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STRING MODEL SOLUTION FOR LINEARIZED GSR QUANTUM THEORY

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ABSTRACT

Generalized special relativistic is linearized to find simple useful quantum equation. This equation is solved by considering particles as strings.

The energy consists of imaginary part accounting for friction. The real part energy is that of harmonic oscillator, the wave function solution for string shows travelling damping wave which can accounts for the inelastic scattering beside travelling amplified wave that can describe lasing process.

KEYWORDS: *generalized special relativity, string, energy quantization, quantization, Inelastic scattering.*

1. INTRODUCTION

The Klein- Gordon equation was first considered as a quantum wave equation by Schrödinger in his search for an equation describing de Broglie waves. The equation is found in his notebooks from late1925, and he appears to have prepared a manuscript applying it to the hydrogen atom. Yet, because it fails to take into account the electron's spin, the equation predicts the hydrogen atoms fine structure incorrectly, including overestimating the overall magnitude of the splitting pattern. The Dirac result is, however, easily recovered if the orbital momentum quantum number is replaced by total angular momentum quantum number In January 1926, Schrödinger submitted for publication instead his equation, a non-relativistic approximation that predicts the Bohr energy levels of hydrogen without fine structure[1,2,3,4,].

In 1927, soon after the Schrödinger equation was introduced, Vladimir Fock wrote an article about its generalization for the case of magnetic fields, where forces were dependent on velocity, and independently derived this equation. Both Klein and Fock used Kaluza and Klein's method. Fock also determined the gauge theory for the wave equation. The Klein- Gordon equation for a free particle has a simple plane wave solution**.**

Despite the remarkable successes Schrödinger and Klein Gordon equation, they suffer from noticeable problems. One of these problems comes from the fact that quantum equations cannot feel the existence of frictional force [5,6].

Attempts were made to account for the effect of friction. One of the earliest attempts was made by Langevin ,where he utilize the relation between classical laws and expectation values to compare classical expression of force with the corresponding expectation value equation in the presence of frictional force. Attempts also are made by M.D. Kostin to show that quantum system losses energy in the presence of friction. Rouman Tsekov also uses frictional quantum equation to describe Brounian motion.

2. Imaginary Friction for Linear GSR:

It is well known that for wave motion the imaginary part in the energy term causes wave amplitude alternation and damping which decreases amplitude and energy. In classical mechanics friction force causes wave damping. The damping can be obtained [7,8,9,10] by assuming

$$E_T = E - iE_d \qquad (2.1.1)$$

Where:

E_T= total energy.

E = particle energy.

E_d= damping energy.

Thus the wave function become:

$$\psi = Ae^{\frac{i}{\hbar}[P_x-(E-iE_d)t]} = Ae^{-E_d t}e^{\frac{i}{\hbar}[P_x-Et]} \qquad (2.1.2)$$

Using eqn. (2.1.10) together with (2.1.1) in addition to equation (2.2.10) yields,

$$E_T = cp + V$$

$$E - iE_d = cp + V$$

$$E = cp + V + iE_d = cp + V + \frac{i\hbar}{\tau} \qquad (2.1.3)$$

Now multiplying both sides by ψ to get:

$$E\psi = cp\psi + V\psi + i\frac{\hbar}{\tau}\ \psi \qquad (2.1.4)$$

Using the relations:

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi \qquad \frac{\hbar}{i}\ \nabla\psi = p\psi \qquad (2.1.5)$$

and substituting it in (2.1.5) yields:

$$i\hbar\frac{\partial\psi}{\partial t} = c\frac{\hbar}{i}\nabla\psi + V\psi + i\frac{\hbar}{\tau}\ \psi \qquad (2.1.6)$$

Considering particles as strings oscillators, the potential takes the form [11,12,13,14,15]:

$$V = \frac{1}{2}kx^2 \qquad (2.1.7)$$

Now consider the solution:

$$\psi = e^{-i\omega t}e^{+ikx}u \qquad (2.1.8)$$

Where:

$$\omega = \frac{E}{\hbar} \qquad K = \frac{P}{\hbar} \qquad (2.1.9)$$

Thus:

$$\frac{\partial\psi}{\partial t} = -i\omega\psi = -\frac{iE}{\hbar}\psi \qquad (2.1.10)$$

$$\nabla\psi = \frac{\partial\psi}{\partial x} = e^{-i\omega t}\left[ike^{ikx}u + e^{ikx}\frac{\partial u}{\partial x}\right] =$$

$$e^{-i\omega t+ikx}[iku + \nabla u] \qquad (2.1.11)$$

Thus inserting (2.1.9, 10, and 11) in (2.1.6) yields:

$$Ee^{-i\omega t}e^{ikx}u = \frac{c\hbar}{i}e^{-i\omega t}e^{ikx}[iku + \nabla u]$$

$$+\left[\frac{1}{2}kx^2u + \frac{i\hbar}{\tau}u\right]e^{-i\omega t}e^{ikx} \qquad (2.1.12)$$

Cancelling exponential terms on both sides yield:

$$Eu = c\hbar ku + \frac{c\hbar}{i}\nabla u + \frac{1}{2}kx^2u + \frac{i\hbar}{\tau}u \qquad (2.1.13)$$

Consider now the solution:

$$u = Ae^{\propto x^3} \qquad (2.1.14)$$

$$\nabla u = \frac{\partial u}{\partial x} = 3 \propto x^2Ae^{\propto x^3} = 3 \propto x^2u \qquad (2.1.15)$$

Substituting (2.1.14 ,15) in (2.1.13) gives:

$$Eu = c\hbar ku - 3 \propto x^2 c\hbar iu + \frac{1}{2}kx^2u + \frac{i\hbar}{\tau}u$$

Therefore;

$$E = c\hbar k + \frac{i\hbar}{\tau} \qquad (2.1.16)$$

And:

$$-c\hbar \propto i = \frac{1}{2}k$$

$$\propto = \frac{-k}{c\hbar i} = \frac{ik}{c\hbar} \qquad (2.1.17)$$

Let the potential vanish and consider another solution:

$$u = Ae^{\propto x} \qquad (2.1.18)$$

$$\nabla u = \propto Ae^{\propto x} = \propto u$$

sub

$$Eu = c\hbar ku + \frac{c\hbar}{\tau} \propto u + \frac{i\hbar}{\tau}u \qquad (2.1.19)$$

Equating real and imaginary parts:

$$E = c\hbar k - c\hbar \propto + \frac{\hbar}{\tau} = 0$$

$$\propto = \frac{1}{c\tau} \qquad (2.1.20)$$

3. Generalized Special Relativistic Energy Friction Equation:

Using the equation below:

$$\nabla u = \frac{\partial u}{\partial x} = 3 \propto x^2 Ae^{\propto x^3} = 3 \propto x^2 u$$

Considers:

$$E^2 + 2VE = c^2p^2 \qquad (3.1.1)$$

Let the friction term be added through energy term by assuming:

$$E \rightarrow E + \frac{i\hbar}{\tau} \qquad (3.1.2)$$

To get:

$$\left(E+\frac{i\hbar}{\tau}\right)^2+2V\left(E+\frac{i\hbar}{\tau}\right)=c^2p^2$$

$$E^2-\frac{\hbar^2}{\tau^2}-\frac{2i\hbar E}{\tau}+2VE+\frac{2i\hbar V}{\tau}=c^2p^2$$

$$E^2\psi-\frac{\hbar^2}{\tau^2}\psi-\frac{2i\hbar}{\tau}E\psi+2VE\psi+\frac{2i\hbar}{\tau}V\psi=c^2p^2\psi \qquad (3.1.3)$$

Now use equation (2.1.9) to get

$$i\hbar\frac{\partial\psi}{\partial t}=E\psi \quad -\hbar^2\frac{\partial^2\psi}{\partial t^2}=E^2\psi$$

$$\frac{\hbar}{\tau}\nabla\psi=p\psi \quad -\hbar^2\nabla^2\psi=p^2\psi \qquad (3.1.4)$$

Sub (3.1.4) in (3.1.3) to get:

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2}-\frac{\hbar^2}{\tau^2}\psi+2\frac{\hbar^2}{\tau}\frac{\partial\psi}{\partial t}+2i\hbar V\frac{\partial\psi}{\partial t}+2\frac{i\hbar}{\tau}V\psi=-c^2\hbar^2\nabla^2\psi \quad (3.1.5)$$

Which is the GSR quantum frictional equation; consider the solution [16, 17, 18]:

$$\psi=e^{-i\omega t}u \qquad \omega=\frac{E}{\hbar} \qquad (3.1.6)$$

$$\left[E^2-\frac{\hbar^2}{\tau^2}-\frac{2i\omega\hbar^2}{\tau}+2\hbar\omega V+\frac{2i\hbar V}{\tau}\right]e^{-i\omega t}u=-c^2\hbar^2\nabla^2ue^{-i\omega t}$$

Therefore:

$$\left[E^2-\frac{\hbar^2}{\tau^2}-\frac{2iE\hbar}{\tau}\right]u=-c^2\hbar^2\nabla^2u-\left(2E+\frac{2i\hbar}{\tau}\right)V \qquad (3.1.7)$$

One can re write this equation in the form:

$$\frac{-\hbar^2}{2m}\nabla^2u+V_ou=E_o \qquad (3.1.8)$$

With:

$$V_o=\frac{-\left(2E+\frac{2i\hbar}{\tau}\right)V}{2mc^2} \qquad (3.1.9)$$

$$E_o = \frac{\left(E^2 - \frac{\hbar^2}{\tau^2} - \frac{2iE\hbar}{\tau}\right)}{2mc^2} \qquad (3.1.10)$$

For particles oscillators in the form of string:

$$V = \frac{1}{2}kx^2 \qquad (3.1.11)$$

$$V_o = \frac{-\left(2E + \frac{2i\hbar}{\tau}\right)kx^2}{4mc^2} = \frac{1}{2}k_o x^2 \qquad (3.1.12)$$

The solution of equation (3.1.8) is:

$$E_o = \left(n + \frac{1}{2}\right)\hbar\omega \qquad (3.1.13)$$

If on agrees with the fact that the relation

$$E = mc^2 \qquad (3.7.14)$$

holds also for quantum system this can enable simplifying equations (3.1.13) (3.1.14) to get

$$V_o = -\left(1 + \frac{i\hbar}{\tau E}\right)V \qquad (3.1.15)$$

$$E_o = \left(\frac{1}{2}E - \frac{\hbar^2}{2\tau^2 E} - \frac{i\hbar}{\tau}\right) \qquad (3.1.16)$$

$$\frac{1}{2}E - \frac{\hbar^2}{2\tau^2 E} - \frac{i\hbar}{\tau} = \left(n + \frac{1}{2}\right)\hbar\omega$$

Thus:

$$E^2 - \frac{\hbar^2}{\tau^2} - \frac{2i\hbar}{\tau}E = 2\left(n + \frac{1}{2}\right)\hbar\omega E$$

$$E^2 - 2\left[\left(n + \frac{1}{2}\right)\hbar\omega + \frac{i\hbar}{\tau}\right)E - \frac{\hbar^2}{\tau^2} = 0 \qquad (3.1.17)$$

$$a = 1 \qquad b = -\left[\left(n + \frac{1}{2}\right)\hbar\omega + \frac{i\hbar}{\tau}\right] \qquad c = -\frac{\hbar^2}{\tau^2}$$

$$E = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (3.1.18)$$

Thus:

$$E = \frac{\left[\left(n+\frac{1}{2}\right)\hbar\omega + \frac{i\hbar}{\tau}\right] \pm \sqrt{\left[\left(n+\frac{1}{2}\right)\hbar\omega + \frac{i\hbar}{\tau}\right]^2 + \frac{4\hbar^2}{\tau^2}}}{2} \qquad (3.1.19)$$

When no friction exists:

$$\gamma = \frac{m}{\tau} \quad , \quad \frac{1}{\tau} = 0 \qquad (3.1.20)$$

Therefore:

$$E = \frac{\left(n+\frac{1}{2}\right)\hbar\omega + \left(n+\frac{1}{2}\right)\hbar\omega}{2}$$

$$E = \frac{2\left(n+\frac{1}{2}\right)\hbar\omega}{2}$$

$$E = \left(n+\frac{1}{2}\right)\hbar\omega \qquad (3.1.21)$$

Which is the ordinary harmonic oscillator solution. when the friction is very effective:

$$\frac{\hbar}{\tau} \gg \left(n+\frac{1}{2}\right)\hbar\omega \qquad (3.1.22)$$

Thus:

$$b = \frac{-i\hbar}{\tau}$$

Hence:

$$E = \frac{i\hbar}{\tau} \pm \sqrt{\frac{-\hbar^2}{\tau^2} + \frac{4\hbar^2}{\tau^2}}$$

Therefore:

$$E = \frac{i\hbar}{\tau} \pm \sqrt{3}\,\frac{\hbar}{\tau} \qquad (3.1.23)$$

Thus the energy can be given by:

$$E = \frac{i\hbar}{\tau}\left(i \pm \sqrt{3}\right) \qquad (3.1.24)$$

Which means that either:

$$E = \frac{i\hbar}{\tau}\left(i + \sqrt{3}\right) = -\frac{\hbar}{\tau} + \frac{i\hbar\sqrt{3}}{\tau}$$

Or

$$E = \frac{i\hbar}{\tau}\left(i - \sqrt{3}\right) = -\frac{\hbar}{\tau} - \frac{i\hbar\sqrt{3}}{\tau} \qquad (3.1.25)$$

4. DISCUSSION

The linearized GSR quantum equation was found for frictional medium, the suggested solution shown by eqns (2.1.8) and (2.1.14), shows complex energy solution. Equation (2.1.17) shows complex value for α, which indicates according to eqns (2.1.8) and (2.1.14) travelling waves with constant spatial [19,20,21].

5. DISTRIBUTION

When the friction effect energy, as shown in section (3.1),eqn (3.1.5) shows the quantum equation. The solution (3.1.6) for string harmonic oscillator shows energy expression (3.1.19) which is very complex. When no friction exist equation (3.1.19) is reduced to eqn (3.1.21) which is ordinary harmonic oscillator solution.

However, when friction dominates the energy become complex as eqns (3.1.24) and (3.1.25). This complex term when inserted in equation (3.1.6) shows decaying wave function. The number of particles,

$$n \sim |u^2| \sim e^{-\frac{\hbar\sqrt{3}}{\tau}t}$$

Which shows decaying process. This agrees with fact the friction decreases flux by inelastic scattering. It is also interested to note that, also

$$n \sim |u^2| \sim e^{\frac{\hbar\sqrt{3}}{\tau}t}$$

Which shows possibility of losing.

6. CONCLUSION

Using linearized GSR energy equation and treating particles as srtings , the quantum model can describe lasing process as well as inelastic scattering process.

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