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INVARIANT DEFINITION OF A TANGENT

Eshbekov Raykhonbek[1], Abdurakhmanov Bobomurod G'ulombek ugli[2]
[1]Teacher, Samarkand State University, Uzbekistan
[2]Student, Samarkand State University, Uzbekistan

INTRODUCTION

When forming and describing mathematical concepts, its definition is of great importance.

It is known that the concept of a tangent to the graph of functions and to any curves is important in mathematics.

But the very introduction or definition of a tangent requires special attention.

The purpose of our study is to introduce the concept of a tangent for graphically defined functions without applying the concepts of the derivative and secant line. We know that any function can be defined analytically, graphically, and tabularly.

First consider the logical requirement for introducing and describing the definition of the concepts to be introduced.

Avicenna (Abu Ali Ibn Sina) was also concerned with the concept of "definition" and its structures.

One of the errors in describing and introducing a new concept through definitions was Avicenna's belief "that an object wants to be cognized through such an object, which cannot be cognized without the aid of the first object.

For example, when defining the sun, one says that "*the sun is a star that rises in the daytime. Hence, "the sun" is defined by means of "the day. But it is impossible to know the day other than by means of the sun, because in fact the day is the time when the sun has risen*" [1] .

Consider two basic definitions of a tangent used in mathematical analysis and in school mathematics courses.

Definition 1.If the function $f: E \to R$ on the set $E \subset R$ is continuous and differentiable at the point x_0 , then the line given by the equation

$$y = f(x_0) + f'(x_0)(x - x_0) \qquad (1)$$

is called a tangent to this function drawn at a point $(x_0;\ f(x_0))$. [2] As you can see, this definition of a tangent is related to the notion of a derivative at a point and the coordinate system XOY , and so this definition is not invariant.

For example, consider a line which is the graph of a function $y = f(x)$ in the XOY coordinate system, if we move to another coordinate system, for example $\tilde{X}O\tilde{Y}$, then its equation will be $\tilde{y} = f(\tilde{x})$, and we cannot claim that the equation

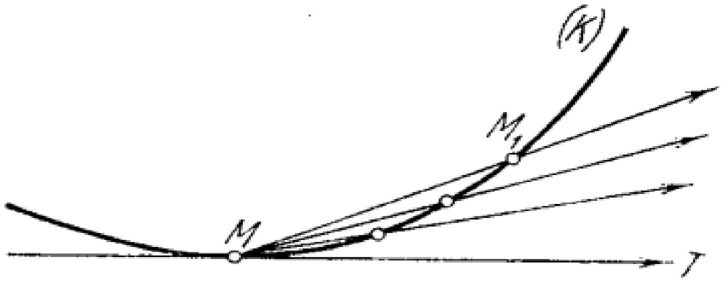
$\tilde{y} = \tilde{f}(x_0) + \tilde{f}(x_0)(\tilde{x} - x_0)$ (2) and equation (1) define the same straight line, i.e. a tangent.

So, this definition is applicable only when the functions are defined analytically and depend on the choice of coordinate system and therefore are not invariant. (*The angular coefficient of a tangent and the derivative of a given function at the same point are equivalent concepts*.)

On the other hand, this definition does not apply to graphically defined functions.

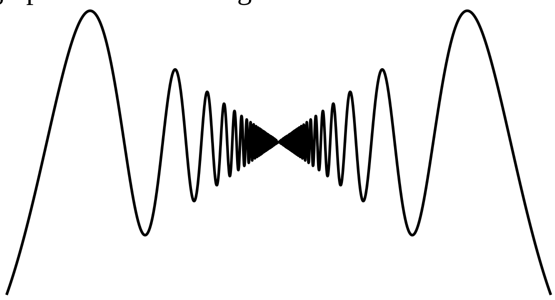
If the function is defined graphically, then we have the following definition of a tangent.

Definition 2. Castellar to the curve K at M is the limiting position MT of the secant MM_1, when the point M_1 along the curve tends to coincide with M. [3].



This definition makes sense if the graph of the function has a simple form.

Consider the graph of the following function:



A function at a point (0;0) has a derivative, so a tangent exists, but it is impossible to construct a secant or a tangent line due to the complexity of the line (suppose we do not know the analytic expression). So our goal is to solve this problem: define the notion of a tangent so that it is invariant. To put it another way, no matter how complex the line is, it would be possible to construct a tangent using this definition.

The definition we will give should not be derived from the definition of the derivative.

To invariantly define a tangent, we need some additional concepts.

2. $K_\varepsilon(M_o, p)$ - CONUS

Let the point M_0 and the line p be on the plane or in space .Let the point M_0 lie on the line p, give $\varepsilon > 0$.

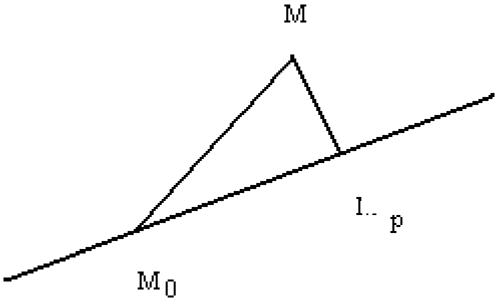
The set of points M, satisfying the inequality

$$|MM_p| \le |M_o M_p| \cdot \varepsilon$$

is called the ε-cone and is denoted by $K_\varepsilon(M_o, p)$

Here by marking the projection M_p of the point M on the line p,

$K_\varepsilon(M_o, p)$ - cone can be represented in the following form:



In what follows, we state the invariant definition of the tangent to the graph of a function at M_o.

4. INVARIANT DEFINITION OF THE TANGENT

Definition 7. For an arbitrary $K_\varepsilon(\mathrm{M}_o, \mathrm{p})$ -conus and a point $M_0 \in \Gamma$ there is a neighborhood $U_\delta(\mathrm{M}_0)$, and all points of the curve Γ of this neighborhood belong to the conus $K_\varepsilon(\mathrm{M}_o, \mathrm{p})$, i.e.

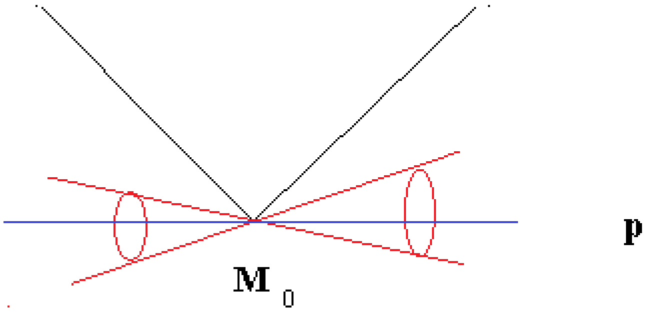
$$\Gamma \cap U_\delta(M_0) \subset K_\varepsilon(M_0, p)$$

then the line p is tangent to the curve Γ at the point M_0 .

For example, consider a graphically defined function. In the figure you can see that this is the graph of the function $y = |x|$.

Although this function is continuous at M_0 , the derivative at this point does not exist.

This can be clearly seen in the figure. You cannot find the vicinity included in the cone $K_\varepsilon(\mathrm{M}_o, \mathrm{p})$ of the point M_0 of the given line, i.e. the graph of the function $y = |x|$.



Theorem 1. If the line G has a tangent at the limit point M_0 then this tangent at this point is unique.

Proof. Suppose the converse view, i.e., the curve G at point m₀ has not one but two tangents p and g . Take $0 < \varepsilon < tg\,\dfrac{\varphi}{2}$ and for lines p and g construct $K_\varepsilon(\mathrm{M}_0, p)$ and $K_\varepsilon(\mathrm{M}_0, q)$. As we know, these two tangents intersect at the point M_0 .

$$K_\varepsilon(\mathrm{M}_0, p) \cap K_\varepsilon(\mathrm{M}_0, q) = \{M_0\}$$

By the invariant definition of a tangent p , there exists such a $\delta_1 > 0$, that . $\Gamma \cap U_{\delta_1}(\mathrm{M}_0) \subset K_\varepsilon(\mathrm{M}_0, p)$

Exactly the same tangent will find $\delta_2 > 0$, which will be true: $\Gamma \cap U_{\delta_2}(\mathrm{M}_0) \subset K_\varepsilon(\mathrm{M}_0, q)$. Let us take $\delta = \min\{\delta_1, \delta_2\}$, then

$$\Gamma \cap U_\delta(\mathrm{M}_0) \subset K_\delta(\mathrm{M}_0, \mathrm{p}) \cap K_\delta(\mathrm{M}_0, g) = \{M_0\}$$

$$U_\delta(\mathrm{M}_0) \cap (\Gamma \setminus \{M_0\}) = \varnothing$$

But by the condition of the theorem, the point M_0 is the limit point of curve D.

So, if there is a tangent at the limit point of the curve, it is unique.

Theorem 2. If a function $y = f(x)$ at the point x_0 is differentiable, then its graph G at the point. $M_0(x_0, f(x_0))$ has an invariant tangent and the angular coefficient of this tangent is equal to the derivative of this function at the point $x_0 = k = f'(x_0)$

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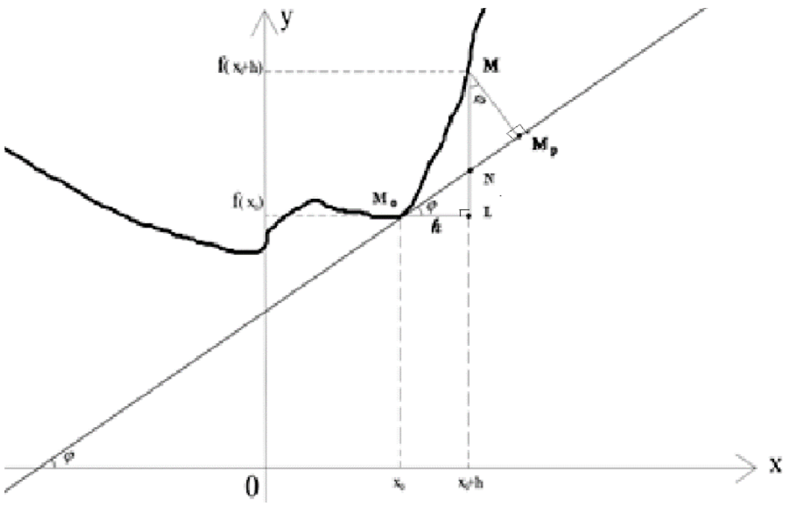
Proof. Suppose that a graphically given function at the point x_0 is differentiable, i.e., it has a tangent in the classical sense at this point. Let us prove that there exists a tangent in the invariant sense at this point.

Let the point M_0 be the limit point of G and through the point M_0 draw the line *p* with the angle factor k, and show that the line *p* is tangent to the line G at the point $M_0 \in \Gamma$.

From the classical definition we know that $tg\varphi = k = f'(x_o)$, then

$$|MM_p| = |MN| \cdot \cos\varphi = O(|MN|), (1)$$

$$|M_0N| = |M_0L| \frac{1}{\cos\varphi} = o(h), (2)$$



As can be seen from the figure

$$|h| = |M_0L|, \cos\varphi \neq 0.$$

From the differentiability of the function at the point x_0, the following equality is true for *y*

$$|MN| = |f(x_0+h) - f(x_0) - f'(x_0) \cdot h| = o(h), (3)$$

By virtue of (1) and (3)

$$|MM_p| = o(h), \cos\varphi = \mathrm{o(h)}..$$

For points $\mathrm{M}, \mathrm{M}_p, \mathrm{N}$ there is an inequality:

$$|NM_p| \leq |MN| + |MM_p|$$

So

$$|NM_p| = o(h) + o(h) = o(h).$$

And there is also equality:

$$|M_0M_p| = |M_0N| + |NM_p| = O(h) + o(h) = O(h).$$

It follows that $h = O(|M_oM_p|)$.

By virtue of these equations, we have

$$|MM_0| = o(h) = o(O(|M_oM_p|)) = o(|M_oM_p|)$$

By definition, in the invariant sense, there exists for $\forall \varepsilon > o$ such $\delta > 0$ where

$$\left|M_0M_p\right| < \delta \Rightarrow \left|M_0M_p\right| < \left|M_0M_p\right| \cdot \varepsilon$$

Then

$$M \in U_\delta(\mathrm{M}_0) \Rightarrow \left|M_0M\right| < \delta \Rightarrow \left|M_0M_p\right| < \delta \Rightarrow M \in K_\varepsilon(\mathrm{M}_0, p)..$$

Thus, we deduced from the differentiability of a graphically defined function at M_0 the existence of a tangent at this point in an invariant sense. Consequently, we conclude that the classical and invariant definitions of the tangent are equivalent.

REFERENCES

1. *1.Ibn Sina. The Book of Knowledge: Selected Philosophical Works, translated from Arabic and Farsi-Dari / Comp. and introduction. V. Kuznetsov, Moscow: EXMO-Press, 1999.*
2. *Fichtenholz G. M. Course of differential and integral calculus: in 3 vols. - M.: Nauka, G. Ed. of Physics and Mathematics, 1967. -. T. 2.*
3. *Alimov. Sh.A., Y.M. Kolyagin, Y.V. Sidorov, etc.Algebra and Beginnings of Analysis. Grade 10-11 /. - M.: Prosveshcheniye; 2nd edition, 2014. - 256 c.*
4. *Shvedov I.A. A Compact Course in Mathematical Analysis. Part 1. Functions of one variable.Novosibirsk State UniversityYear of issue: 2003.*