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EMBEDDED EIGENVALUES OF THE DIRAC OPERATOR WITH MASS

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SUMMARY

In this paper, a Dirac operator with mass is constructed by the Gelfand – Levitan method, the continuous spectrum of which contains a countable number of positive eigenvalues, and sufficient conditions for the resulting potential to belong to space are found and the dependence of the location of the embedded eigenvalues of the Dirac operator with mass on general boundary conditions.

KEY WORDS: ***operator, vector functions, spectrum, Parseval's equality, problem, system, root.***

Consider the Dirac operator self-adjoint in the space of vector functions $L_2^2(0,\infty)$ generated by the differential expression

$$Dy \equiv B\frac{dy}{dx} + mTy + \Omega(x)y = \lambda\, y, \quad 0 < x < \infty \tag{1}$$

and the boundary condition

$$y_1(0) = 0, \tag{2}$$

where

$$y = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \ \Omega(x) = \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix}, \tag{3}$$

m – constant positive number (mass), - real continuous functions.

Introduce the splitting up of the operator spectrum D

$$\sigma(D) = p\sigma(D) \cup c\sigma(D) \cup pc\sigma(D),$$

where $p\sigma(D)$ – point spectrum, $c\sigma(D)$ – continuous spectrum, $pc\sigma(D)$ – a set of eigenvalues located on a continuous spectrum, i.e. nested eigenvalues.

Let us denote by $\varphi(x,\lambda) = \left(\varphi_1(x,\lambda), \varphi_2(x,\lambda)\right)^T$ the solution of the system of equations (1), satisfying the initial conditions

$$\varphi_1(0,\lambda) = 0, \qquad \varphi_2(0,\lambda) = 1. \tag{4}$$

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Let $f(x)=\left(f_1(x),\ f_2(x) \right)^T$ – an arbitrary real vector-function from the class $L_2^2(0,\infty)$. If we put

$$F(\lambda)=\int_0^\infty [f_1(x)\varphi_1(x,\lambda)+ f_2(x)\varphi_2(x,\lambda)]dx,$$

then there is a monotonically increasing function $\rho(\lambda),\ -\infty<\lambda<\infty,$ independent of $f(x)$ called the spectral function of problem (1), (2), such that has a place Parseval's equality

$$\int_0^\infty [f_1^{\,2}(x)+ f_2^{\,2}(x)]\,dx=\int_{-\infty}^\infty F^2(\lambda)\,d\rho(\lambda). \quad (5)$$

At $\Omega(x)\equiv 0$ operator D we will designate through D_0, i.e.

$$D_0 y\equiv B\frac{dy}{dx}+mTy=\lambda y,\quad 0<x<\infty \quad (6)$$

$$y_1(0)=0. \quad (7)$$

Let us denote by $\theta^0(x,\lambda)$ and $\varphi^0(x,\lambda)$ system solutions (6), satisfying the following initial conditions:

$$\theta^0(0,\lambda)=\begin{pmatrix}1\\0\end{pmatrix},\quad \varphi^0(0,\lambda)=\begin{pmatrix}0\\1\end{pmatrix}.$$

It is easy to deduce that

$$\theta^0(x,\lambda)=\begin{pmatrix}\cos\left(\sqrt{\lambda^2-m^2}\right)x\\ (\lambda-m)\dfrac{\sin\left(\sqrt{\lambda^2-m^2}\right)x}{\sqrt{\lambda^2-m^2}}\end{pmatrix},$$

$$\varphi^0(x,\lambda)=\begin{pmatrix}-(\lambda+m)\dfrac{\sin\left(\sqrt{\lambda^2-m^2}\right)x}{\sqrt{\lambda^2-m^2}}\\ \cos\left(\sqrt{\lambda^2-m^2}\right)x\end{pmatrix}.$$

From the condition

$$\psi^0(x,z)=\theta^0(x,z)+m_0(z)\varphi^0(x,z)\in L_2^2(0,\infty),\quad \operatorname{Im} z>0$$

unequivocally determined by the Weil – Titchmarsh function of the problem (6), (7):

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$$m_0(z) = -i\frac{\sqrt{z^2-m^2}}{z+m}\ , \quad \operatorname{Im} z > 0 .$$

Here the root is understood in the analytical sense, and the branch is taken that satisfies the condition $\sqrt{z^2-m^2} = z + \overset{=}{o}(1), \quad |z| \to \infty.$

Using the Weil – Titchmarsh function $m_0(z)$ and the formula

$$\rho_0(\lambda) - \rho_0(\mu) = \frac{1}{\pi} \lim_{v\to+0} \int_{\mu}^{\lambda} \left\{ -\operatorname{Im}\left[m_0(u+iv) \right] \right\} du$$

find the derivative of the spectral function of the problem (6), (7)

$$\rho_0'(\lambda) = \begin{cases} -\dfrac{1}{\pi}\cdot\dfrac{\sqrt{\lambda^2-m^2}}{\lambda+m}, & \lambda < -m, \\ 0, & -m<\lambda<m, \\ \dfrac{1}{\pi}\cdot\dfrac{\sqrt{\lambda^2-m^2}}{\lambda+m}, & \lambda > m. \end{cases} \tag{8}$$

In formula (8), the roots are understood in the arithmetic sense. The spectrum of problem (6), (7) is purely continuous and consists of the set $(-\infty, -m] \cup [m, \infty)$, that is

$$c\sigma(D_0) = (-\infty, -m] \cup [m, \infty), \quad p\sigma(D_0) = \varnothing, \quad pc\sigma(D_0) = \varnothing .$$

Lemma 1. If $\varphi(x,\lambda)$ – solution of equation (1) with initial conditions (4), then there exists a matrix $K(x,t) = \left\| K_{ij}(x,t) \right\|_{i,j=1,2}$ such that

$$\varphi(x,\lambda) = \varphi^0(x,\lambda) + \int_0^x K(x,t)\varphi^0(t,\lambda)\,dt, \tag{9}$$

moreover $K(x,t)$ is the solution to the problem

$$B\frac{\partial K}{\partial x} + \frac{\partial K}{\partial t}B + mTK - mKT = -\Omega(x)K, \tag{10}$$

$$K(x,x)B - BK(x,x) = \Omega(x), \tag{11}$$

$$K_{11}(x,0) = K_{21}(x,0) = 0. \tag{12}$$

And vice versa, if the matrix-function $K(x,t)$ is the solution to the problem (10) –(12), then the vector function $\varphi(x,\lambda)$, certain by the formula (9) is a solution to equation (1) with initial conditions (4).

Theorem 1. Matrix-function $K(x,y)$ satisfies the Gelfand – Levitan linear integral equation

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$$F(x,y) + K(x,y) + \int_0^x K(x,s)F(s,y)ds = 0, \quad 0 \le y \le x,$$

where

$$F(x,y) = \frac{\partial^2}{\partial x \partial y} f(x,y),$$

$$f(x,y) = \int_{-\infty}^{\infty} \left\{ \int_0^x \varphi^0(t,\lambda)dt \right\} \cdot \left\{ \int_0^y \varphi^0(t,\lambda)dt \right\}^T d\tau(\lambda), \qquad (13)$$

$$\tau(\lambda) = \rho(\lambda) - \rho_0(\lambda).$$

Theorem 2. In order for a monotonically increasing function $\rho(\lambda)$ to be a spectral function of a boundary value problem of the form(1), (2) with a continuous matrix-function $\Omega(x)$, it is necessary and sufficient that the following conditions are satisfied:

I. If $g(x)$ is an arbitrary real continuous finite vector-function and

$$\int_{-\infty}^{\infty} G^2(\lambda)d\rho(\lambda) = 0, \qquad (14)$$

where $G(\lambda) = \int_0^{\infty} [g_1(\lambda)\varphi_1^0(x,\lambda) + g_2(x)\varphi_2^0(x,\lambda)]\, dx$, то $g(x) \equiv 0$.

II. The matrix-function $f(x,y)$ defined by formula (13) has a continuous second derivative $f''_{xy} = F(x,y)$, moreover $F_{11}(x,0) = 0$ $F_{21}(x,0) = 0$.

Theorem 3. Let $\{\lambda_n\}_{n=1}^{\infty}$ и $\{a_n\}_{n=1}^{\infty}$ – a sequence of positive numbers satisfying the conditions:

1^0. $m < \lambda_1 < \lambda_2 < \ldots < \lambda_n < \ldots$;

2^0. $\sum_{n=1}^{\infty} a_n = c < \infty;$

3^0. $\sum_{n=1}^{\infty} \lambda_n a_n = \tilde{c} < \infty;$

4^0. $\sum_{n=1}^{\infty} a_n \gamma_n = d < 1,$

where $\gamma_n = \dfrac{2}{min\left\{\left|\lambda_{n-1} - \lambda_n\right|, \left|\lambda_{n+1} - \lambda_n\right|\right\}} \cdot \sqrt{\dfrac{\lambda_1 + m}{\lambda_1 - m}}$.

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Then there is a unique continuous matrix $\Omega(x)$ of the form (3) such that

$$\rho(\lambda)=\begin{cases} -\frac{1}{\pi}\int\limits_{-m}^{\lambda}\frac{\sqrt{t^2-m^2}}{t+m}dt, & \lambda\le -m, \\ 0 & -m<\lambda\le m, \\ \frac{1}{\pi}\int\limits_{m}^{\lambda}\frac{\sqrt{t^2-m^2}}{t+m}dt+\sum\limits_{n=1}^{\infty}a_nH(\lambda-\lambda_n), & \lambda>m, \end{cases} \qquad (15)$$

($H(\lambda)$ – Heaviside function) is the spectral function of the operator D defined by formulas (1) - (3), moreover $pc\sigma(D)=\{\lambda_1,\lambda_2,\ldots,\lambda_n,\ldots\}$

Proof. Let us show that the function $\rho(\lambda)$ defined by formula (15) satisfies all conditions of Theorem 2.

It is easy to see that

$$\rho(\lambda)=\rho_0(\lambda)+\sum_{n=1}^{\infty}a_nH(\lambda-\lambda_n). \qquad (16)$$

From the positivity of the numbers a_n and the monotonic increase $\rho_0(\lambda)$, the monotonic increase follows $\rho(\lambda)$. Substituting (16) into (14), we have

$$\int_{-\infty}^{\infty}G^2(\lambda)d\rho_0(\lambda)+\sum_{n=1}^{\infty}a_nG^2(\lambda_n)=0.$$

Hence it follows that $\int_{-\infty}^{\infty}G^2(\lambda)d\rho_0(\lambda)=0$.

From Parseval's equality written for problem (6), (7) we have:

$$\int_{-\infty}^{\infty}[\,g_1^2(x)+g_2^2(x)]\,dx=0,$$

hence, by virtue of the continuity of the vector-function $g(x)$, we obtain $g(x)\equiv 0$. Hence, condition I of Theorem 2 is satisfied.

Consider the matrix function

$$F(x,y)=\sum_{n=1}^{\infty}a_n\varphi^0(x,\lambda_n)\left(\varphi^0(y,\lambda_n)\right)^T. \qquad (17)$$

From Condition 20, by virtue of the Weierstrass criterion, it follows that the series in (17) converges uniformly. Obviously, the function $F(x,y)$ is an continuous. From equality $\varphi^0(0,\lambda_n)=0$ followed by unity $F_{11}(x,0)=0,\quad F_{21}(x,0)=0$. Hence, condition II of Theorem 2 is also satisfied.

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Therefore, it is a spectral function of an operator D of the form (1) - (3), the continuous spectrum of which fills the intervals $(-\infty, m]$ and $[m, \infty)$ and has embedded eigenvalues λ_n, $n = 1, 2, \ldots$.

To find the coefficients $p(x),\ q(x)$ of the operator D, consider the Gel'fand – Levitan integral equation

$$K(x,y) + F(x,y) + \int_0^x K(x,t)F(t,y)dt = 0, \qquad 0 \le y \le x, \tag{18}$$

where the kernel $F(x,y)$ has the form (17). For each fixed $x \ge 0$ integral equation (18) has a unique solution.

Substituting (17) into (18), we deduce that the solution $K(x,y) = \left\| K_{ij}(x,y) \right\|_{i,\,j=1,2}$ equation (18) has the form

$$K(x,y) = \sum_{n=1}^{\infty} \begin{pmatrix} A_n(x) \\ B_n(x) \end{pmatrix} \cdot \left(\varphi^0(y, \lambda_n)\right)^T . \tag{19}$$

For vector functions $\begin{pmatrix} A_n(x) \\ B_n(x) \end{pmatrix}$, $n = 1, 2, \ldots$ we obtain the system of equations

$$\begin{pmatrix} A_n(x) \\ B_n(x) \end{pmatrix} + a_n \sum_{i=1}^{\infty} \begin{pmatrix} A_i(x) \\ B_i(x) \end{pmatrix} J_{i,n}(x) + a_n \varphi^0(x, \lambda_n) = 0, \quad n = 1, 2, \ldots, \tag{20}$$

where

$$J_{i,n}(x) = \begin{cases} \dfrac{\varphi_1^0(x,\lambda_i)\varphi_2^0(x,\lambda_n) - \varphi_1^0(x,\lambda_n)\varphi_2^0(x,\lambda_i)}{\lambda_i - \lambda_n}, & i \ne n, \\[2ex] \dfrac{x}{\lambda_n - m}\left(\lambda_n + m\dfrac{\varphi_1^0(x,\lambda_n)\varphi_2^0(x,\lambda_n)}{\lambda_n + m}\right), & i = n. \end{cases}$$

The solution to system (20) exists and is unique, since this system is equivalent to the Gel'fand – Levitan integral equation (18).

Further, by formula (11) we define the matrix-function $K(x,y)$, and the coefficients $p(x)$ and $q(x)$ the operator D are found by the formulas

$$p(x) = -K_{12}(x,x) - K_{21}(x,x), \qquad q(x) = K_{11}(x,x) - K_{22}(x,x),$$

or

$$p(x) = -\sum_{n=1}^{\infty}\left(A_n(x)\varphi_2^0(x,\lambda_n) + B_n(x)\varphi_1^0(x,\lambda_n)\right),$$
$$q(x) = \sum_{n=1}^{\infty}\left(A_n(x)\varphi_1^0(x,\lambda_n) + B_n(x)\varphi_2^0(x,\lambda_n)\right). \qquad (21)$$

Now, consider the family of self-adjoint operators D_α in $L_2^2(0,\infty)$ generated by the differential expression

$$D_\alpha y \equiv B\frac{dy}{dx} + mTy + \Omega(x)y = \lambda y, \qquad 0 < x < \infty \qquad (22)$$

and the boundary condition

$$y_1(0)\cos\alpha + y_2(0)\sin\alpha = 0, \qquad \alpha \in [0,\pi), \qquad (23)$$

where $p(x)$ and $q(x)$ are defined by equalities (21).

Theorem 4. When $\alpha \in (0,\pi)$, the operator D_α has no nested eigenvalues lying on $(-\infty,m]\cup[m,\infty)$.

Remark 1. If $ctg\alpha \geq 0$, then the operator D_α has no eigenvalues and on the interval $(-m,m)$.

Remark 2. If $ctg\alpha < 0$ then the operator D_α can have eigenvalues on the interval $(-m,m)$.

Example. Consider the Di-cancer self-adjoint $L_2^2(0,\infty)$ operator generated by the differential expression

$$Dy \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} + \begin{pmatrix} m & 0 \\ 0 & -m \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \lambda\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \quad 0 < x < \infty$$

and the boundary condition

$$y_1(0) = 0,$$

where m- is a constant positive number, $p(x),\ q(x)$ are real continuous functions.

Let's admit

$$\rho(\lambda) = \rho_0(\lambda) + aH(\lambda - m),$$

where $a,\ m > 0$. It's obvious that

$$c\sigma(D) = (-\infty,\ -m] \cup [m,\ \infty),\ pc\sigma(D) = \{m\},\ p\sigma(D) = \varnothing.$$

In this case $\sigma(\lambda) = \rho(\lambda) - \rho_0(\lambda)$ it has the form $\sigma(\lambda) = aH(\lambda - m)$ and, therefore

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$$F(x,y)=a\varphi^0(x,\lambda_n)\,(\varphi^0(y,\lambda_n))^T=a\begin{pmatrix}-2mx\\1\end{pmatrix}(-2my,\;\;1).$$

The integral Gelfand – Levitan equation (18) will be an equation with a degenerate kernel $F(x,y)$ and will take the form:

$$K(x,y)+a\left[\begin{pmatrix}-2mx\\1\end{pmatrix}+\int_0^x K(x,t)\begin{pmatrix}-2mt\\1\end{pmatrix}dt\right](-2my,\;\;1)=0, \qquad (24)$$

so the solution $K(x,t)=\left\|K_{ij}(x,t)\right\|_{i,j=1,2}$ equations (25):

$$K(x,y)=-\frac{3}{4am^2x^3+3ax+3}\begin{pmatrix}4m^2xy & -2mx\\-2my & 1\end{pmatrix}. \qquad (25)$$

Let's find the coefficients $p(x)$ and $q(x)$ the operator D.

By virtue of (11) and (25)

$$p(x)=-K_{12}(x,x)-K_{21}(x,x)=-\frac{12amx}{4am^2x^3+3ax+3},$$

$$q(x)=K_{11}(x,x)-K_{22}(x,x)=-\frac{12am^2x^2-3a}{4am^2x^3+3ax+3}.$$

So, we have constructed the following Dirac operator

$$Dy\equiv\begin{pmatrix}0&1\\-1&0\end{pmatrix}\begin{pmatrix}y_1'\\y_2'\end{pmatrix}+\begin{pmatrix}m&0\\0&-m\end{pmatrix}\begin{pmatrix}y_1\\y_2\end{pmatrix}+$$

$$+\frac{3a}{4am^2x^3+3ax+3}\begin{pmatrix}-4mx & 1-4m^2x^2\\1-4m^2x^2 & 4mx\end{pmatrix}\begin{pmatrix}y_1\\y_2\end{pmatrix}=\lambda\begin{pmatrix}y_1\\y_2\end{pmatrix},\;\;0<x<\infty \qquad (26)$$

with the boundary condition

$$y_1(0)=0. \qquad (27)$$

The solution $\varphi(x,\lambda)$ to system (26) at has $\lambda=m$ the following form:

$$\varphi(x,m)=\frac{3}{4am^2x^3+3ax+3}\begin{pmatrix}-2mx\\1\end{pmatrix}.$$

It is easy to check that $\varphi_1(0,m)=0$ and $\varphi(x,\,m)\in L_2^2(0,\infty)$, therefore, is an eigen function corresponding to an eigenvalue $\lambda=m$.

The Weyl – Titchmarsh function of problem (26), (27) is given by the formula

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$$m(z) = -i\frac{\sqrt{z^2 - m^2}}{z+m} + \frac{a}{m-z} .$$

Now, consider a family of self-adjoint operators D_α in $L_2^2(0,\infty)$ generated by the differential expression

$$D_\alpha y \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} + \begin{pmatrix} m & 0 \\ 0 & -m \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} +$$

$$+\frac{3a}{4am^2x^3 + 3ax + 3}\begin{pmatrix} -4mx & 1-4m^2x^2 \\ 1-4m^2x^2 & 4mx \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \lambda\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \ 0 < x < \infty \qquad (28)$$

and the boundary condition

$$y_1(0)cos\alpha + y_2(0)sin\alpha = 0, \quad \alpha \in (0,\pi) . \qquad (29)$$

Let us find the Weyl – Titchmarsh function of problem (28), (29). For this we use the formula

$$m_\alpha(z) = \frac{-sin\alpha + m(z)cos\alpha}{cos\alpha + m(z)\, sin\alpha} ,$$

that is

$$m_\alpha(z) = \frac{-1 + \left(-i\frac{\sqrt{z^2-m^2}}{z+m} + \frac{a}{m-z}\right) ctg\alpha}{ctg\,\alpha - i\frac{\sqrt{z^2-m^2}}{z+m} + \frac{a}{m-z}} .$$

The eigenvalues of the operator D_α are defined as the poles of the function $m_\alpha(z)$ from the equation.

$$-i\frac{\sqrt{z^2-m^2}}{z+m} + \frac{a}{m-z} + ctg\alpha = 0. \qquad (30)$$

Obviously, this equation has no roots belonging to $(-\infty,-m]\cup[m,\infty)$. Therefore, the real roots of equation (30) can be located only on the interval $(-m,m)$.

At $-m < z < m$ equation (30) is transformed to the form

$$\frac{\sqrt{m^2-z^2}}{z+m} + \frac{a}{m-z} + ctg\alpha = 0. \qquad (31)$$

By replacing

$$\sqrt{\frac{m-z}{z+m}} = \frac{1}{y}, \quad y > 0, \quad z = \frac{m(1-y^2)}{1+y^2} \qquad y \qquad (32)$$

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equation (31) can be written with respect to the new variable y in the form:

$$\frac{1}{y}+\frac{a(1+y^2)}{2my^2}+ctg\alpha=0,$$

or

$$(a+2mctg\alpha)y^2+2my+a=0. \quad (33)$$

The discriminant of this quadratic equation is $\Delta=4m^2-4a(a+2mctg\alpha)$.

Consider the following cases:

1. If $a+2mctg\alpha=0$, then equation (33) has one real solution

$$y=-\frac{a}{2m}<0;$$

2. If $ctg\alpha=\frac{m^2-a^2}{2ma}$, then equation (33) has one real solution $y=-\frac{a}{m}<0$;

3. If $ctg\alpha<\frac{m^2-a^2}{2ma}$, $ctg\alpha>-\frac{a}{2m}$, then equation (33) has two real solutions

$$y_1=-\frac{m+\sqrt{m^2-a^2-2mactg\alpha}}{a+2mctg\alpha}<0,$$

$$y_2=\frac{\sqrt{m^2-a^2-2mactg\alpha}-m}{a+2mctg\alpha}<0.$$

4. If $ctg\alpha>\frac{m^2-a^2}{2ma}$, then equation (33) has no real solutions;

In cases 1–4, equation (33) has no real solution satisfying the condition $y>0$. Hence, in these cases, the operator D_α has no eigenvalues on the interval $(-m,m)$ either. Hence it follows that

$$c\sigma(D_\alpha)=(-\infty,-m]\cup[m,\infty),\ \ pc\sigma(D_\alpha)=\varnothing,\ \ p\sigma(D_\alpha)=\varnothing.$$

5. If $ctg\alpha<\frac{m^2-a^2}{2ma}$, $ctg\alpha<-\frac{a}{2m}$, то

$$y_1=\frac{m+\sqrt{m^2-a^2-2mactg\alpha}}{-(a+2mctg\alpha)}>0, \qquad y_2=\frac{m-\sqrt{m^2-a^2-2mactg\alpha}}{-(a+2mctg\alpha)}<0.$$

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Substituting the value in (32), we calculate the root of the equation (31) z_0, which belongs to the interval $(-m,m)$:

$$z_0 = \frac{a^2 - m^2 + 3mactg\alpha + 2m^2ctg^2\alpha - m\sqrt{m^2 - a^2 - 2mactg\alpha}}{m + 2mctg^2\alpha + actg\alpha + \sqrt{m^2 - a^2 - 2mactg\alpha}}.$$

In this case, the spectrum is split as follows:

$$c\sigma(D_\alpha) = (-\infty, -m] \cup [m, \infty), \qquad pc\sigma(D_\alpha) = \varnothing, \qquad p\sigma(D_\alpha) = \{z_0\}.$$

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