.15. Let X be a bounded implicative BCIK-algebra and I be a non-principal nodal ideal of X. Then for any x∈ X. Then for any x∈ X, exactly one of x and N_x belongs to I.
Proof. If x ∉ I and N_x ∉ I for some x∈ X, then x ∧ N_x=x*(x*(1*x))=x*x=0∈I, which implies x∈ I or N_x∈ I , which is a contradiction . If x∈ I and N_x∈ I for some x∈ X, then 1∈ I as I is an ideal, this impossible. Summarizing the above facts obtain that exactly one of x, N_x belongs to I.

Proposition 3.16. If positive implicative BCIK-algebra X has n node, then it has at least n nodal ideals.
Proof. If x is a node of X, then (x] is a nodal ideal. Now, let x and y two nodes of X. If (x]=(y], then x∈(y] and yx∈(y], since X is a positive implicative BCIK-algebra, thus x*y=0 and y*x=0, so x≤y and y≤x, thus x=y. Therefore, if X has n node, then it has at least n nodal ideals.

Example 3.17. In Example 3.3,(a), let I={0} and J={0,c} be ideals of X. Then I⊆J and I is a nodal ideal of X but J is not nodal ideal of X. So extension property for nodal ideal in X is not valid.

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Theorem 3.18. If I and J are two nodal ideals of X, then
(i) $I \cap J$ is a nodal ideal of X,
(ii) $I \cup J$ is a nodal ideal of X.

Theorem 3.19. For any X, $(N(X), \cap, \cup, (0], X)$ is a bounded infinitely distributed lattices, i.e. it is bounded lattices and satisfies for any $I, J_i \in N(X) (i \in A)$, $I \cap (\cup \{J_i : i \in A\}) = (\cup \{I \cap J_i : i \in A\})$.

Proof. By Theorem 3.20, $(N(X), \cap, \cup, (0], X)$ is a bounded lattice. Let $x \in I \cap (\cup \{J_i : i \in A\})$, then $x \in I$ and $x \in \cup \{J_i : i \in A\}$, so there exist a $j \in A$ such that $x \in J_i$, thus $x \in I \cap J_i$, thus $x \in I \cap J_i$, so $x \in \cup \{I \cap J_i : i \in A\}$. Now let $x \in \cup \{I \cap J_i : i \in A\}$, so there exist $j \in A$ such that $x \in I \cap J_i$, thus $x \in I$ and $x \in J_i$, then $x \in \cup \{J_i : i \in A\}$, thus $x \in I \cap (\cup \{J_i : i \in A\})$, therefore $I \cap (\cup \{J_i : i \in A\}) = (\cup \{I \cap J_i : i \in A\})$.

Example 3.20. (a) In the general every nodal ideal is not an implicative ideal. In Example 3.3, (b) I={0,1,2} is a nodal ideal but is not an implicative ideal, because $3*(4*3)=0 \in I$ but $3 \notin I$.
(b) In the general every nodal ideal is not an commutative ideal. In Example 3.3, (b), I={0,1,2} is a nodal ideal but is not a commutative ideal.
(c) In the general every nodal ideal is not an normal ideal. In Example 3.3, (b), I={0,1,2} is a nodal ideal but is not a normal ideal, because $4*(4*3) \in I$ but $3*(3*4) \notin I$.
(d) In the general every nodal ideal is not an obstinate ideal. In Example 3.3,(b), I={0,1,2} is a nodal ideal but is not an obstinate ideal.
(e) In the general every nodal ideal is not a nodal ideal. In Example 3.3,(a), I={0,c} is a normal ideal but is not a nodal ideal.
(f) In the general every positive implicative ideal is not a nodal ideal. In Example 3.3,(d), I={0,a} is a positive implicative ideal but is not a nodal ideal.
(g) In the general every maximal ideal is not a nodal ideal. In Example 3.3, (a), I={0,c} is a maximal ideal but is not a nodal ideal.
(h) In the general every obstinate ideal is not a nodal ideal. In Example 3.3, (d), I={0,a} is an obstinate ideal but is not a nodal ideal.
(i) In the general every nodal ideal is not a Varlet ideal. In Example 3.3, (d), I={0,a,b} is a nodal ideal but is not a Varlet ideal.
(j) In the general every Varlet ideal is not a nodal ideal. In Example 3.3, (a), I={0,a} is a Varlet ideal but is not a nodal ideal.

Proposition 3.21. Let (x,*,0) and (X',*',0') be two positive implicative BCIK-algebra and $f: X \to X'$ a homomorphism. Then the following are satisfied:
(a) If f is injective and $J \in N(X')$, then $f^{-1}(J) = \{x \in X : f(x) \in J\} \in N(X)$,
(b) If f is bijective and $I \in N(X)$, then $f(I) \in N(X')$.

Proof.(a): Since f(0)=0', so $0 \in f^{-1}(J)$, thus $f^{-1}(J) \neq \emptyset$. Let $x*y \in f^{-1}(J)$ and $y \in f^{-1}(J)$. Then $f(x)*'f(y)=f(x*y) \in J$. It follows that $f(x) \in J$. So $x \in f^{-1}(J)$. This says that $f^{-1}(J)$ is an ideal of X.
Now, let $x \in f^{-1}(J)$ and $y \notin f^{-1}(J)$. Then $f(x) \in J$ and $f(y) \notin J$. Since J is a nodal ideal of positive implicative BCIK-algebra X', then $f(x)<f(y)$, thus $f(x*y)=f(x)*'f(y)=0$, since f is injective, we get $x*y=0$, thus $x<y$, so $f^{-1}(J)$ is a nodal ideal of X.
(b): Since f(0)=0', so $0' \in f(I)$. If $x,y \in X'$, $x*y \in f(I)$ and $y \in f(I)$, then there exist $a \in X$ and $b \in I$ such that f(a)=x and f(b)=y. Also since $x*y \in f(I)$, there exists $c \in I$ such that $f(c)=x*y=f(a)*f(b)=f(a*b)$, since f is injective, a*b=c, so $a \in I$, thus $f(a)=x \in f(I)$, therefore f(I) is an ideal of X'.
Let $x,y \in X'$, $x \in f(I)$ and $y \notin f(I)$, then there exists a $a \in I$ and $b \in X-I$ such that f(a)=x and f(b)=y. Since I is anodal ideal of X, so a<b, then a*b=0, thus f(a)=x and f(b)=y. Since I a nodal ideal of X, so a<b, then a*b=0, thus $f(a)*'f(b)=f(a*b)=f(0)=0'$, so $f(a)<f(b)$, then x<y, therefore f(I) is nodal ideal of X' .

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4. CONGRUENCE RELATION ON BCIK-ALGEBRA RESPECT A NODAL IDEAL

For every nodal ideal ideal I of X, we define θ_I if and only if x*y ЄI and y*XЄI. θ_I is congruence relation on X. For xЄX, let C_x be the equivalence class of modulo θ_I and X/I be the equivalence set X/ θ_I. We define * on X/I, by $C_x*C_y=C_{x*y}$.

Theorem 4.1. Let I be a non-principal nodal ideal of lower BCIK-semi lattices X and for any x,yϵX, (X*Y)□ (Y*X)=0. Then (X/I,*,C_o) is a BCIK-chain.
Proof. Let C_x,C_y ЄX/I. Since (x*y) ⋀(y*x)=0ЄI and by Theorem 3.11, x*yЄI or y*xЄI, and so $C_x * C_y =C_{x*y}=C_o$ or$C_y*C_x =C_{x*y}=C_o$, equivalently, for any x,yЄX, $C_x\leq C_y$ or $C_y\leq C_x$. That is to say that X/I is a BCIK-chain.

Theorem 4.2. Suppose I_1 and I_2 are nodal ideals of BCIK-algebra X_1 and X_2, respectively. Then I_1xI_2 is a nodal ideals of the direct product of BCIK-algebras X_1 and X_2.
Proof. Let I_i (i=1,2) be a nodal ideal of X_i(i=1,2)and x*yЄI_1xI_2, yЄI_1xI_2. Then (x*y)(i)=x(i)*y(i),y(i)ЄI_i, Where x(i) is the i-projection of x. So x(i)ЄI_i. This shows that xЄI_1xI_2. Obviously, 0ЄI_1xI_2. Hence I_1xI_2 is an ideal of X_1xX_2. Now, let x,yЄX_1xX_2,xЄX_1xX_2 and y∉X_1xX_2. So x(i)ЄI_i, thus y(i) ∉I_i, thus x(i)<y(i), then x<y. Therefore I_1xI_2 is a nodal ideal of the direct product X_1xX_2.

5. ASSOCIATED GRAPHS

In what follows, let X denote a BCIK-algebra unless otherwise specified.
For any subset A of X, we will use the notations r(A) and l(A) to denote the sets
r(A):={xЄX/ a*x=0, for all aЄA},
l(A):={xЄX/ x*a=0, for all aЄA}

Proposition 5.1. Let A and B be subsets of X, then
(1) A⊆l(r(A)) and A⊆r(l(A)),
(2) If A⊆B, then l(B) ⊆l(A) and r(B) ⊆r(A),
(3) l(A)=l(r(l(A))) and r(A)=r(l(r(A))).

Proof. Let aЄA and xЄl(A), then x*a=0, and so aЄr(l(A)). This says that A⊆r(l(A)). Dually, A⊆r(l(A)). Hence, A⊆r(l(A))=A⊆l(lrA)).
Assume that A⊆B and xЄl(B), then x*b=0 for all bЄB, which implies from A⊆B that x*b=0 for all bЄA. Thus, xЄl(a), which shows that l(B) ⊆l(A). Similarly, we have r(B) ⊆r(A). Thus l(B) ⊆l(A)=r(B) ⊆r(A).
Using A⊆r(l(A))=A⊆l(lrA)) and l(B) ⊆l(A)=r(B) ⊆r(A), we have l(r(l(A))) ⊆l(A) and r(l(r(A)) ⊆r(A). If we apply A⊆r(l(A))=A⊆l(lrA)) to l(A) and r(A), then $l(A) \subseteq l\left(r\left(l(A)\right)\right)$ and $r(A) \subseteq r(l(r(A))$. Hence $l(A) \subseteq l\left(r\left(l(A)\right)\right)$ and $r(A) \subseteq r(l(r(A))$.

Table 1: *-operation.

*	0	a	b	c	D
0	0	0	0	0	0
a	a	0	0	0	0
b	b	a	0	a	0
c	c	a	a	0	0
D	D	B	A	B	0

Table 2:*-operation.

*	0	1	a	b	C
0	0	0	c	c	A
1	1	0	c	c	A
A	a	a	0	0	C
B	b	a	1	0	C
C	c	c	a	a	0

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Definition 5.2. A nonempty subset I on X is called a quasi-ideal of X if it satisfies $(\forall x \in X)(\forall y \in I)$ (x$*\ y = 0 \Longrightarrow x \in I$)

Example 5.3. Let X={0,a,b,c,d} be a set with ∗-operation given by Table 1, then (X; ∗,0) is a BCIK-algebra (see[3,4,5]). The set I:={0,a,b} is a quasi-ideal of X.

Obviously, every quasi-ideal I of a BCIK-algebra X contains the zero element 0. The following example shows that there exists a quasi-ideals l of a BCIK-algebra X such that 0∉I.

Example 5.4: Let X={0,1,a,b,c} be a set with the ∗-operation given by Table 1, then (X; ∗,0) is a BCIK-algebra (see[3,4,5]). The set I:={0,1,a} is a quasi-ideal of X containing the zero element 0, but the set J:={a,b,c} is a quasi-ideal of X, but the converse is not true . In fact, the quasi-ideal I:={a,b,c} in Example 5.3. is not an ideal of X. Also, quasi-ideals I and J in Example 5.4 are not ideals of X.

Definition 5.5. A (quasi-)ideal I of X is said to be l-prime if it satisfies

(i) I is proper, that is, I≠X,

(ii) $(\forall x, y \in I)$(l({x ∗ y} $\subseteq I \Longrightarrow x \in I\ or x \in I$)

Example 5.6. Consider the BCIK-algebra X={0,a,b,c,d} with the operation ∗ which is given by Table 3, then the set I={0,a,c} is an l-prime ideal of X.

Theorem 5.7. A proper (quasi-) ideal I of X is l-prime if and only if it satisfies$l(\{x_1, \dots . x_n\}) \subseteq I \Longrightarrow (\exists i \in \{1, \dots . , n\})\ (x_i \in I), \forall x_i, \dots . , x_n \in X$.

Table 3:*-operation.

*	0	a	b	c	D
0	0	0	0	0	0
a	a	0	a	0	0
b	b	B	0	b	0
c	c	a	c	0	A
d	d	d	d	D	0

Table 4:*-operation.

*	0	1	2	a	B
0	0	0	0	a	A
1	1	0	1	b	A
2	2	2	0	a	A
a	a	A	a	0	0
b	b	A	b	1	0

Proof. Assume that I is an l-prime (quasi-) ideal of X. We proceed by induction on n. If n=2, then the result is true. Suppose that the statement holds for n-1. Let $x_1, \dots . x_n \in X$ be such that $l(\{x_1, \dots . , x_{n-1}, x_n\}) \subseteq I$. If y∈ $l(\{x_1, \dots . , x_{n-1}\})$, then $l(\{y, x_n\}) \subseteq l(\{l(\{x_1, \dots . , x_{n-1}, x_n\}) \subseteq I$. Assume that $x_n \notin I$, then $y \in I$ by the l-primeness of I, which shows that $l(\{x_1, \dots . , x_{n-1}\})$⊆ I. Using the induction hypothesis, we conclude that $x_i \in I$ for some $i \in \{1, \dots . , n-1\}$. The converse is clear.

For any $x \in X$, we will use the notation Z_x to denote the set of all elements $y \in X$ such that $l(\{x, y\}) = \{0\}$, that is, $z_x \coloneqq \{y \in X / l(\{x, y\}) = \{0\}\}$, which is called the set of zero divisors of x.

Lemma 5.8. If X is a BCIK-algebra, then $l(\{x, y\}) = \{0\}$ for all $x \in X$.

Proof. Let $x \in X$ and a∈ $l(\{x, 0\})$, then $a * x = 0 = a * a$, and so $l(\{x, y\}) = \{0\}$ for all $x \in X$.

If X is a BCIK-algebra, then Lemma 5.8 does not necessarily hold. In fact, let X={0,1,2,a,b} be a set with the ∗-operation given by Table 4, then $(X;*,0)$ is a BCIK-algebra (see[3,4,5]). Note that $l(\{x, y\}) = \{0\}$ for all $x \in \{1,2\}$ and $l(\{x, y\}) = \emptyset$ for all $x \in \{a, b\}$.

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Corollary 5.9. If X is a BCIK-algebra, then $l(\{x,y\}) = \{0\}$ for all $x \in X$ with $l(\{x,y\}) \neq \emptyset$.

Lemma 5.10. If X is a BCIK-algebra, then $l(\{x,y\}) = \{0\} for$ all $\in X_+$, where $X_+$is the BCIK-part of X.

Proof. Straightforward.

Lemma 3.11. For any elements a and b of a BCIK-algebra X, if $a * b = 0$, then $l(\{a\}) \subseteq I(\{b\})$ and $Z_b \subseteq Z_a$.

Table 5: *-operation.

*	0	a	b	C
0	0	0	0	0
A	A	0	a	A
B	B	b	0	B
C	C	C	C	0

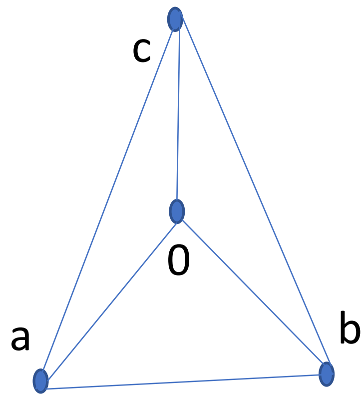

Figure 1: Associated graph $\Gamma(X)$ of X.

Proof. Assume that $a * b = 0$. Let $x \in l(\{a\})$, then $x * a = 0$, and so $0 = (x * b) * (x * a) = (x * b) * 0 = x * b$. Thus, $x \in l(\{b\})$, which shows that $l(\{a\}) \subseteq l(\{b\})$. Obviously, $Z_b \subseteq Z_a$.

Theorem 5.12. For any element x of a BCIK-algebra, the set of zero divisors of x is a quasi-ideal of X containing the zero element 0. Moreover, if Z_x is maximal in$\{Z_a | a \in X, Z_a \neq X\}$, then Z_x is l-prime.

Proof. By lemma 5.8, we have $0 \in Z_x$. Let $a \in X$ and $b \in Z_x$ be such that $a * b = 0$. Using Lemma 3.11, we have $l(\{x,a\}) = l(\{x\}) \cap l(\{a\}) \subseteq l(\{x\}) \cap l(\{b\}) = l(\{x,b\}) = \{0\}$, and so $l(\{x,a\}) = \{0\}$. Hence, $a \in Z_x$. Therefore , Z_x is a quasi-ideal of X. Let $a, b \in X$ be such that $l(\{a,b\}) \subseteq Z_x$ and $a \notin Z_x$, then $l(\{a,b,x\}) = 0$, Let $0 \neq y \in l(\{a,b\})$ be an arbitrarily element, then $l(\{b,y\}) \subseteq l(\{a,b,x\}) = \{0\}$, and so $l(\{b,y\}) = \{0\}$, that is, $b \in Z_x$. Since $y \in l(\{a,x\})$, we have $y * x = 0$. It follows from Lemma 3.11 that $Z_x \subseteq Z_y \neq X$ so from the maximality of Z_x it follows that $Z_x = Z_y$. Hence, $\in Z_x$, which shows that Z_x is l-prime.

Definition 5.13. By the associated graph of a BCIK-algebra X, denoted $\Gamma(X)$, we mean the graph whose vertices are just the elements of X, and for distinct $x, y \in \Gamma(X)$, there is an edge connecting x and y, denoted by x-y if and only if $l(\{x,y\}) = \{0\}$.

Example 5.14. Let X={0,a,b,c} be a set with the $*$-operation given by Table 5, then X is a BCIK-algebra (see[3,4,5]). The associated graph $\Gamma(X)$ of X is given by the Figure 1.

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Table 6: *-operation.

*	0	a	b	c	D
0	0	0	0	0	0
A	A	0	a	0	A
B	B	b	0	b	0
C	C	a	c	0	C
D	D	d	d	d	0

Table 7: *-operation.

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	0	1	1
2	2	1	0	2	2
3	3	3	3	0	3
4	4	4	4	4	0

Example 5.15. Let X={0,a,b,c,d} be a set with the $*$-operation given by Table 6, then X is a BCIK-algebra ([see 3,4,5]). By Lemma 5.8, each nonzero point is adjacent to 0. Note that $l(\{a,b\}) = l(\{a,d\}) = l(\{b,c\}) = l(\{c,d\}) = \{0\}, l(\{a,c\}) = \{0,a\}, and\ l(\{b,d\}) = \{0,b\}$. Hence the associated graph $\Gamma(X)$ of X is given by the Figure 2.

Example 5.16. Let X={0,1,2,3,4} be a set with the $*$-operation given by Table 7, then X is a BCIK-algebra ([see3,4,5]). By Lemma5.8, each nonzero point is adjacent to 0. Note that l({1,2})={0,1}, that is, 1 is not adjacent to 2 and $l(\{1,3\}) = l(\{1,4\}) = l(\{2,3\}) = l(\{2,4\}) = l(\{3,4\}) = \{0\}$. Hence, the associated graph $\Gamma(X)$ of X is given by Figure 3.

Example 5.17. Consider a BCIK-algebra X={0,1,2,a,b} with the $*$-operation given by Table 4, then$l(\{1,a\}) = l(\{1,b\}) = l(\{2,a\}) = l(\{2,b\}) = \emptyset$, $l(\{a,b\}) = \{a\}, and\ l(\{1,2\}) = \{0\}$. Since $X_+ = \{0,1,2\}$, we know from Lemma 3.10 that two points 1 and 2 adjacent to 0. The associated graph $\Gamma(X)$ of X is given by Figure 4.

Theorem 5.18. Let $\Gamma(X)$ be the associated graph of a BCIK-algebra X. For any $x, y \in \Gamma(X), if\ Z_x\ and\ Z_y$ are distinct l-prime quasi-ideals of X, then is an edge connecting x and y.

Proof. It is sufficient to show that $l(\{x,a\}) = \{0\}. If\ l(\{x,y\}) \neq \{0\}$, then $x \notin Z_y and\ y \notin Z_x$. For any $a \in Z_x$, we have $L(\{x,a\}) = \{0\} \subseteq Z_y$.Since Z_y is l-prime, it follows that $a \in Z_x$ so that $Z_x \subseteq Z_y$. Similarly, $Z_y \subseteq Z_x$. Hence, $Z_y = Z_x$, which is a contradiction. Therefore, x is adjacent to y.

Theorem 5.19. The associated graph of a BCIK-algebra is connected in which every nonzero vertex is adjacent to 0.

Proof. It follows from Lemma 5.8.

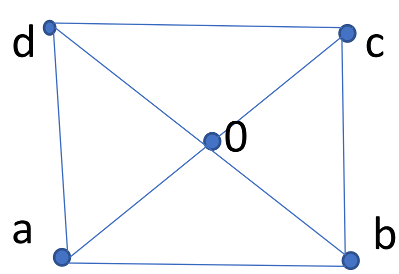

Figure 2: Associated graph $\Gamma(X)$ of X

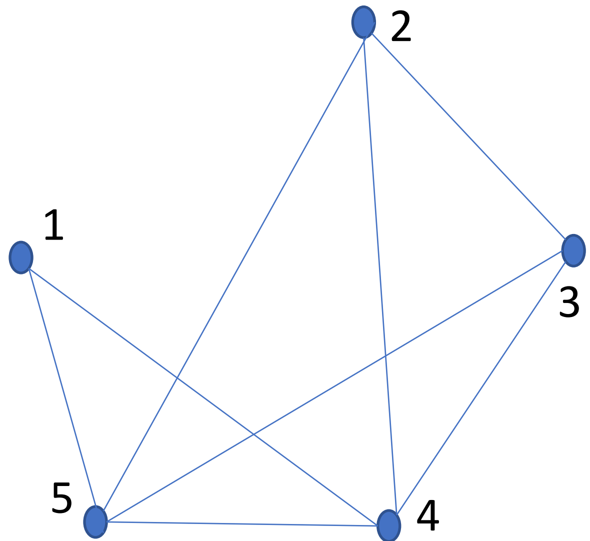

Figure 3: Associated graph $\Gamma(X)$ of X

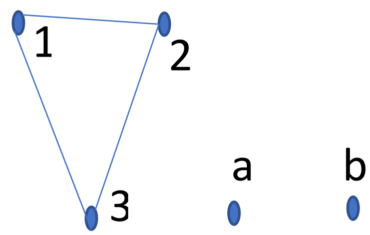

Figure 4: Associated graph $\Gamma(X)$ of X

6. CONCLUSION AND FUTURE RESEARCH

Derivation is a very interesting and are of research in the theory of algebraic structures in mathematics. The theory of derivations of algebraic structures is a direct descendant of the development of classical Galosis theory. The notions of BCIK-algebras were formulated S Rethina Kumar. The concept of nodal filter for a meet semi lattice introduced in 1973 by J. C. Varlet, then Nodal Filters of BL-algebras introduced in 2014 by R. Tayebi Khorami and A. Borumand Saeid.

In this paper, we have introduced the associative graph of a BCIK-algebra with several examples. We have shown that the associative graph of a BCIK-algebra is connected, but the associative graph of a BCIK-algebra is not connected.

In our future work is to study how to induce BCIK-algebra from the given graph (with some additional conditions):

(1) Developing the properties of quasi ideal of BCIK-algebra,
(2) Finding useful results on other structures,
(3) Constructing the related logical properties of such structures.

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