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IMPROVEMENT OF DEGREE OF CONFIDENCE IN CRITICAL PATH METHOD USING Z NUMBER

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ABSTRACT
Recently, Z-number has been introduced by Zadeh to model fuzzy numbers with a degree of confidence. Linguistic decision-making problems can be easily and effectively handled using it. In this work, we have tried to propose a new methodology for estimating the expected project completion time under an uncertain environment of Z-number.

KEYWORDS: ***Algorithm, Machine Learning, z number***

I. INTRODUCTION
The real world is full of unpredictable event. Much of the information on which decisions are based is uncertain. Humans have a remarkable capability to make rational decisions based on information that is uncertain, imprecise, and/or incomplete. The formalization of this capability, at least to some degree, is a challenge that is hard to meet. Zadeh proposed a notion, called Z-number that tries to overcome formalizing the above-mentioned challenge [1]. A Z-number is an ordered pair of fuzzy numbers (A, B), where A represents a value of the variable and B represents an idea of certainty or probability. In the past few decades, the classical fuzzy set has been exploited in the development of various fields such as fuzzy control, and fuzzy decision making, and resulted in tremendous success. However, the reliability of the information provided is not well taken into consideration. Compared with the classical fuzzy number, Z-number has more ability to describe the perception of humans. Both moderations, as well as accuracy, can be portrayed by it. The following table describes Z-number in comparison to deterministic and fuzzy concepts.

Deterministic	**Fuzzy**	**Z- NUMBER**
- RAM is 25 years - SHYAM is 3 years older than RAM ***SHYAM is (25+3) years old***	- RAM is young - SHYAM is a few years older than Ram ***SHYAM is (young + few) years old***	- RAM is young , sure - SHYAM is few years older than RAM, sure ***SHYAM is (young, sure)* (older, sure) years old***

Z-numbers can be utilized in order to use ambiguous details in real life. Taking into consideration the instance, it can be stated that, in risk assessment, the privation of intensity of a factor is very depressed. In relation to the Z number, it can be delineated as very depressed.

II. SURVEY OF PROBLEM AREA
A project is defined as a combination of interrelated activities all of which must be executed in a certain order to achieve a set of goals. In a large and complex project involving a number of interrelated activities requiring men, machines, and material it is impossible for management to control and plan. Hence, the strategy of scheduling network was utilized to organize and design a project. There are 2 types of network scheduling techniques one is CPM and other is PERT. At first critical path method CPM was developed that consisted of routine tasks whose resource requirements and duration were known with certainty. In a word, it is a deterministic tool used for planning and control techniques. But later on, under the uncertain conditions where time estimations are uncertain, a probabilistic tool is used for planning and control technique. Here have developed a methodology using the concept of Z-number in estimating the expected completion time.

III. PROBLEM SOLVING METHODOLOGY
A fuzzy set A is defined on a universe X may be defined as
$A = \{\langle x; \text{U}\, A(x)\rangle \mid x \in X\}$ (1)
Where U A: $X \rightarrow [0; 1]$ is the membership function A. The membership value describes the degree of belongingness of x $\in X$ in A. Dedicated specialists in the applications of reality may exhibit their expressions with the assistance of fuzzy numbers.
For example, in estimating a day, an expert's estimation may be expressed as follows:
1. least number of days required is 20 (optimistic time)
2. most probable number of days required is 30 (most likely time) and
3. Maximum number of days required can be 40 (pessimistic time).

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These can be represented as a triangular fuzzy numbers (20, 30, and 40). A as a triangular fuzzy number can be elucidated by a triplet (a1, a2, a3), in which the determination of membership can be done as the equation which is illustrated in the following-

$$\mu_A(x) = \begin{cases} 0;\ X \in (-\infty, a_1) \\ \frac{x-a_1}{a_2-a_1};\ X \in (a_1, a_2) \\ \frac{C-x}{a_3-a_2};\ X \in (a_2, a_3) \end{cases}$$

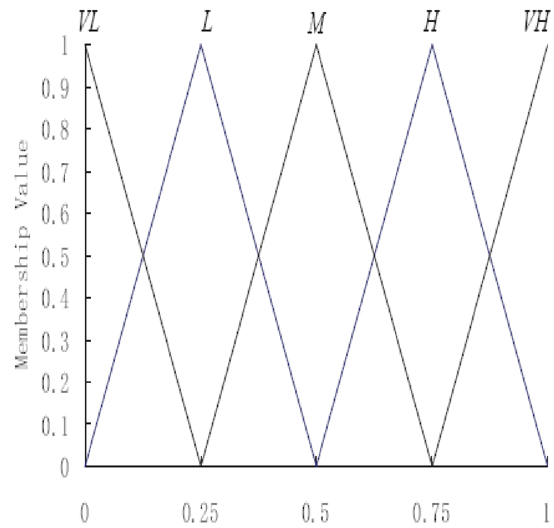
Let X be the universe of discourse, which include five linguistic variables describing the degree of security, X = {Very Low; Low; Medium; High; Very High}, assuming that only two adjacent linguistic variables have the overlap meanings. Let A be a fuzzy set of the universe of discourse X subjectively defined as follow:

$$f_{Very\,Low}(x) = -4x+1,\ 0 \le x \le 0.25$$

$$f_{Low}(x) = \begin{cases} 4x, & 0 \le x \le 0.25 \\ -4x+2, & 0.25 \le x \le 0.5 \end{cases}$$

$$f_{Medium}(x) = \begin{cases} 4x-1, & 0.25 \le x \le 0.5 \\ -4x+3, & 0.5 \le x \le 0.75 \end{cases}$$

$$f_{High}(x) = \begin{cases} 4x-2, & 0.5 \le x \le 0.75 \\ -4x+4, & 0.75 \le x \le 1 \end{cases}$$



A Z-number is an ordered pair of fuzzy numbers denoted as *Z=(A, R)*. The first component *A*, is a restriction on the values, is a real-valued uncertain variable *X*. The second component *R*is a measure of reliability for the first component, which can be
Described as in Fig. 3. Z-numbers can be utilized in order to use ambiguous details in real life. For simplicity, *A*
and *B* are assumed to be triangular fuzzy numbers as defined in Eq. (3) and Fig. 2.
Let A = (a1, a2, a3) and B = (b_1, b_2, b_3) be two triangular fuzzy numbers. The graded mean
integration representation of triangular fuzzy numbers *A* and B can be obtained respectively.
Let, A= (a1, a2, a3) and B= (b1, b2, b3) be two triangular fuzzy number,
P(A)= 1/6(a1+4 X a2+a3)

P(B)=1/6(b1+4 X b2+ b3)

The above-mentioned concepts can be applied to solve real-life problems of decision-making. In this work, the problem of estimating the project completion time is considered to demonstrate the effectiveness of the Z-number.

Planning a project usually involves dividing it into a number of small tasks that can be assigned to individuals or teams. The schedule of the project is dependent on the continuation of these pieces of work as well as the order in which they are laid out.

Network Dependence diagram: - Any schematic display of the logical relationship of project activities.

Forward Pass: - It goes from the task with no predecessors to the final task with no successors, visiting every node in every path and setting the ES and EF dates on the tasks.

Backward Pass:- It goes from the end task to the starting task, observing each node in the whole path by setting the LS and LF dates on the tasks.

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Critical Path Method:-

It is a procedure for using a network dependency diagram to identify those tasks that must be completed within a predefined time to successfully complete the project [4].

System Design

A project is defined as a combination of interrelated activities all of which must be executed in a certain order to achieve a set of goals. In a large and complex project involving a numberof interrelated activities requiring men, machines,and materials it is impossible for management to control and plan. So network scheduling technique was used for planning and scheduling the project. There are 2 types of network scheduling techniques one is CPM and other is PERT. At first critical path method CPM was developed that consisted of routine tasks whose resource requirements and duration were known with certainty. In a word, it is a deterministic tool used for planning and control techniques. But later on, under the uncertain condition where time estimations are uncertain, a probabilistic tool is used for planning and control techniques. Here we have developed a methodology using the concept of Z-number in estimating the expected completion time.

Construction of the fuzzy decision-making matrix [2]

In a certain company, each employee provides the optimistic, pessimistic, and most likely time to complete their part, Together with the completion times, each employee provides the confidence level of their respective estimations using a linguistic variable (high or medium or very high or low) under fuzzy notion. This confidence or sureness of the estimations is termed the certainty factor. With this given information, we have constructed a matrix.

Let the matrix M be the decision-making matrix, where m is the basic element of the matrix, where m_{i1}, m_{i2}, m_{i3}

defines pessimistic time, most likely time, and optimistic time respectively and m_{i4} defines the certainty factor, where (value of i=1--n) and certainty factor may be (very low, low, medium, high, very high).

Conversion of the pessimistic, most likely, and optimistic time

To avoid the complexity of mathematical operations in a decision process, the min-max normalization is used here.

The lowest value is set to 0 and the max value is set to 1.

This provides an easy way to compare values that are measured using different scales (for example degrees Celsius and degrees Fahrenheit) or different units of measure (speed and distance).

The formula to obtain the corrected value=(the value - the minimum) / (the range of values).

Suppose there are different readings of temperature(20, 24, 26, 27, 30).

The minimum value is 20. The maximum value is 30. The data has

a range of 10 degrees. (26 - 20) / 10 = 0.6.

Transformation of the linguistic value to numerical value:

For complex engineering systems, two kinds of information are available; numerical data received from sensor measurement, and linguistic rules obtained from human operators and domain experts. However, some of the information obtained in this manner are hybrid, that is, their components are not homogeneous but a blend of direct and indirect, numerical and linguistic information. In order to integrate the above linguistic and numerical information forhybrid intelligent control, which employs an integration of fuzzy logic; these linguistic variables should be converted into numerical values under the frame of fuzzy (values lie between 0 to 1).

For example, if the linguistic variable is “high” then according to the membership function of linguistics, the numerical value is (0*:*5*;* 0*:*75*;* 1) according to (Definition-2, figure-2).

Conversion of the Z-numbers to crisp number

The combination between Restriction *A* and Reliability *R* can be denoted by the following equation according to the canonical representation of multiplication operation on the triangular fuzzy number which is introduced by Chou [3].

$$
\begin{aligned}
w\left(Z_{ij}\right) &= w\left(\tilde{A}, \tilde{R}\right) \\
&= \tilde{A} \otimes \tilde{R} \\
&= \left(a_{ij}^{l}, a_{ij}^{m}, a_{ij}^{u}\right) \otimes \left(r_{ij}^{l}, r_{ij}^{m}, r_{ij}^{u}\right) \\
&= \left(\left(a_{ij}^{l} + 4 \times a_{ij}^{m} + a_{ij}^{u}\right)/6\right) \times \left(\left(r_{ij}^{l} + 4 \times r_{ij}^{m} + r_{ij}^{u}\right)/6\right)
\end{aligned}
$$

Activity specification

We can use the Work Breakdown Structure (WBS) to identify the activities involved in the project.

Network diagram

In this step, the correct activity sequence is established taking into account the followings:

1. Tasks on which the current task depends
2. Tasks that are parallel to the current task
3. Tasks that depend on the current task

Once the activity sequence is correctly identified, the network diagram can be drawn. Although the early diagrams were drawn on paper, there is a number of computer software, such as Primavera, for this purpose.

Earliest occurrence time

Forward pass calculations are used to find the earliest occurrence times of events (Ei). It goes from the Leftmost position to the Rightmost position. The start value is equal to zero. (Earliest occurrence time of 1st event = 0).

When two or more activities merge in an event, the maximum value is taken as the earliest occurrence time for that event.

Forward pass time= Earliest time of Tail event + Activity time.

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Latest occurrence time

Backward pass calculations are from Right to Left. It is used to find the most recent occurrence times of events (Li). For the last event, the Latest time = the Earliest time.

If there is more than one activity coming back in an event, in backward pass we take minimum value. Backward pass time= Latest time of Head event – Activity time

Critical path

Critical path indicates the path containing all critical activities. Here, Earliest starting and Latest finishing times are equal.

A real life implementation with Z-numbers:-

Suppose a project runs by 9 employer in an organization. They are working independently on different phases of the project to complete it. Each employee provides the optimistic, pessimistic and most likely time to complete their part, which is shown in Table 1. Together with the completion times, each employee provides the confidence level of their respective estimations using linguistic variable (high or medium or very high or low) under fuzzy notion. This confidence or sureness of the estimations is termed as certainty factor in the table.

EMPLOYEE	PRECIDING ACTIVITY	PESSIMISTIC TIME	MOST LIKELY TIME	OPTIMISIC TIME	CERTAINTY FACTOR
A		9	10	12	HIGH
B		20	24	25	HIGH
C	A	15	15	15	MEDIUM
D	A	70	100	120	VERY HIGH
E	B	60	70	100	MEDIUM
F	D,E	70	80	90	HIGH
G	C,F	4	5	6	HIGH
H	D,E	1	8	10	HIGH
I	B	1	4	7	HIGH

Table 1. An Application with Z number

Following the MIN-MAX normalization[4.2], pessimistic time, optimistic time and most likely time are transformed in the range of [0,1] and are shown in Table 2.

EMPLOYEER	PESSIMISTIC TIME	MOST LIKELY TIME	OPTIMISIC TIME	CERTAINTY FACTOR
A	.06	.07	.09	HIGH
B	.16	.19	.20	HIGH
C	.12	.12	.12	MEDIUM
D	.58	.83	1	VERY HIGH
E	.50	.58	.83	MEDIUM
F	.58	.66	.75	HIGH
G	.02	.03	.04	HIGH
H	.05	.06	.07	HIGH
I	0	.02	.05	HIGH

Table 2. Min-Max Normalization

In accordance to the membership function [denoted by Eq. (3) and described by Fig.2,] the linguistic variable can be converted to numerical value, which is described as Table 3.

EMPLOYER	PESSIMISTIC TIME	MOST LIKEL YTIME	OPTIMISIC TIME	CERTAINTY FACTOR
A	.06	.07	.09	(.75,1,1)
B	.16	.19	.20	(.5,.75,1)
C	.12	.12	.12	(.5,.75,1)
D	.58	.83	1	(.25,.5,.75)
E	.5	.58	.83	(.75,1,1)
F	.58	.66	.75	(.5,.75,1)
G	.02	.03	.04	(.5,.75,1)
H	.05	.06	.07	(.5,.75,1)
I	0	.02	.05	(.5,.75,1)

Table 3. Numerical linguistic value

Conversion of the Z-numbers to crisp number is done according to [4.4] and shown in Table-4.

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EMPLOYEE		
A	(.06,.07,.09) ⊗ (.75,1,1)	.06
B	(.16,.19,.20) ⊗ (.5,.75,1)	.14
C	(.12,.12,.12) ⊗ (.5,.75,1	.09
D	(.58,.83,1) ⊗ (.25,.5,.75)	.41
E	(.5,.58,.83) ⊗ (.75,1,1)	.58
F	(.58,.66,.75) ⊗ (.5,.75,1)	.49
G	(.02,.03,.04) ⊗(.5,.75,1)	.02
H	(.05,.06,.07) ⊗ (.5,.75,1)	.04
I	(0,.02,.05) ⊗ (.5,.75,1)	.01

Table 4. Conversion of Z number to Crisp Number

The project consist of set of activities and there are some precedence relationship among these activities. Based on this precedence relationship we are able to draw this network. After drawing the network we write down also the duration associated with each activity from TABLE-4.

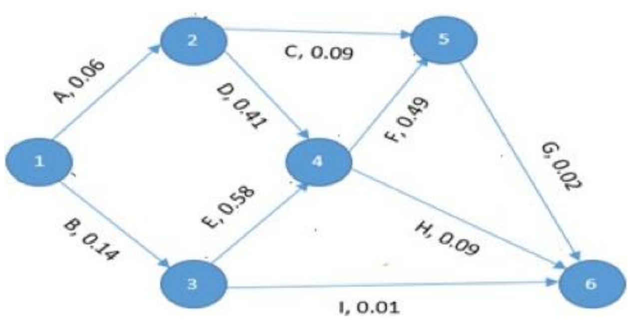

Fig.1

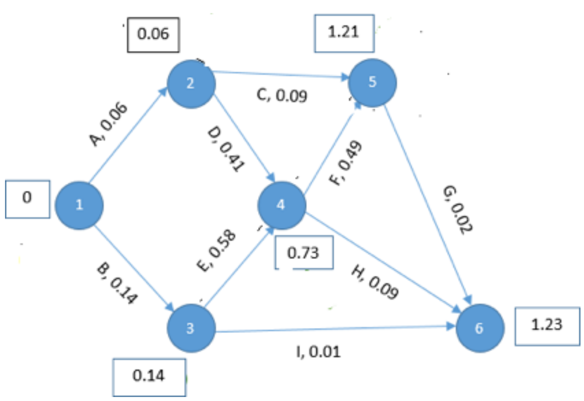

Fig. 2

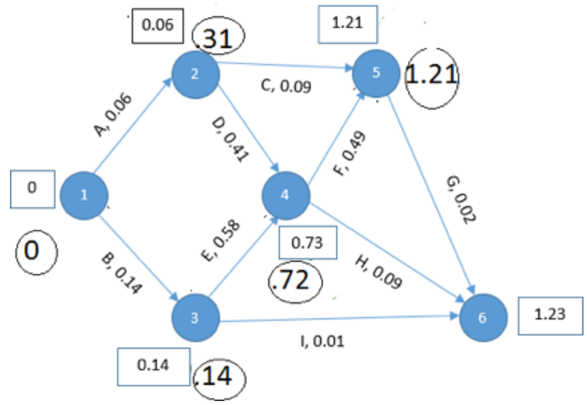

Fig. 3

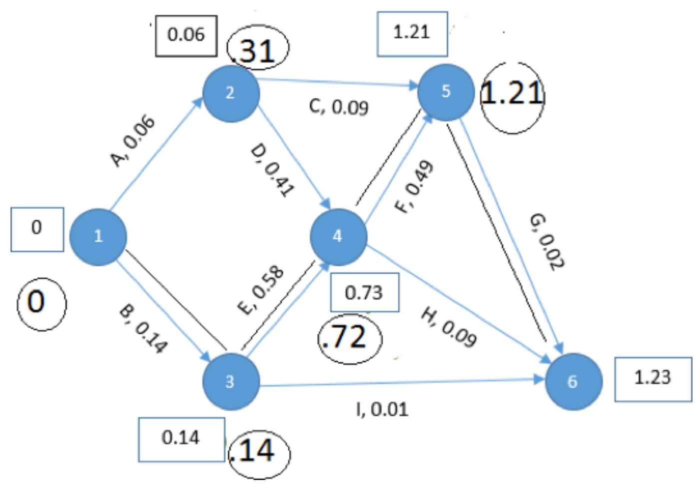

Fig. 4

IV. RESULT

Project completion day is 1.23days. We have calculated the raw project completion days. Then again using the min-max algorithm [definition 4.2] we transformed it into the actual project completion days, and the actual estimation of project completion days is 147 days.

The critical path is 1-3-4-5-6.

V. CONCLUSION

The fuzzy set has been widely applied in the process of decision-making since uncertainty and complexity is a pervasive phenomena in the real world. But a problem is that the reliability of information is not taken into consideration efficiently. Z-number [6] is a new notion proposed by Zadeh and has more ability to describe uncertain knowledge. We have seen a real-life application of Z-number [7].

Zadeh is reckoned as a notable person who is well known for his establishment of fuzzy sets. In addition, the concept of the fuzzy set has helped in enhancing the notion of a typical fuzzy set and has been utilized in an effective manner in various sectors like control

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systems based on fuzzy logic.

However, the ultimate aim is to establish a mathematical model to simulate the intelligence of human beings in spite of little progress. Both accuracies as well as moderation can be taken into account and has more benefits to delineating the information which is uncertain in nature.

Many works like uncertainty measurement and distance of Z number remain to be figured out.

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