ISSN (Online): 2455-3662
EPRA International Journal of Multidisciplinary Research (IJMR) - Peer Reviewed Journal
Volume: 7 | Issue: 7 | July 2021|| Journal DOI: 10.36713/epra2013 || SJIF Impact Factor 2021: 8.047 || ISI Value: 1.188

Black-Hole Having Electric Hairs for Positive Cosmological Constant $\Lambda > 0$ on a Cosmic Horizon Scale $\sim 1/\sqrt{\Lambda}$

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Article DOI: https://doi.org/10.36713/epra7771
DOI No: 10.36713/epra7771

ABSTRACT

This paper is a technical review for a more deliberate paper (Bhattacharya & Lahiri, 2007) where it has been shown that on a positive cosmological scale with $\Lambda > 0$ having a cosmic horizon scale $\sim 1/\sqrt{\Lambda}$, there exists the soft electric hairs for the solution having the T_{00} components of the stress-energy tensor $T_{\mu\nu}$ i.e., $\rho = 0$ on black hole horizon B_H having the maximum density at black hole singularity B_S where cosmic horizon C_H and black hole horizon B_H has only been considered.

KEYWORDS: ***Black Hole – Cosmic Horizon – TOV Limit – Stress-Energy Tensor – Positive Cosmological Constant***

METHODOLOGY: In this paper we will use the formulations used in Ref. [1] of BHs having soft hairs on a positive cosmological constant scales. Any star having a larger limit than the Tolman-Oppenheimer-Volkoff limit [2] could turn to a collapsed form of gravitational singularity with the BH solutions of the prescribed Einstein-Maxwell gravitation and electromagnetism theory could be identified by means of three observable parameters as mass, charge and angular momentum. The No-hair theorem [3] stands for that, BH does not contain any other informations apart form these 3 externally observable parameters. The 11 components of the BH, the mass-energy (1 component), linear momentum (3 components), angular momentum (3 components), position (3 components), electric charge (1 components) when scaled through the reference frame by adopting the linear momentum and position to 0, and orienting the spin along the +Z axis, 3 components remain as of mass, spin and charge. However, in the case of a positive cosmological constant $\Lambda > 0$ having a cosmic horizon scale $\sim 1/\sqrt{\Lambda}$ there are some soft electric hairs over the horizons [4] provided the stress-energy counterpart $T_{00} = \rho$ taken the values in $B_S \ll B_H$ with ρ in B_H is vanishing probing the way for an external hairy charge above the horizon.

Denoting space-time with at least 2 horizons B_H and C_H where there are no globally defined timelike killing vectors ξ^μ between those horizons satisfying the norm $\sigma(r) = \sqrt{-\xi^\mu \xi_\mu}$ having the radial coordinate r dividing the manifold into 3 static regions, r_S, B_H and C_H with a spatial hypersurface coordinate bundles Ξ_C where the regions lies in the extreme future of Ξ_C in the limit $C_H > r_S > C_H$ such that the points of Ξ_C do not lie to the past of $r_S > C_H$ where the no-hair theorems have been computed on spacelike hypersurface Ξ_C as classical fields between the two regions B_H and C_H. The Lie derivative of the norm $\sigma(r)$ must vanish along the Killing vector field $\xi^\mu \xi_\mu$ as regards to the stress-energy tensor $T_{\mu\nu}$ of the Einstein field equations $G_{\mu\nu} = 8\pi T_{\mu\nu} - \Lambda g_{\mu\nu}$ remaining in a bounded form.

If we denote the projection vector $\varpi^\mu_\mu, \ldots \varpi^\nu_\nu$ with an induced connection bundle $\widetilde{\nabla}_\mu$ over an antisymmetric tensor field ζ having a vanishing Lie derivative along its components ξ^μ given as,

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$$\tilde{\nabla}_a(\sigma\kappa^{a\mu\ldots\nu}) = \sigma\left(\nabla_a\zeta^{a\mu'\ldots\nu'}\right)\varpi_\mu^\mu, \ldots \varpi_\nu^\nu$$

With $\kappa^{a\mu\ldots\nu}$ being the connection on Ξ_C. This is important as we will use this convention for the rest of the paper with the scalar field ϕ having its first derivative potential or equations of motion as $V'(\phi)$ along with its convex form, the second derivative $V''(\phi)$ satisfying the relation $V''(\phi) \geq 0$ where the norm taken with the first derivative of the potential $V'(\phi)$ as $\sigma V'(\phi)$ given the implicit derivative of the connection bundle with the normed-connection bundle scalar form as [5],

$$\tilde{\nabla}_\mu\left(\sigma\tilde{\nabla}^\mu\phi\right) = \sigma V'(\phi)$$

Assuming the potential having mass, when integrated over spacelike hypersurfaces Ξ_C gives an additional relation over the boundary domains as $\partial\Xi_C$ of the form,

$$\int_{\partial\Xi_C}\sigma V'(\phi)m^\mu\tilde{\nabla}_\mu(\phi) = -\int_{\Xi_C}\sigma\left[V''(\phi)\tilde{\nabla}^\mu\phi\tilde{\nabla}_\mu\phi + V'^2\phi\right]$$

Which for an additional massive fields for a massless potential ϕ_0 multiplying with the Ξ_C integral reduces to,

$$\int_{\partial\Xi_C}\sigma V'(\phi)m^\mu\tilde{\nabla}_\mu(\phi) = -\phi_0\int_{\Xi_C}\sigma\left[V''(\phi)\tilde{\nabla}^\mu\phi\tilde{\nabla}_\mu\phi + V'^2\phi\right]$$

Where it has been shown that for a boundary domain $\partial\Xi_C$, with anything that is massless scalar ϕ_0 looses further gravitational potential taking the explicit form $-\phi_0 \ll \phi_0$ with the stress-energy tensor being $\tilde{\nabla}^\mu\phi\tilde{\nabla}_\mu\phi \approx T_{\mu\nu}$ being bounded at the horizons satisfying the Schwarz inequality theorem as,

$$|m^a\tilde{\nabla}_a\phi|^2 \leq (m^a m_a)\left(\tilde{\nabla}^a\phi\tilde{\nabla}_a\phi\right) \equiv T_{\mu\nu}$$

Here, as the norm $\sigma(r) = 0$, the $\partial\Xi_C$ integral vanishes with the stress-energy term $\tilde{\nabla}^a\phi\tilde{\nabla}_a\phi$ being non-negative with the convex form also being a satisfactory non-negative solution as $V''(\phi) \geq 0$ as we see before, illuminating the fact that the potential is constant everywhere with its bare minimum lies at the hypersurface $\partial\Xi_C$. This gives us a cumulative formulation as,

$$\int_{\partial\Xi_C}\sigma V'(\phi)m^\mu\tilde{\nabla}_\mu(\phi) = -\phi_0\int_{\Xi_C}\sigma\left[V''(\phi)\tilde{\nabla}^\mu\phi\tilde{\nabla}_\mu\phi + V'^2\phi\right] + \int_{\Xi_C}V'(\phi_0)m^\mu\tilde{\nabla}_\mu(\phi)$$

Showing without the norm relation,

$$\int_{\Xi_C}V'(\phi_0)m^\mu\tilde{\nabla}_\mu(\phi) > -\phi_0\int_{\Xi_C}\sigma\left[V''(\phi)\tilde{\nabla}^\mu\phi\tilde{\nabla}_\mu\phi + V'^2\phi\right]$$

Thereby denoting the boundary integral $\partial\Xi_C$ non-negative with no curvature singularity with only a coordinate singularity.

Specifying the electric potential $\phi = \sigma^{-1}\xi_\mu A^\mu$ with the electric field being $e^\mu = \sigma^{-1}\xi_\nu F^{\mu\nu}$ giving the equation of motion with the mass m associated with the potential ϕ along with the connection bundle $\tilde{\nabla}_\mu$ in the form,

$$\tilde{\nabla}_\mu e^\mu = m^2\phi$$

Giving the electric field for e^μ covering the Lie derivative parameter $\square_\xi\eta = 0 = \dot{\eta}$, along with the charge $e^\mu = 0$ with the T_{00} component ρ decreasing from B_S to B_H giving us three relations as,

$$\tilde{\nabla}_\mu e^\mu - q^2\rho^2\left(\phi + \frac{1}{qv}\square_\xi\eta\right) = 0$$

$$m^2\phi - q^2\rho^2\left(\phi + \frac{1}{qv}\square_\xi\eta\right) = 0$$

Where we can strictly see the mass m being positive definite has a magnitude of $\left|q^2\rho^2\left(\phi + \frac{1}{qv}\square_\xi\eta\right)\right|$ where a static spherically symmetric BH which has no electric charge, the gauge field can become massive via spontaneous symmetry breaking pervading the possibility of the electric field integral over the boundary surface $\partial\Xi_C$ and the spacelike hypersurface Ξ_C in the relation as,

$$\int_{\partial\Xi_C}\sigma\left(\phi + \frac{1}{\sigma qv}\dot{\eta}\right)e^\mu n_\mu + \int_{\Xi_C}\sigma\left[e^\mu e_u + q^2\rho^2\left(\phi + \frac{1}{\eta qv}\dot{\eta}\right)^2\right] = 0$$

Where as $e^\mu e_u$ appears in the stress-energy tensor $T_{\mu\nu}$, the Schwarz inequality can be given by,

$$\left|n^\mu n_\mu(\phi + (qv\sigma)^{-1}\dot{\eta})\right|^2 \leq \left(n^\mu n_\mu\right)\left(q^2\rho^2(\phi + (qv\sigma)^{-1}\dot{\eta})\right)$$

Admits that Ξ_C integral can only be non-zero if $(\phi + (qv\sigma)^{-1}\dot{\eta})$ diverges on at least one horizon, giving the value $\rho = 0$ abiding that BH can have hairy electric implants on their horizon.

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