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CO – NEIGHBOUR GRAPHS

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ABSTRACT

Let G(V,E) be a connected graph. For a vertex v in V, the set of all neighbours of v is called an open neighbourhood of v and is denoted by N(v). The closed neighbourhood of v is defined by N[v] = N(v) $\cup$ {v}. The co – neighbour graph CN(G) of a graph is defined as a graph with the same vertex set as that of G and two vertices in CN(G) are adjacent if and only if N[u] $\cup$ N[v] $\subset$ V(G). In this paper, we introduce this concept and study some properties of co – neighbour graph of a graph.

KEYWORDS: *Co – neighbour graph, copairable vertices, self centered graphs.*

AMS Subject Classification Code **(2000)**: 05C (Primary)

1 INTRODUCTION

Throughout this paper we consider only finite, simple, undirected graphs. For notations and terminology, we follow [2]. Let G(V,E) be a graph of order n. For any vertex v $\in$ V, the *open neighbourhood* N(v) of v is the set of all vertices adjacent to v. That is, N(v) = {u $\in$ V / uv $\in$ E}. The *closed neighbourhood* of v is defined by N[v] = N(v) $\cup$ {v}. A *full vertex* is a vertex in G which is adjacent to all other vertices of G. A *1 – factor* is a 1 – regular spanning subgraph of G and it is denoted by F.

The *distance* d(u,v) between two vertices u and v is the length of a shortest path between them. The *eccentricity* e(u) of a vertex u is the distance of a farthest vertex from u. The *radius* rad(G) of G is the minimum eccentricity and the *diameter* diam(G) of G is the maximum eccentricity in G. A vertex v is called an *eccentric vertex* of a vertex u if d(u,v) = e(u). A vertex u with e(u) = rad(G) is called a *central vertex*. The set of all central vertices is denoted by cen(G). A graph G for which rad(G) = diam(G) is called a *self – centered graph* of radius rad(G). Or equivalently a graph is *self – centered* if all of its vertices are central vertices.

A subset S of V is called a *dominating set* of G if every vertex in V – S is adjacent to at least one vertex in S. The *domination number* γ(G) is the minimum cardinality taken over all dominating sets in G. One can refer [3] for further reading on domination in graphs.

For any two distinct vertices u and v in G, u is said to be *copairable* with v if N(u) = $N(v)^c$ in G. A vertex in G is called a *copairable vertex*[1] if it is copairable with a vertex in G. For example, a graph with copairable vertices u and v is shown in Figure 1. A connected graph G of order at least 2 is said to be a *copairable graph* if every vertex of G is copairable. For example, $K_{n,m}$ is a copairable graph of order n, for any n, m $\geq$ 1.

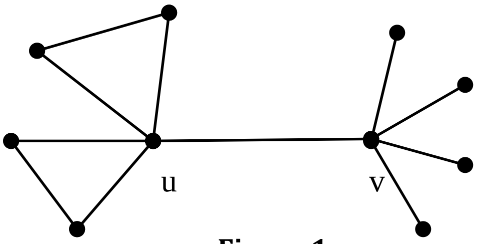

Figure 1

In this paper, we introduce a new type of graphs called co – neighbour graph which is defined as follows:

The *co – neighbour graph* CN(G) of a graph G is a graph with the same vertex set as that of G and two vertices u and v are adjacent in CN(G) if and only if there exists a vertex w in G such that $w \notin N[u] \cup N[v]$ in G. In other words, u and v are adjacent in CN(G) if and only if $N[u] \cup N[v] \subset V(G)$. For example, a graph G and its co – neighbour graph CN(G) are shown in Figure 2.

G **CN(G)**

Figure 2

We study some properties of co – neighbour graphs in this paper.

2 MAIN RESULTS

The following facts can be easily verified for a co – neighbour graph.

Fact 2.1 A full vertex in G is an isolated vertex in CN(G).

Fact 2.2 Copairable vertices are independent in CN(G).

Fact 2.3 $CN(K_n) = K_n^c$.

Fact 2.4 $CN(K_{1,n}) = K_1 \cup K_n$.

Fact 2.5 $CN(K_{m,n}) = \begin{cases} K_m \cup K_n & if\ m,n > 2 \\ K_4^c & if\ m,n = 2. \\ K_{m+n}^c & if\ m,n < 2 \end{cases}$

Fact 2.6 $CN(P_n) = CN(C_n) = K_n$ if $n > 6$.

Fact 2.7 $CN(P_n) = K_n^c$ if $n < 4$; $CN(P_4) = 2K_2$; $CN(P_5) = K_1 \vee P_4$; $CN(P_6) = K_6 - e$.

Fact 2.8 $CN(C_n) = K_n^c$ if n = 3 or 4; $CN(C_5) = C_5$; $CN(C_6) = K_6 - F$.

There exist some graphs G such that $CN(G) \cong G$. For example consider C_5. $CN(C_5) = C_5$. The following theorem discusses the conditions under which $CN(G) \cong G$.

Theorem 2.9 Let G be a disconnected graph. Then $CN(G) \cong G$ if and only if G is a disjoint union of two complete graphs.

Proof Let G be any disconnected graph with components $G_1, G_2, \ldots, G_n$. Assume that $CN(G) \cong G$. That implies, for any $u \in G_i$ and $v \in G_j$, $(i \neq j)$, $uv \notin E(CN(G))$ and so we conclude that $N[u] \cup N[v] = V(G)$. This is possible only when G contains exactly two components with u and v as full vertices in their respective components. Since u and v are arbitrary, we have $G = K_n \cup K_m$. Also any two vertices in the same component of G have all vertices in the other component as common non neighbour and hence adjacent in CN(G) too.

And the converse is obvious.

■

Theorem 2.10 A connected graph G is isomorphic to its co – neighbour graph CN(G) if and only if $\overline{G}$ is a triangle free self centered graph of radius two.

Proof Let G be any connected graph. And suppose that $G \cong CN(G)$. Therefore by Fact 2.1, G does not contain a full vertex. Then any two adjacent vertices in G are also adjacent in CN(G). This implies that for any two vertices $u, v \in V(G)$, if $uv \in E(G)$, then there exists a vertex w in G such that $w \notin N[u] \cup N[v]$. Hence $d(u,v) = 2$ in $\overline{G}$ for any two adjacent vertices u and v in G. Therefore $diam(\overline{G}) = 2$. Since G is connected, $\overline{G}$ does not contain a full vertex and so $rad(G) = 2$. Hence G is a self centered graph of radius 2.

In addition, for any two non adjacent vertices u and v in G, $N[u] \cup N[v] = V(G)$. Hence there is no common neighbour for any two adjacent vertices in $\overline{G}$. Therefore $\overline{G}$ is triangle free.

Conversely, if $\overline{G}$ is triangle free, then no two non adjacent vertices in G has a common non neighbour and hence not adjacent in CN(G) also. Since $\overline{G}$ is self centered of radius two, every two non adjacent vertices in $\overline{G}$ has a common neighbour and thereby $uv \in G$ implies that $uv \in CN(G)$ also. Therefore $G \cong CN(G)$.

■

Theorem 2.11 For a disconnected graph G of order n, $CN(G) \ncong K_n$ if and only if G has exactly two components each having a full vertex in it.

Proof Let G be any disconnected graph of order n. Assume that $CN(G) \ncong K_n$. If G has more than two components then obviously every two vertices has a common non neighbour which is a contradiction since $CN(G) \ncong K_n$. So G has exactly two components. Clearly every two vertices in the same component of G are adjacent in CN(G).

In particular, if any one of the two components, say G_1, does not contain a full vertex, then every vertex v in G_1 has a non neighbour in it which becomes the common non neighbour for v and all other vertices in G_2. Hence $CN(G) \cong K_n$, which is a contradiction. Hence each of the two components in G has a full vertex in it.

Conversely let G be a disconnected graph containing exactly two components each with a full vertex, say u and v. Clearly $uv \notin E(CN(G))$ and so $CN(G) \ncong K_n$. ■

Theorem 2.12 The co – neighbour graph CN(G) of a graph G of order $n \geq 3$, is isomorphic to its complement $\overline{G}$ if and only if $G \cong K_n$ or K_n^c or $K_{n,m}$, where $n, m \geq 1$ and $n \neq 2 \neq m$.

Proof Let G be any graph with at least three vertices. Assume that $CN(G) \cong \overline{G}$. Then for every two adjacent vertices u and v in G, $uv \notin E(CN(G))$ and so $N[u] \cup N[v] = V(G)$. Hence the vertices u and v have no common neighbour in $\overline{G}$. Therefore $d(u,v) > 2$ in $\overline{G}$ for every two non adjacent vertices in it.

Since $CN(G) \cong \overline{G}$, $uv \notin E(G)$ implies that $uv \in E(CN(G))$. In addition u and v have a common non neighbour in $\overline{G}$. Thus every edge in $\overline{G}$ lies in a triangle. In addition we can note that $\overline{G}$ does not contain P_3 as an induced subgraph, otherwise there exist two non adjacent vertices at a distance 2 in $\overline{G}$ which is a contradiction. Hence we conclude that every two vertices in each component of $\overline{G}$ are adjacent. Also since every edge if exists lies in a triangle, each component contains at least three vertices or every component is an isolated vertex.

The graphs satisfying above conditions are K_n or K_n^c or $K_{n,m}$, where $n, m \geq 1$ and $n \neq 2 \neq m$. And the converse is obvious.

■

Theorem 2.13 The co – neighbour graph CN(G) of a graph G is isomorphic to K_n if and only if $\gamma(G) > 2$.

Proof Let G be a graph for which the co – neighbour graph $CN(G) \cong K_n$. If possible let $\gamma(G) \leq 2$. Suppose $\gamma(G) = 1$, then G contains a full vertex and hence CN(G) contains an isolated vertex which is a contradiction. If $\gamma(G) = 2$, let {u,v} be a minimal dominating set of G. Then u and v have no common non neighbour in G and hence $uv \notin E(CN(G))$ which is also a contradiction. Hence we can conclude that $\gamma(G) > 2$.

Conversely suppose G is a graph with $\gamma(G) > 2$. If possible let $CN(G) \ncong K_n$. Then CN(G) contains at least two vertices u and v such that $uv \notin E(CN(G))$. Now consider u and v in G. They have no common non neighbour in G. In other words, every vertex of G is either a neighbour of u or v or both. Then clearly {u,v} is a minimal dominating set of G which is a contradiction. Hence the theorem.

■

Corollary 2.14 If $diam(G) > 4$, then $CN(G) \cong K_n$.

Proof Let G be a graph with diam(G) > 4. Then G does not contain a full vertex and so $\gamma(G) \neq 1$. Suppose $\gamma(G) = 2$. Then there exist two vertices u and v such that $N[u] \cup N[v] = V(G)$. Let x and y be any two vertices in G. If both are neighbours of u,(or v) then $d(x,y) \leq 2$. If not, $d(x,y) \leq 4$. That implies diam(G) ≤ 4, which is a contradiction. Hence $\gamma(G) > 2$ and by the above theorem, $CN(G) \cong K_n$. ■

The converse of the above corollary need not be true. For example, for the graph shown in Figure 3, $CN(G) \cong K_n$ but diam(G) = 4. But for graphs with diameter less than 3, domination number is less than or equal to 2 that implies $CN(G) \ncong K_n$. ■

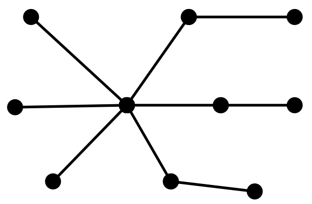

Figure 3

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