ISSN (Online): 2455-3662
EPRA International Journal of Multidisciplinary Research (IJMR) - Peer Reviewed Journal
Volume: 8| Issue: 5| May 2022|| Journal DOI: 10.36713/epra2013 || SJIF Impact Factor 2022: 8.205 || ISI Value: 1.188

ON NOTEWORTHY APPLICATIONS OF DIFFERENTIAL EQUATIONS WITH LEGUERRE POLYNOMIAL

Dinesh Verma[1], Atul Kumar Rai[2], Vipin Dixit[3]

[2]*Associate Professor*

[3]*Assistant Professor*

[2,3]*Kothiwal Institute of Technology and Professional Studies, Moradabad*

Article DOI: https://doi.org/10.36713/epra10109
DOI No: 10.36713/epra10109

ABSTRACT

The Kamal Transform is a mathematical tool used in solving the differential equations. Kamal Transform makes it easier to solve the problem in engineering application and make differential equations simple to solve. In this paper, we will discuss analytic solutions of differential equations including leguerre polynomial by using kamal transform.

KEY WORDS: ***Kamal Transform, Leguerre Polynomial, Differential Equations.***

INTRODUCTION

The Kamal Transform has been applied in different areas of science, engineering and technology. The Kamal Transform is applicable in so many fields and effectively solving linear differential equations. Ordinary linear differential equation with constant coefficient and variable coefficient can be easily solved by the Kamal Transform without finding their general solutions [1, 2, 3]. The Leguerre polynomial of nth order generally solved by adopting Laplace Transform, Elzaki Transform [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,16,17]. This paper presents Analysis Kamal Transfrm of Leguerre polynomial of nth order the application of Kamal Transform in solving the differential equations including Leguerre Polynomial.

DEFINITIONS

Kamal Transform

If the function $\hbar(y)$, $y \geq 0$ is having an exponential order and is a piecewise continuous function on any interval, then the Kamal transform of $\hbar(y)$ is given by

$$K\{\hbar(z)\} = \bar{\hbar}(p) = \int_0^\infty e^{-\frac{z}{p}} \hbar(y)dy.$$

The Kamal Transform [1, 2, 3] of some of the functions are given by

- $K\{z^n\} = n!\, p^{n+1}$
, where $n = 0,1,2,..$
- $K\{e^{az}\} = \frac{P}{1-ap}$,
- $K\{sinaz\} = \frac{ap^2}{1+a^2p^2}$,
- $K\{cosaz\} = \frac{P}{1+a^2p^2}$,
- $K\{sinhaz\} = \frac{ap^2}{1-a^2p^2}$,
- $K\{coshaz\} = \frac{p}{1-a^2p^2}$.

Inverse Kamal Transform

The Inverse Kamal Transform of some of the functions are given by

- $K^{-1}\{p^{n+1}\} = \frac{z^n}{n!}$
- $K^{-1}\left\{\frac{P}{1-ap}\right\} = e^{az}$,
- $K^{-1}\left\{\frac{p^2}{1+a^2p^2}\right\} = \frac{1}{a} sinaz$,
- $K^{-1}\left\{\frac{P}{1+a^2p^2}\right\} = cosaz$,
- $K^{-1}\left\{\frac{p^2}{1-a^2p^2}\right\} = \frac{1}{a} sinhaz$,

- $K^{-1}\left\{\frac{P}{1-a^2p^2}\right\} = \cos az,$

FORMULATION

Leguerre Polynomial

The Laguerre polynomial is defined as [18,19,20,21,22,23,24,25]

$$L_n(u) = \frac{e^u}{n!}\frac{d^n}{du^n}(e^{-u}u^n)$$

We know that by the definition of Kamal Transform

$$K\{F(t)\} = \int_0^\infty e^{-t/p}F(t)dt$$

Therefore,

$$L\{L_n(t)\} = \int_0^\infty e^{-t/p}\left\{\frac{e^t}{n!}\frac{d^n}{dt^n}(e^{-t}t^n)\right\}dt$$

$$= \frac{1}{n!}\int_0^\infty e^{-(\frac{1}{p}-1)t}\left\{\frac{d^n}{dt^n}(e^{-t}t^n)\right\}dt$$

$$= \frac{1}{n!}\left[\left(\frac{1}{p}-1\right)\int_0^\infty e^{-(\frac{1}{p}-1)t}\frac{d^{n-1}}{dt^{n-1}}(e^{-t}t^n)dt\right]$$

Integrating again,

$$\frac{(\frac{1}{p}-1)^2}{n!}\int_0^\infty e^{-(\frac{1}{p}-1)t}\frac{d^{n-2}}{dt^{n-2}}(e^{-t}t^n)dt$$

Integrating n again,

$$= \frac{(\frac{1}{p}-1)^n}{n!}\int_0^\infty e^{-(\frac{1}{p}-1)t}(e^{-t}t^n)dt$$

$$= \frac{(\frac{1}{p}-1)^n}{n!}\left[\int_0^\infty e^{-t/p}(t^n)dt\right]$$

$$= \frac{(\frac{1}{p}-1)^n}{n!}K\{t^n\}$$

But by the definition of Kamal Transformation

$$K\{F(t)\} = \int_0^\infty e^{-t/p}F(t)dt$$

Hence,

$$\frac{(\frac{1}{p}-1)^n}{n!}K(t^n)$$

$$= \frac{(\frac{1}{p}-1)^n}{n!}.n!\ p^{n+1}$$

Hence,

$$K\{L_n(t)\} = p(1-p)^n$$

Module-I

Solve the differential equations

$$(D^2+4)y = L_2(t)$$

with initial conditions

$y(0) = 0\,, y'(0) = 1$

Given equation can be written as

$$y'' + 4y = L_2(t)$$

Taking Kamal Transform on sides

$$k\{y''\} + 4k\{y\} = k\{L_2(t)\}$$

Because Leguerre polynomial of order 2 is

$$L_2\{t\} = \frac{1}{2}\{2-4t+t^2\}$$

Now,

$$\frac{\bar{y}(p)}{p^2} - \frac{1}{p}y(0) - y'(0) + 4\bar{y}(p)] = p(1-p)^2$$

Applying initial conditions, we get

$$[\frac{\bar{y}(p)}{p^2} - 1 + 4\bar{y}(p)] = p(1-p)^2$$

$$\left[\frac{1}{p^2}+4]\right]\bar{y}(p) = p(1-p)^2 + 1$$

$$\bar{y}(p) = \frac{p^3}{1+4p^2} + \frac{p^5}{1+4p^2} - \frac{2p^4}{1+4p^2} + \frac{p^2}{1+4p^2}$$

$$\bar{y}(p) = \left[\frac{1}{4}p - \frac{1}{4}\frac{p}{1+4p^2}\right] + \left[\frac{1}{4}p^3 - \frac{1}{16}p + \frac{1}{16}\frac{p}{1+4p^2}\right] - 2\left[\frac{1}{4}p^2 - \frac{1}{4}\ \frac{p^2}{1+4p^2}\right] + \frac{p^3}{1+4p^2}$$

Applying inverse Kamal Transform

ISSN (Online): 2455-3662
EPRA International Journal of Multidisciplinary Research (IJMR) - Peer Reviewed Journal
Volume: 8| Issue: 5| May 2022|| Journal DOI: 10.36713/epra2013 || SJIF Impact Factor 2022: 8.205 || ISI Value: 1.188

$$y=\frac{1}{4}-\frac{1}{4}cos2t+\frac{t^2}{8}-\frac{1}{16}+\frac{1}{16}cos2t-\frac{1}{2}t+\frac{1}{4}sin2t+\frac{1}{2}sin2t$$

$$y=\frac{3}{16}+\frac{t^2}{8}-\frac{1}{2}t-\frac{3}{16}cos2t+\frac{3}{4}sin2t$$

Module-II

Solve the differential equations

$$(D^2+D)y=L_1(t)$$

with initial conditions

$y(0)=0\,,y'(0)=1$

Given equation can be written as

$$y''+y'=L_1(t)$$

Taking Kamal Transform on sides

$$k\{y''\}+k\{y'\}=k\{L_1(t)\}$$

Because Leguerre polynomial of order 1 is

$$L_1\{t\}=\{1-t\}$$

$$\left[\frac{\bar{y}(p)}{p^2}-\frac{1}{p}y(0)-y'(0)\right]-\left[\frac{\bar{y}(p)}{p}-y(0)\right]=p-p^2$$

Applying initial conditions, we get

$$\left[\frac{1}{p^2}+\frac{1}{p}\right]\bar{y}(p)=p-p^2+1$$

$$\bar{y}(p)=\frac{p^3}{1+p}-\frac{p^4}{1+p}+\frac{p^2}{1+p}$$

$$\bar{y}(p)=\left[p^2-p+\frac{p}{1+p}\right]-[p^3-p^2+p-\frac{p}{1+p}]+p-\frac{p}{1+p}$$

Applying inverse Kamal Transform

$$y=t-1+e^{-t}-\frac{t^2}{2}+t-1+e^{-t}+1-\ e^{-t}$$

$$y=2t-1+e^{-t}-\frac{t^2}{2}$$

Module-III

Solve the differential equations

$(D^2+\beta^2D)y=L_1(t)$,

with initial conditions $y(0)=0\,,y'(0)=0$

Given equation can be written as

$$y''+\beta^2y'=L_1(t)$$

Taking Kamal Transform on sides

$$E\{y''\}+\beta^2E\{y'\}=E\{L_1(t)\}$$

Because Leguerre polynomial of order 1 is

$$L_1\{t\}=\{1-t\}$$

$$\left[\frac{\bar{y}(p)}{p^2}-\frac{1}{p}y(0)-y'(0)\right]-\beta^2\left[\frac{\bar{y}(p)}{p}-y(0)\right]=p-p^2$$

Applying initial conditions, we get

$$\left[\frac{1}{p^2}+\frac{\beta^2}{p}\right]\bar{y}(p)=p-p^2$$

$$\bar{y}(p)=\frac{p^3}{1+\beta^2p}-\frac{p^4}{1+\beta^2p}$$

$$\bar{y}(p)=-\left[\frac{1}{m^2}p^3-\frac{1}{m^4}p^2+\frac{1}{m^6}-\frac{1}{m^6}\frac{p}{1+m^2p}\right]+\frac{1}{m^2}p^2-\frac{1}{m^4}p+\frac{1}{m^4}.\frac{p}{1+m^2p}$$

Applying inverse Kamal Transform

$$y=-\left(\frac{1}{m^6}+\frac{1}{m^4}\right)+\left(\frac{1}{m^2}+\frac{1}{m^4}\right)t-\frac{1}{m^2}\frac{t^2}{2}+(\frac{1}{m^6}+\frac{1}{m^4})e^{-m^2t}$$

$$y=\left(e^{-m^2t}-1\right)\left(\frac{1}{m^6}+\frac{1}{m^4}\right)-\frac{t^2}{2m^2}+\left(\frac{1}{m^2}+\frac{1}{m^4}\right)t$$

CONCLUSION

The conclusion of this paper is that, the Analytical solution of differential equations including Leguerre polynomial has been Examined by the application of new integral transform 'Kamal Transform' and represent the Kamal Transform for analyzing the Applications of Differential Equations with Leguerre Polynomial. A new and different integral transform is introduced for getting the result of Applications of Differential Equations with Leguerre Polynomial.

REFERENCES

1. *Dinesh Verma ,Elzaki Transform Approach to Differential Equatons with Leguerre Polynomial, International Research Journal of Modernization in Engineering Technology and Science (IRJMETS)" Volume-2, Issue-3, March 2020.*
2. *Dinesh Verma, Aftab Alam, Analysis of Simultaneous Differential Equations By Elzaki Transform Approach, Science, Technology And Development Volume Ix Issue I January 2020.*

3. *Sunil Shrivastava, Introduction of Laplace Transform and Elzaki Transform with Application (Electrical Circuits),International Research Journal of Engineering and Technology (IRJET) ,volume 05 Issue 02 , Feb-2018.*
4. *Updesh Kumar, and Dinesh Verma, Ana;yzation of physical sciences problems, EPRA International Journal of Multidisciplinary Research (IJMR)" Volume-8, Issue-4, April- 2022, eISSN 2455-3662; PP: 174-178.*
5. *Dinesh Verma and Rahul Gupta ,Delta Potential Response of Electric Network Circuit, Iconic Research and Engineering Journal (IRE) Volume-3, Issue-8, February 2020.*
6. *B.V.Ramana, Higher Engineering Mathematics.*
7. *Erwin Kreyszig, Advanced Engineering Mathematics, Wiley, 1998.*
8. *.Dinesh Verma, Relation between Beta and Gamma functionby using Laplace transformation, Researcher, 10(7), 2018.*
9. *Dinesh Verma, Elzaki Transform of some significantInfinite Power Series, International Journal of Advance Research and InnovativeIdeas in Education(IJARIIE)" Volume-6, Issue-1, February 2020.*
10. *Dinesh Verma and Amit Pal Singh, Applications of Inverse Laplace Transformations, Compliance Engineering Journal, Volume-10, Issue-12, December 2019.*
11. *Dinesh Verma, A Laplace Transformation approach to Simultaneous Linear Differential Equations, New York Science Journal, Volume-12, Issue-7, July 2019.*
12. *Dinesh Verma, Signification of Hyperbolic Functions and Relations, International Journal of Scientific Research & Development(IJSRD), Volume-07, Issue-5, 2019.*
13. *Dinesh Verma and Rahul Gupta,Delta Potential Response of Electric Network Circuit, Iconic Research and Engineering Journal (IRE)" Volume-3, Issue-8, February 2020.*
14. *Dinesh Verma and Amit Pal Singh ,Solving Differential Equations Including Leguerre Polynomial via Laplace Transform, International Journal of Trend in scientific Research and Development (IJTSRD),Volume-4, Issue-2, February 2020.*
15. *Dinesh Verma, Rohit Gupta and Amit Pal Singh,Analysis of Integral Equations of convolution via Rssidue Theorem Approach, International Journal of analytical and experimental modal" Volume-12, Issue-1, January 2020.*
16. *Dinesh Verma and Rohit Gupta, A Laplace Transformation of Integral Equations of Convolution Type, International Journal of Scientific Research in Multidisciplinary Studies" Volume-5, Issue-9, September 2019.*
17. *Dinesh Verma, A Useful technique for solving the differential equation with boundary values,Academia Arena" Volume-11, Issue-2, 2019.*
18. *Dinesh Verma , Relation between Beta and Gamma function by using Laplace Transformation, Researcher Volume-10, Issue-7, 2018.*
19. *Dinesh Verma , An overview of some special functions, International Journal of Innovative Research in Technology (IJIRT), Volume-5, Issue-1, June 2018.*
20. *Dinesh Verma, Applications of Convolution Theorem, International Journal of Trend in Scientific Research and Development (IJTSRD)" Volume-2, Issue-4, May-June 2018.*
21. *Dinesh Verma , Solving Fourier Integral Problem by Using Laplace Transformation ,International Journal of Innovative Research in Technology (IJIRT), Volume-4, Issue-11, April 2018.*
22. *Dinesh Verma ,Applications of Laplace Transformation for solving Various Differential equations with variable co-efficient, International Journal for Innovative Research in Science and Technology (IJIRST), Volume-4, Issue-11, April 2018.*
23. *Rohit Gupta, Dinesh Verma and Amit Pal Singh , Double Laplace Transform Approach to the Electric Transmission Line with Trivial Leakages through electrical insulation to the Ground, Compliance Engineering Journal Volume-10, Issue-12, December 2019.*
24. *Dinesh Verma and Rohit Gupta, Application of Laplace Transformation Approach to Infinite Series, International Journal of Advance and Innivative Research(IJAIR)" Volume-06, Issue-2, April-June,2019.*
25. *Rohit Gupta, Rahul Gupta and Dinesh Verma ,Laplace Transform Approach for the Heat Dissipation from an Infinite Fin Surface , Global Journal of Engineering Science and Researches (GJESR),Volume-06, Issue-2(February 2019).*