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BAIRE SPACES VIA BIPOLAR SINGLE VALUED NEUTROSOPHIC SET

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ABSTRACT

In this paper, we introduce bipolar single valued neutrosophic Baire and bipolar single valued neutrosophic pre Baire spaces in bipolar single valued neutrosophic topological spaces. We also examine some of their properties and characterizations.

KEYWORDS: ***Bipolar single valued neutrosophic Baire space and Bipolar single valued neutrosophic pre Baire space.***

1. INTRODUCTION

Fuzzy topology was introduced by C.L.Chang [3] in 1967 after the introduction of fuzzy sets by L.A.Zadeh [16] in 1965. In 1994, W.R.Zhang [17] who introduced the notion of a bipolar fuzzy set. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] in 1986 as a generalization of fuzzy sets. Smarandache [12] introduced the neutrosophic set which is the base for the new mathematical theories. Neutrosophic topological spaces were presented by Salama et al. [11]. Single-valued neutrosophic sets were proposed by Wang et al.[15] by simplifying the Neutrosophic set. Single-valued neutrosophic topological space was given by YL Liu and HL Yang [9] and discussed the relationships between single valued neutrosophic approximation spaces and single valued neutrosophic topological spaces. Bipolar single-valued neutrosophic set was introduced by Mohana et al. [10] and also they give bipolar single-valued neutrosophic topological spaces. The concept of Baire space in fuzzy settings was introduced and studied by G.Thangaraj and S.Anjalmose[13]. Fuzzy Pre-Baire spaces was also investigated by G.Thangaraj and S.Anjalmose[14]. In Intuitionstic fuzzy, Dhavaseelan[7] was gave the concept of Intuitionstic fuzzy Baire spaces. Caldas et al [2] gave the Neutrosophic resolvable and Neutrosophic irresolvable space. Dhavaseelan et. al[8] introduced the concept of Neutrosophic Baire spaces.Here in this paper, we introduce the

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concept of bipolar single valued neutrosophic Baire space and bipolar single valued neutrosophic pre Baire space in bipolar single valued neutrosophic topological spaces.

2. PRELIMINARIES

2.1 Definition [12]: Let a universe U of discourse. Then K={<x, $T_K(x)$, $I_K(x)$, $F_K(x)$>x∈X} defined as a neutrosophic set where truth-membership function T_K, an indeterminacy-membership function I_K and a falsity-membership function F_K. T_K, I_K, F_K are real or non-standard elements of] 0^-, 1^+ [. No restriction on the sum of $T_K(x)$, $I_K(x)$ and $F_K(x)$, so $0^- \leq \sup T_K(x) \leq \sup I_K(x) \leq \sup F_K(x) \leq 3^+$.

2.2 Definition [11]: A Neutrosophic topology [NT for short] is a non-empty set X is a family of Neutrosophic subsets in X satisfying the following axioms:

(NT_1) 0_N, $1_N \in \tau$,

(NT_2) $G_1 \cap G_2 \in \tau$, for any G_1, $G_2 \in \tau$,

(NT3) $\cup G_i \in \tau$, for every $\{G_i : i \in J\} \subseteq \tau$.

The pair (X, τ) is called a Neutrosophic topological space (NTS for short). The elements of τ are called Neutrosophic open sets [NOS for short]. A complement C(A) of a NOS A in NTS (X, τ) is called a Neutrosophic closed set [NCS for short] in X.

2.3 Definition [15]: Let a universe X of discourse. Then A_{NS}={<x,$T_A(x)$,$I_A(x)F_A(x)$>x∈X} defined as a single-valued neutrosophic set(SVNS in short) where truth-membership function T_A:X→[0,1],an indeterminacy-membership function I_A: X→[0,1] and a falsity-membership function F_A: X → [0,1].No restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0 \leq \sup T_A(x) \leq \sup I_A(x) \leq \sup F_A(x) \leq 3$. $\tilde{A}$=<T, I, F> is denoted as a single-valued neutrosophic number.

2.4 Definition [9]: A Single-valued neutrosophic topology on a non-empty set U is a family τ of SVNSs in U that satisfies the following conditions:

(T_1) $\tilde{\phi}$, $\tilde{U} \in \tau$,

(T_2) $\tilde{A} \cap \tilde{B} \in \tau$ for any $\tilde{A}$, $\tilde{B} \in \tau$,

(T_3) $\cup_{i \in I} \tilde{A}_i \in \tau$ for any $\tilde{A}_i \in \tau$, i∈ I, where I is an index set. The pair (U,τ) is called Single valued neutrosophic topological space and each SVNS $\tilde{A}$ in τ is referred to as a single valued neutrosophic open set in (U, τ). The complement of a single valued neutrosophic open set in (U, τ) is called a single valued neutrosophic closed set in (U, τ).

2.5 Definition [6]: In *X*, a bipolar neutrosophic set B *is* defined in the form B=<x, ($T^+(x)$, $I^+(x)$, $F^+(x)$, $T^-(x)$, $I^-(x)$, $F^-(x)$):x∈X> where T^+, I^+, F^+: X→ [1, 0] and T^- , I^- , F^- : X [-1, 0].The positive membership degree denotes the truth membership $T^+(x)$, indeterminate membership I^+ (x) and false membership F^+ (x) of an element x∈X corresponding to the set *A* and the negative membership degree denotes the truth membership $T^-(x)$, indeterminate membership

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$I^-(x)$ and false membership $F^-(x)$ of an element $x \in X$ to some implicit counter-property corresponding to a bipolar neutrosophic set .

2.6 Definition [8]: Let (X, T) be a neutrosophic topological space. A neutrosophic set A in (X, T) is called neutrosophic first category if A = $\bigcup_{i=1}^{\infty} B_i$, where B_i's are neutrosophic nowhere dense sets in (X,T). Any other neutrosophic set in (X,T) is said to be of Neutrosophic second category.

2.7 Definition [8]: A neutrosophic topological space (X, T) is called neutrosophic first category space if the neutrosophic set 1_N is a neutrosophic first category set in (X, T).That is, $1_N = \bigcup_{i=1}^{\infty} A_i$ where A_i's are neutrosophic nowhere dense sets in (X, T). Otherwise (X, T) will be called a neutrosophic second category space.

2.8 Definition [8]: Let A be a neutrosophic first category set in (X, T). Then $\overline{A}$ is called a neutrosophic residual set in (X, T).

2.9 Definition [8]: Let (X, T) be a neutrosophic topological space. Then (X, T) is said to neutrosophic Baire space if $N\,int(\bigcup_{i=1}^{\infty} A_i) = 0_N$, where A_i's are neutrosophic nowhere dense sets in (X, T).

2.10 Definition [14]: Let (X, T) be. A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy pre- dense if there exists no fuzzy pre-closed set μ in (X, T) such that $\lambda < \mu < 1$. That is $pcl(\lambda) = 1$.

2.11 Definition [14]: Let (X, T) be a fuzzy topological space. A fuzzy set λ in (X, T) is called a fuzzy pre- nowhere dense if there exists no fuzzy pre-open set μ in (X, T) such that $\mu \subset pcl(\lambda)$. That is $pint(pcl(\lambda)) = 0$.

2.12 Definition[14]: Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy pre-first category if $\lambda = \bigcup_{i=1}^{\infty} \lambda_i$, where λ_i's are fuzzy pre-nowhere dense sets in (X, T). Any other fuzzy set in (X,T) is said to be of fuzzy pre-second category.

2.13 Definition [14]: Let λ be a fuzzy pre- first category set in fuzzy topological space (X, T). Then 1-λ is called a fuzzy pre-residual set in (X, T).

2.14 Definition [14]: A fuzzy topological space (X, T) is called a fuzzy pre- first category space if the fuzzy set 1_X is a fuzzy pre- first category set in (X, T).That is, $1_X = \bigcup_{i=1}^{\infty} \lambda_i$ where λ_i's are fuzzy pre-nowhere dense sets in (X, T). Otherwise (X, T) will be called a fuzzy pre-second category space.

2.15 Definition [14]: Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy Baire space if $int(\bigcup_{i=1}^{\infty} \lambda_i) = 0$, where λ_i's are fuzzy nowhere dense sets in (X, T).

2.16 Definition [14]:. Let (X, T) be a fuzzy topological space. Then (X, T) is called a fuzzy pre- Baire space if

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$\text{int}(\bigcup_{i=1}^{\infty}\lambda_i) = 0$, where λ_i's are fuzzy pre-nowhere dense sets in (X, T).

2.17 Definition [10]: A Bipolar Single-Valued Neutrosophic set (BSVNs) S in X is defined in the form of

$$\text{BSVN (S)} = \langle x, (T^{+}_{BSVN}, I^{+}_{BSVN}, F^{+}_{BSVN}, T^{-}_{BSVN}, I^{-}_{BSVN}, F^{-}_{BSVN} : x \in X \rangle \rightarrow \text{(I)}$$

where $(T^{+}_{BSVN}, I^{+}_{BSVN}, F^{+}_{BSVN}): X \rightarrow [0,1]$ and $(T^{-}_{BSVN}, I^{-}_{BSVN}, F^{-}_{BSVN}): X \rightarrow [-1,0]$ .In this Definition, there T_{BSVN}^{+} and T_{BSVN}^{-} are acceptable and unacceptable in past. Similarly I_{BSVN}^{+} and I_{BSVN}^{-} are acceptable and unacceptable in future. F_{BSVN}^{+} and F_{BSVN}^{-} are acceptable and unacceptable in present respectively.

2.18 Definition [10]: Let two bipolar single-valued neutrosophic sets $BSVN_1(S)$ and $BSVN_2(S)$ in X defined as

$BSVN_1(S) = \langle x, T^{+}_{BSVN}(1), I^{+}_{BSVN}(1), F^{+}_{BSVN}(1), T^{-}_{BSVN}(1), I^{-}_{BSVN}(1), F^{-}_{BSVN}(1)) : x \in X \rangle$ and

$BSVN_2(S) = \langle x, T^{+}_{BSVN}(2), I^{+}_{BSVN}(2), F^{+}_{BSVN}(2), T^{-}_{BSVN}(2), I^{-}_{BSVN}(2), F^{-}_{BSVN}) : x \in X \rangle$. Then the operators are defined as follows:

(i) Complement

$$BSVN^{c}(S) = \{\langle x, (1-T^{+}_{BSVN}), (1-I^{+}_{BSVN}), (1-F^{+}_{BSVN}), (-1-T^{-}_{BSVN}), (-1-F^{-}_{BSVN}) : x \in X \rangle\}$$

(ii) Union of two BSVN

$BSVN_1(S) \cup BSVN_2(S) =$

$$\left\langle \begin{matrix} \max(T^{+}_{BSVN}(1), T^{+}_{BSVN}(2)), \min(I^{+}_{BSVN}(1), I^{+}_{BSVN}(2)), \min(F^{+}_{BSVN}(1), F^{+}_{BSVN}(2)) \\ \max(T^{-}_{BSVN}(1), T^{-}_{BSVN}(2)), \min(I^{-}_{BSVN}(1), I^{-}_{BSVN}(2)), \min(F^{-}_{BSVN}(1), F^{-}_{BSVN}(2)) \end{matrix} \right\rangle$$

(iii) **Intersection of two BSVN**

$BSVN_1(S) \cap BSVN_2(S) =$

$$\left\langle \begin{matrix} \min(T^{+}_{BSVN}(1), T^{+}_{BSVN}(2)), \max(I^{+}_{BSVN}(1), I^{+}_{BSVN}(2)), \max(F^{+}_{BSVN}(1), F^{+}_{BSVN}(2)) \\ \min(T^{-}_{BSVN}(1), T^{-}_{BSVN}(2)), \max(I^{-}_{BSVN}(1), I^{-}_{BSVN}(2)), \max(F^{-}_{BSVN}(1), F^{-}_{BSVN}(2)) \end{matrix} \right\rangle$$

2.19 Definition [10]: Let two bipolar single-valued neutrosophic sets be $BSVN_1$ and $BSVN_2$ in X defined as Then $S_1 \subseteq S_2$ if and only if

$T^{+}_{BSVN}(1) \le T^{+}_{BSVN}(2)$, $I^{+}_{BSVN}(1) \ge I^{+}_{BSVN}(2)$, $F^{+}_{BSVN}(1) \ge F^{+}_{BSVN}(2)$,

$T^{-}_{BSVN}(1) \le T^{-}_{BSVN}(2)$, $I^{-}_{BSVN}(1) \ge I^{-}_{BSVN}(2)$, $F^{-}_{BSVN}(1) \ge F^{-}_{BSVN}(2)$ for all $x \in X$.

2.20 Definition [10]: A bipolar single-valued neutrosophic topology (BSVNT) on a non-empty set X is a τ of BSVN sets satisfying the axioms

(i) $0_{BSVN}, 1_{BSVN} \in \tau$

(ii) $S_1 \cap S_2 \in \tau$ for any $S_1, S_2 \in \tau$

(iii) $\cup S_i \in \tau$ for any arbitrary family $\{S_i : i \in j\} \in \tau$. The pair (X,τ) is called BSVN topological

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space (BSVNTS). Any BSVN set in τ is called as BSVN open set(BSVNOs) in X. The complement S^c of BSVN set in BSVN topological space (X,τ) is called a BSVN closed set(BSVNCs).

2.21 Definition [4]: Let 0_{BSVN} and 1_{BSVN} be BSVNS in X defined as

0_{BSVN} = {<x, 0, 1, 1, -1, 0, 0: x ∈ X >} is said to be Null or Empty bipolar single valued neutrosophic set.

1_{BSVN} = {<x, 1, 0, 0, 0, -1, -1: x ∈ X >} is said to be Absolute or Unit bipolar single valued neutrosophic set.

2.22 Definition [4]: Let (X,τ) be a BSVN topological space (BSVNTS) and BSVN (S) be a BSVN set in X. Then the closure and interior of Sss is defined as BSVN int (S) = U {F: F is a BSVN open set (BSVNOs) in X and F⊆S} and BSVN cl (S) = ∩ {F: F is a BSVN closed set (BSVNCs) in X and S⊆F}.

2.23 Proposition [4]: Let S be any BSVNS in X. Then

(1) BSVN int (S^c) = (BSVN cl (S))^c and

(2) BSVN cl (S^c) = (BSVN int (S))^c.

2.24 Definition [5]: An bipolar single valued neutrosophic set (BSVNs) S in bipolar single valued neutrosophic topological space (BSVNT) (X, τ) is called bipolar single valued neutrosophic dense (BSVN dense) if there exists no bipolar single valued neutrosophic closed set T in (X, τ) such that S ⊆ T ⊆ 1_{BSVN}. (i.e.) BSVN cl(S) = 1_{BSVN}.

2.25 Definition [5]: Let (X, τ) be a bipolar single valued neutrosophic topological space. (X, τ) is called bipolar single valued neutrosophic resolvable (BSVN resolvable) if there exists a bipolar single valued neutrosophic dense set S in (X, τ) such that BSVN cl (S^c) = 1_{BSVN}. Otherwise, (X, τ) is called bipolar single valued neutrosophic irresolvable (BSVN irresolvable).

2.26 Definition [5]: A BSVNTS (X, τ) is called bipolar single valued neutrosophic submaximal (BSVN submaximal) space if each BSVNs S in (X, τ) such that BSVN cl(S) = 1_{BSVN}, then S ∈ τ.

2.27 Definition [5]: A BSVNs S in BSVNTS (X, τ) is called bipolar single valued neutrosophic nowhere dense(BSVN nowhere dense) set if there exist no BSVN open set U in (X, τ) such that U ⊆ BSVN cl(S). That is BSVN int (BSVN cl(S)) = 0_{BSVN}.

2.28 Definition [5]: A bipolar single valued neutrosophic Topological space (X, τ) is called a bipolar single valued neutrosophic almost resolvable (BSVN almost resolvable) space if $\bigcup_{i=1}^{\infty} S_i$ = 1_{BSVN}, where S_i 's are BSVNs's in (X, τ) are such that BSVN int (S_i) = 0_{BSVN}. Otherwise (X, τ) is called bipolar single valued neutrosophic almost irresolvable (BSVN almost irresolvable) space.

2.29 Definition [5]: An BSVNTS (X, τ) is called a bipolar single valued neutrosophic hyper connected (BSVN hyper connected) space if every BSVNOs is BSVN dense in (X, τ). That is BSVN cl (S_i) = 1_{BSVN}, for all $S_i \in$ τ.

2.30 Definition [5]: A BSVNs S in a BSVNTS (X, τ) is called BSVN-G_δ if S=$\bigcap_{i=1}^{\infty} S_i$, where each $S_i \in$ τ.

2.31 Definition [5]: A BSVNs S in a BSVNTS (X, τ) is called BSVN-F_σ if S=$\bigcup_{i=1}^{\infty} S_i$, where each $S_i \in$ τ.

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2.32 Definition [5]: A BSVNTS (X, τ) is called BSVN P-space; if countable intersection of BSVNOs's in (X, τ) is bipolar single valued neutrosophic open. That is, every non-zero BSVN- G_δ set in (X, τ) is bipolar single valued neutrosophic open in (X, τ).

3. BIPOLAR SINGLE VALUED NEUTROSOPHIC BAIRE SPACE

Definition 3.1 Let (X, τ) be a bipolar single valued neutrosophic topological space. A bipolar single valued neutrosophic set S in (X, τ) is called bipolar single valued neutrosophic first category if S = $\bigcup_{i=1}^{\infty} T_i$, where T_i's are bipolar single valued neutrosophic nowhere dense sets in (X, τ). Any other bipolar single valued neutrosophic set in (X, τ) is said to be bipolar single valued neutrosophic second category.

Definition 3.2 The Complement of bipolar single valued neutrosophic First category sets in (X, τ) is a bipolar single valued neutrosophic residual set in (X, τ).

Definition 3.3 A bipolar single valued neutrosophic topological space (X, τ) is called bipolar single valued neutrosophic first category space if the bipolar single valued neutrosophic set 1_{BSVN} is a bipolar single valued neutrosophic first category set in (X, τ).That is, $1_{BSVN} = \bigcup_{i=1}^{\infty} S_i$, where S_i's are bipolar single valued neutrosophic nowhere dense sets in (X, τ). Otherwise (X, τ) will be called an bipolar single valued neutrosophic second category space.

Proposition 3.4 If S be a bipolar single valued neutrosophic first category set in (X, τ), then $S^c = \bigcap_{i=1}^{\infty} T_i$, where BSVN cl (T_i) = 1_{BSVN}.

Definition 3.5 Let (X, τ) be a bipolar single valued neutrosophic topological space. Then (X, τ) is said to bipolar single valued neutrosophic Baire space if BSVN int ($\bigcup_{i=1}^{\infty} S_i$) = 0_{BSVN}, where S_i's are bipolar single valued neutrosophic nowhere dense sets in (X, τ).

Example 3.6 Let X = {p, q}. Define the bipolar single valued neutrosophic sets S, T and P as follows

$$S=\begin{Bmatrix} < p,(0.1,0.3,0.5,-0.7,-0.8,-0.1) > \\ < q,(0.2,0.4,0.6,-0.8,-0.2,-0.4) > \end{Bmatrix}$$ .Then τ = {0_{BSVN}, 1_{BSVN}, S} is a BSVNT.

$$T=\begin{Bmatrix} < p,(0.8,0.7,0.8,-0.7,-0.2,-0.3) > \\ < q,(0.1,0.7,0.9,-0.2,-0.7,-0.2) > \end{Bmatrix} \quad P=\begin{Bmatrix} < p,(0.2,0.1,0.5,-0.3,-0.9,-0.1) > \\ < q,(0.8,0.3,0.1,-0.3,-0.5,-0.6) > \end{Bmatrix}$$

Now S^c, T, P^c are bipolar single valued neutrosophic nowhere dense sets in (X, τ). Also BSVN int ($S^c \cup T \cup P^c$) = 0_{BSVN}. Hence (X, τ) is a bipolar single valued neutrosophic Baire space.

Proposition 3.7 If BSVN int ($\bigcup_{i=1}^{\infty} S_i$) = 0_{BSVN}, where BSVN int (S_i) = 0_{BSVN} and $S_i \in \tau$, then (X, τ) is an bipolar single valued neutrosophic Baire space.

Proposition 3.8 If BSVN cl ($\bigcap_{i=1}^{\infty} S_i$) = 1_{BSVN}, where S_i's are bipolar single valued neutrosophic dense and bipolar single valued neutrosophic open sets in (X, τ), then (X, τ) is an bipolar single valued neutrosophic Baire Space.

Proposition 3.9 Let (X, τ) be a bipolar single valued neutrosophic topological space. Then the following are equivalent

(i) (X, τ) is a bipolar single valued neutrosophic Baire space.

(ii) BSVN int (S) = 0_{BSVN}, for every bipolar single valued neutrosophic first category set S in (X, τ).

(iii) BSVN cl (T) = 1_{BSVN}, for every bipolar single valued neutrosophic residual set T in (X, τ).

Proposition 3.10 A bipolar single valued neutrosophic topological space (X, τ) is a bipolar single valued neutrosophic Baire space if and only if ($\bigcup_{i=1}^{\infty} S_i$) = 1_{BSVN}, where S_i's is a bipolar single valued neutrosophic closed set in (X, τ) with BSVN int (S_i) = 0_{BSVN}, implies that BSVN int ($\bigcup_{i=1}^{\infty} S_i$) = 0_{BSVN}.

Proposition 3.11 If the bipolar single valued neutrosophic topological space (X, τ) is a bipolar single valued neutrosophic first category, then (X, τ) is a bipolar single valued neutrosophic almost resolvable space.

Proposition 3.12 If BSVN cl (BSVN int(S)) = 1_{BSVN}, for each bipolar single valued neutrosophic dense set S in a bipolar single valued neutrosophic almost resolvable space (X, τ), then (X, τ) is a bipolar single valued neutrosophic first category space.

Proposition 3.13 If BSVN cl (BSVN int(S)) = 1_{BSVN}, for each bipolar single valued neutrosophic dense set S in a bipolar single valued neutrosophic almost resolvable space (X, τ), then (X, τ) is not a bipolar single valued neutrosophic Baire space.

Proposition 3.14 If (X, τ) is a bipolar single valued neutrosophic second category space, then (X, τ) is a bipolar single valued neutrosophic almost resolvable space.

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Proposition 3.15 If the bipolar single valued neutrosophic almost resolvable space (X, τ) is a bipolar single valued neutrosophic submaximal space, then (X, τ) is a bipolar single valued neutrosophic first category space.

Proposition 3.16 If the bipolar single valued neutrosophic almost irresolvable space (X, τ) is a bipolar single valued neutrosophic submaximal space, then (X, τ) is a bipolar single valued neutrosophic second category space.

Proposition 3.17 If the bipolar single valued neutrosophic almost irresolvable space (X, τ) is a bipolar single valued neutrosophic submaximal space, then (X, τ)is not a bipolar single valued neutrosophic Baire space.

Theorem 3.18 If the BSVNTS (X, τ) is a bipolar single valued neutrosophic Baire space, then (X, τ) is an bipolar single valued neutrosophic almost irresolvable space.

Proposition 3.19 If $(\bigcap_{i=1}^{\infty} S_i) = 0_{BSVN}$, where S_i's are bipolar single valued neutrosophic residual sets in a bipolar single valued neutrosophic Baire space (X, τ), then (X, τ) is a bipolar single valued neutrosophic almost resolvable space.

Proposition 3.20 If $\bigcup_{i=1}^{\infty} S_i = 1_{BSVN}$, wher S_i's are non-zero bipolar single valued neutrosophic open sets in a bipolar single valued neutrosophic topological space (X, τ), then (X, τ) is a bipolar single valued neutrosophic almost irresolvable space.

Proposition 3.21 If each BSVN G_δ set is bipolar single valued neutrosophic open and bipolar single valued neutrosophic dense set in a bipolar single valued neutrosophic topological space (X, τ) , then (X, τ) is a bipolar single valued neutrosophic almost irresolvable space.

4. BIPOLAR SINGLE VALUED NEUTROSOPHIC PRE-DENSE AND PRE NOWHERE DENSE

Definition 4.1 An bipolar single valued neutrosophic set (BSVNs) S in bipolar single valued neutrosophic topological space (BSVNT) (X, τ) is called bipolar single valued neutrosophic pre-dense (BSVN dense) if there exists no bipolar single valued neutrosophic pre- closed set T in (X, τ) such that $S \subseteq T \subseteq 1_{BSVN}$. (i.e.)
BSVN pcl(S) = 1_{BSVN}.

Example 4.2 Let X = {p, q}. Define the bipolar single valued neutrosophic sets S and T as follows

$$S=\left\{\begin{matrix} < p,(0.3,0.4,0.5,-0.6,-0.7,-0.8) > \\ < q,(0.9,0.1,0.2,-0.3,-0.4,-0.5) > \end{matrix}\right\}$$

.Then τ = {0_{BSVN}, 1_{BSVN}, S} is a BSVNT.

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T= $\begin{Bmatrix} < p,(0.1,0.3,0.2,-0.4,-0.5,-0.7) > \\ < q,(0.6,0.8,0.2,-0.1,-0.4,-0.1) > \end{Bmatrix}$

Here 1_{BSVN}, S, T, T^c are non-zero bipolar single valued neutrosophic pre-open sets in (X, τ). Then 1_{BSVN}, S are bipolar single valued neutrosophic pre-dense sets in (X, τ).

Definition 4.3 A BSVNs S in BSVNTS (X, τ) is called bipolar single valued neutrosophic pre-nowhere dense set if there exist no BSVN pre-open set U in (X, τ) such that U $\subseteq$ BSVN pcl(S). That is

BSVN pint (BSVN pcl(S)) = 0_{BSVN}.

Example 4.4 Let X = {p, q}. Define the bipolar single valued neutrosophic sets S and T as follows

S= $\begin{Bmatrix} < p,(0.5,0.4,0.1,-0.6,-0.5,-0.4) > \\ < q,(0.5,0.1,0.1,-0.3,-0.1,-0.2) > \end{Bmatrix}$ T= $\begin{Bmatrix} < p,(0.4,0.5,0.3,-0.6,-0.3,-0.1) > \\ < q,(0.2,0.3,0.6,-0.4,-0.2,-0.1) > \end{Bmatrix}$

Then τ = {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT.

R= $\begin{Bmatrix} < p,(0.1,0.5,0.5,-0.7,-0.3,-0.4) > \\ < q,(0.5,0.9,0.2,-0.3,-0.2,-0.1) > \end{Bmatrix}$

Here 1_{BSVN}, S^c, T^c, R, R^c are non-zero bipolar single valued neutrosophic pre-closed sets in (X, τ). Then S^c, T^c are bipolar single valued neutrosophic pre-nowhere dense sets in (X, τ).

Theorem 4.5 Let S is a bipolar single valued neutrosophic pre-nowhere dense set in (X, τ), then BSVN pint (S) = 0_{BSVN}.

Proof. If S is a bipolar single valued neutrosophic pre-nowhere dense set in (X, τ). Then BSVN pint(S) $\subseteq$ BSVN pint (pcl(S)) = 0_{BSVN}. Hence BSVN pint (S) = 0_{BSVN}.

Theorem 4.6 Let S be a bipolar single valued neutrosophic set. If S is a bipolar single valued neutrosophic pre-closed set in (X, τ) with BSVN pint (S) = 0_{BSVN}, then S is a bipolar single valued neutrosophic pre-nowhere dense set in (X, τ).

Theorem 4.7 If S is a bipolar single valued neutrosophic pre-dense and bipolar single valued neutrosophic pre-open set in a BSVNTS (X, τ) and if T $\subseteq$ S^c, then T is a bipolar single valued neutrosophic pre-nowhere dense set in (X, τ) .

Theorem 4.8 If S is a bipolar single valued neutrosophic pre-nowhere dense set in a bipolar single valued neutrosophic topological space (X, τ), then S^c is a bipolar single valued neutrosophic pre-dense set in (X, τ).

Theorem 4.9 If bipolar single valued neutrosophic pre-nowhere dense set S in a bipolar single valued neutrosophic topological space (X, τ) is a Bipolar single valued neutrosophic closed set, then S is a bipolar single valued neutrosophic nowhere dense set.

Theorem 4.10 If bipolar single valued neutrosophic nowhere dense set S in a bipolar single valued neutrosophic topological space (X, τ) is a Bipolar single valued neutrosophic pre-closed set, then S is a bipolar single valued neutrosophic pre-nowhere dense set.

Definition 4.11 Let (X, τ) be a bipolar single valued neutrosophic topological space. A bipolar single valued neutrosophic set S in (X, τ) is called bipolar single valued neutrosophic pre-first category if S = $\bigcup_{i=1}^{\infty} T_i$, where T_i's are bipolar single valued neutrosophic pre-nowhere dense sets in (X, τ). Any other bipolar single valued neutrosophic set in (X, τ) is said to be bipolar single valued neutrosophic pre-second category.

Example 4.12 Let X = {p, q}. Define the bipolar single valued neutrosophic sets S and T as follows

$$S=\begin{Bmatrix} < p,(0.5,0.4,0.1,-0.6,-0.5,-0.4) > \\ < q,(0.5,0.1,0.1,-0.3,-0.1,-0.2) > \end{Bmatrix} \quad T=\begin{Bmatrix} < p,(0.4,0.5,0.3,-0.6,-0.3,-0.1) > \\ < q,(0.2,0.3,0.6,-0.4,-0.2,-0.1) > \end{Bmatrix}$$

Then τ = {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT.

$$R=\begin{Bmatrix} < p,(0.1,0.5,0.5,-0.7,-0.3,-0.4) > \\ < q,(0.5,0.9,0.2,-0.3,-0.2,-0.1) > \end{Bmatrix}$$

.Here 1_{BSVN}, S^c, T^c, R, R^c are non-zero bipolar single valued neutrosophic pre-closed sets in (X, τ). Then S^c, T^c are bipolar single valued neutrosophic pre-nowhere dense sets in (X, τ). Then ($S^c \cup T^c$) = T^c. therefore T^c is a bipolar single valued neutrosophic pre-first category.

Proposition 4.13 If S be a BSVN pre-first category set in BSVNTS (X, τ), then $S^c = \bigcap_{i=1}^{\infty} T_i$ where BSVN pcl (T_i) = 1_{BSVN}.

Definition 4.14 The Complement of bipolar single valued neutrosophic pre-first category sets in (X, τ) is a bipolar single valued neutrosophic pre-residual set in (X, τ).

Definition 4.15 Let (X, τ) be a bipolar single valued neutrosophic topological space. Then (X, τ) is said to bipolar single valued neutrosophic pre-Baire space if BSVN pint ($\bigcup_{i=1}^{\infty} S_i$) = 0_{BSVN}, where S_i's are bipolar single valued neutrosophic pre-nowhere dense sets in (X, τ).

Example 4.16 Let X = {p, q}. Define the bipolar single valued neutrosophic sets S and T as follows

S= $\begin{Bmatrix} < p,(0.5,0.4,0.1,-0.6,-0.5,-0.4) > \\ < q,(0.5,0.1,0.1,-0.3,-0.1,-0.2) > \end{Bmatrix}$ T= $\begin{Bmatrix} < p,(0.4,0.5,0.3,-0.6,-0.3,-0.1) > \\ < q,(0.2,0.3,0.6,-0.4,-0.2,-0.1) > \end{Bmatrix}$

Then τ = {0_{BSVN}, 1_{BSVN}, S, T} is a BSVNT.

R= $\begin{Bmatrix} < p,(0.1,0.5,0.5,-0.7,-0.3,-0.4) > \\ < q,(0.5,0.9,0.2,-0.3,-0.2,-0.1) > \end{Bmatrix}$. Here 1_{BSVN}, S^c, T^c, R, R^c are non-zero bipolar single valued neutrosophic pre-closed sets in (X, τ). Then S^c, T^c are bipolar single valued neutrosophic pre-nowhere dense sets in (X, τ). Then BSVN pint (T^c) = 0_{BSVN}. Therefore it is a bipolar single valued neutrosophic pre-Baire space.

Proposition 4.17 If BSVN pint ($\bigcup_{i=1}^{\infty} S_i$) = 0_{BSVN}, where BSVN pint (S_i) = 0_{BSVN} and S_i's are BSVN pre-closed sets in (X, τ), then (X, τ) is a bipolar single valued neutrosophic pre-Baire space.

Proposition 4.18 If BSVN pcl ($\bigcap_{i=1}^{\infty} S_i$) = 1_{BSVN}, where S_i's are bipolar single valued neutrosophic pre-dense and bipolar single valued neutrosophic pre-open sets in (X, τ), then (X, τ) is a bipolar single valued neutrosophic pre-Baire Space.

Proposition 4.19 Let (X, τ) be a bipolar single valued neutrosophic topological space. Then the following are equivalent

(i) (X, τ) is a bipolar single valued neutrosophic pre-Baire space.

(ii) BSVN pint (S) = 0_{BSVN}, for every bipolar single valued neutrosophic pre-first category set S in (X, τ).

(iii) BSVN pcl (T) = 1_{BSVN}, for every bipolar single valued neutrosophic pre-residual set T in (X, τ).

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