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CERTAIN DEFINITE INTEGRAL INVOLVING STRUVE AND BESSEL FUNCTIONS

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ABSTRACT
In this paper we have developed certain definite integral. These integral involved Bessel functions of first kind and second kind and also involved Struve function.
KEY WORDS : ***Struve function,Bessel function.***
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1. INTRODUCTION

Struve functions are solutions of the non-homogeneous Bessel's differential equation:

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-\alpha^2)y=\frac{4(\frac{x}{2})^{\alpha+1}}{\sqrt{\pi}\,\Gamma(\alpha+\frac{1}{2})}(1.1)$$

and are defined as:

$$H_\alpha(x)=\frac{2(\frac{x}{2})^\alpha}{\Gamma(\alpha+\frac{1}{2})\Gamma(\frac{1}{2})}\int_0^{\frac{\pi}{2}}sin(x\cos\theta)sin^{2\alpha}(\theta)\,d\theta(1.2)$$

Modified Struve function is:

$$L_\alpha(x)=I_{-\alpha}(x)-\frac{2(\frac{x}{2})^\alpha}{\Gamma(\alpha+\frac{1}{2})\Gamma(\frac{1}{2})}\int_0^\infty sin(xu)(1+u^2)^{\alpha-\frac{1}{2}}\,du(1.3)$$

Bessel functions of the first kind, denoted as $J_\alpha(x)$, are solutions of Bessel's differential equation that are finite at the origin ($x=0$) for integer or positive α, and diverge as x approaches zero for negative non-integer α (See[12]). It is possible to define the function by its Taylor series expansion around $x=0$.

$$J_\alpha(x)=\sum_{m=0}^{\infty}\frac{(-1)^m}{m!\,\Gamma(m+\alpha+1)}(\frac{x}{2})^{2m+\alpha}(1.4)$$

where $\Gamma(z)$ is the gamma function, a shifted generalization of the factorial function to non-integer values. The Bessel function of the first kind is an entire function if α is an integer.

The Bessel functions are valid even for complex arguments x, and an important special case is that of a purely imaginary argument(See[12]). In this case, the solutions to the Bessel equation are called the modified Bessel functions (or occasionally the hyperbolic Bessel functions) of the first and second kind. The first kind of modified Bessel function is defined as

$$I_\alpha(x) = \iota^{-\alpha} J_\alpha(\iota x) = \sum_{m=0}^{\infty} \frac{1}{m!\,\Gamma(m+\alpha+1)} \left(\frac{x}{2}\right)^{2m+\alpha} (1.5)$$

The Bessel function of second kind is defined as

$$Y_\upsilon(x) = \frac{1}{\pi}\int_0^{\pi} \sin(x\sin t - \upsilon t)\, dt - \frac{1}{\pi}\int_0^{\infty} [e^{\upsilon t} + (-1)^{\upsilon} e^{-\upsilon t}] e^{-x\sinh t}\, dt (1.6)$$

The second kind of modified Bessel function is defined as

$$K_\upsilon(x) = \int_0^{\infty} \cosh(\upsilon t)\, exp(-x\cosh t)\, dt \qquad x > 0 (1.7)$$

2. Main Formulae of the Integration

$$\int_0^{\infty} \frac{\sin(ax)}{\sqrt{x^2+1}}\, dx = \frac{\pi\, a[I_0(a) - L_0(|a|)]}{2|a|}.(2.1)$$

$$\int_0^{\infty} \frac{\sin(ax)}{\sqrt{x^2-1}}\, dx = \frac{\pi\, a[J_0(a) - \iota H_0(|a|)]}{2|a|}.(2.2)$$

$$\int_0^{\infty} \frac{\cos(ax)}{\sqrt{x^2-1}}\, dx = \frac{-\iota\pi[J_0(a) - \iota Y_0(|a|)]}{2}.(2.3)$$

$$\int_0^{\infty} \frac{\cos(ax)}{\sqrt{x^2+1}}\, dx = K_0(|a|).(2.4)$$

$$\int_0^{1} \frac{\sin(ax)}{\sqrt{x^2-1}}\, dx = -\frac{1}{2}\,\iota\,\pi\, H_0(a).(2.5)$$

$$\int_0^{1} \frac{\cos(ax)}{\sqrt{x^2-1}}\, dx = -\frac{1}{2}\,\iota\,\pi\, J_0(a).(2.6)$$

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