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FRACTIONAL CALCULUS—THEORY AND APPLICATIONS

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ABSTRACT

Fractional calculus has made great strides in many fields of mathematics and science over the past several years. On the one hand, more modern definitions of fractional derivatives and integrals have extended the traditional definitions in some way. Furthermore, a current field of study in mathematical analysis is the thorough consideration of the functional features of these new concepts. Systems involving differential equations with fractional-order operators have undergone thorough analytical and numerical investigation, and prospective applications in the sciences and in technology have been suggested. This Special Issue's goal is to provide a specialized venue for the discussion of new developments in the theory of fractional calculus and their applications. We encourage authors to submit excellent reports on a variety of fascinating research topics, including the analysis of fractional-order differential/integral equations, the evaluation of novel definitions of fractional derivatives, numerical approaches to fractional-order equations, and applications to physical systems governed by fractional differential equations.

The ten papers in the current special issue address the following subjects.

- ***Fractional-order differential/integral equations.***
- ***Existence and regularity of solutions.***
- ***Numerical methods for fractional equations.***
- ***Analysis of convergence and stability.***
- ***Applications to science and technology.***

INTRODUCTION

The authors of one of the articles in this Special Issue [1] took into account a fractional-order malaria pestilence system. The Jacobean matrix technique was used to examine the stability of the model at equilibrium locations . It was clarified how the fundamental reproduction number, R0, affected the infection dynamics and stability analysis. The findings showed that when R0 1, the system under consideration is locally asymptotically stable at the steady-state solution without sickness. When R0 > 1, the endemic equilibrium produced a similar outcome. At both stable states, the underlying system displayed global stability. A stochastic model was then created from the fractional-order system. The stochastic epidemic model's non-parametric perturbation variant was created and computationally investigated for a more realistic understanding of the disease dynamics. The model was solved using the Runge-Kutta method, generic stochastic fractional Euler method, and a newly suggested numerical method. The positive feature of the continuous system was not preserved using the conventional procedures. The positivity was maintained, however, using the proposed stochastic fractional nonstandard finite-difference technique. A conclusion was reached on the nonstandard finite-difference scheme's roundedness. Numerical simulations were used to validate all of the analytical conclusions.

HYPOTHESIS

The article [2] is devoted to studying GPU-based modeling for a parallel fractional- order derivative model of the spiral-plate heat exchanger. As pointed out by the authors, a spiral-plate heat exchanger with two fluids is a compact plant that only requires a small space and is excellent in high heat-transfer efficiency. However, the spiral-plate heat exchanger is a nonlinear plant with uncertainties, considering the difference between the Group of Seven's (G7) economic growth was additionally predicted using a fractional-order gradient descent approach in [3]. More specifically, this work developed a model of economic growth from 1973 to 2016 for all G7 nations, according to which the GDP is correlated with land area, arable land, population, school attendance, gross capital formation, exports of goods and services, general government, final consumer spending, and broad money. The model parameters to fit the GDP and forecast GDP from 2017 to 2019 were estimated using the fractional-order gradient descent and the integer-order gradient descent. The outcomes demonstrated that the fractional-order gradient descent's convergence rate is quicker and has a superior fitting accuracy and prediction effect.

In [4], the authors studied the approximate and analytic solutions of the time-fractional intermediate diffusion wave equation associated with the Fokker–Planck operator. More precisely, the time-fractional wave equation associated with the space-fractional Fokker– Planck operator and with the time-fractional-damped term were studied in this work. The concept of the Green function was implemented to drive the analytic solution of the three-term time-fractional equation. The explicit expressions for the green function of the three-term time-fractional wave equation with constant coefficients was also studied fortwo physical and biological models. The explicit analytic solutions for the two studied models were expressed in terms of the Weber, hypergeometric, exponential, and Mittag– Leffler functions. The relation to the diffusion equation was given therein. The asymptotic behaviors of the Mittag–Leffler function, the hypergeometric function, and the exponential functions were compared numerically. The Grünwald–Letnikov scheme was then used to derive the approximate difference schemes of the Caputo time-fractional operator and the Feller–Riesz space-fractional operator. The explicit difference scheme was numerically studied, and the simulations of the approximate solutions were plotted for different values of the fractional orders.

LITERATURE REVIEW

On the other hand, the authors of [5] reported on some new fractional estimates of inequalities for LR-*p*-convex interval-valued functions by means of pseudo order relation. Interval analysis provides tools to deal with data uncertainty. In general, interval analysis is typically used to deal with the models whose data are composed of inaccuracies that may occur from certain kinds of measurements. In this context, both the inclusion relation ( ) and the pseudo-order relation ($_p$) are two different concepts. By using the latter relation, the authors introduce the new class of non convex functions known as LR-*p*-convex interval- valued functions (LR-*p*-convex-IVFs). With the help of this relation, they establish a strong relationship between LR-*p*-convex-IVFs and Hermite–Hadamard-type inequalities (HH- type inequalities) via the Katugampola fractional integral operator. The results include a wide class of new and known inequalities for LR-*p*-convex-IVFs and their variant forms as special cases. Useful examples that demonstrate the applicability of the theory proposed in this study were given in that study.

Sequential Riemann–Liouville and Hadamard–Caputo fractional differential systems with nonlocal coupled fractional integral boundary conditions were studied in [6]. In that work, the authors investigated the existence of solutions for a fractional differential system that contains mixed Riemann–Liouville and Hadamard–Caputo fractional derivatives, complemented with nonlocal coupled fractional integral boundary conditions. They derived necessary conditions for the existence and uniqueness of solutions of those system by using standard fixed-point theorems, such as Banach contraction mapping principle and the Leray–Schauder alternative. Numerical examples illustrating the theoretical results were also presented.

In [7], a numerical method for solving a fractional diffusion-wave and nonlinear Fred- holm and Volterra integral equations with zero absolute error was presented. The method was based on Euler wavelet approximation and matrix inversion of *M M* collocation points. The proposed equations were presented based on the Caputo fractional derivative, and the authors reduced the resulting system to a system of algebraic equations by implementing the Gaussian quadrature discretization. The reduced system was generated via the truncated Euler wavelet expansion. Several examples with known exact solutions were solved with zero absolute error. This method was also applied to the Fredholm and Volterra nonlinear integral equations and achieved the desired absolute error for all tested examples. The new numerical scheme is appealing in terms of its efficiency and accuracy in the field of numerical approximation.

On the other hand, some non-instantaneous impulsive boundary-value problems containing Caputo fractional derivatives of a function with respect to another function as well as Riemann–Stieltjes fractional integral boundary conditions were considered in [8]. In that work, the authors studied existence and uniqueness results for a new class of boundary- value problems consisting of non-instantaneous impulses and Caputo fractional derivative of a function with respect to another function, supplemented with Riemann–Stieltjes fractional integral boundary conditions. The existence of a unique solution was obtained via Banach's contraction mapping principle, while an existence result is established by using Leray–Schauder nonlinear alternative. Examples illustrating the main results were also constructed. In article [9], the authors considered a retarded linear fractional differential system with distributed delays and Caputo-type derivatives of incommensurate orders. For this system, several a priori estimates for the solutions, applying the two traditional approaches (Gronwall's inequality and integral representations of the solutions) were obtained. As an application of the obtained estimates, different sufficient conditions that guarantee finite- time stability of the solutions were established. A comparison of the obtained different conditions was made with respect to the estimates and norms used.

DISCUSSION AND CONCLUSION

Finally, a fractional coupled hybrid Sturm–Liouville differential equation with a multi- point boundary coupled hybrid condition was presented in [10]. It is worth recalling here that the Sturm–Liouville differential equation is an important tool for physics, applied mathematics, and other fields of engineering and science and has wide applications in quantum mechanics, classical mechanics, and wave phenomena. In this paper, the authors investigated the coupled hybrid version of the Sturm–Liouville differential equation. They studied the existence of solutions for the coupled hybrid Sturm–Liouville differential equa- tion with multi-point boundary-coupled hybrid condition. Furthermore, they investigated the existence of solutions for the coupled hybrid Sturm–Liouville differential equation with an integral boundary coupled hybrid condition. To close that work, the authors gave an application and some examples to illustrate their results.

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