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RESPONSE OF A FIRST-ORDER SERIES SYSTEM TO THE TRANSFER FUNCTION, THE LIQUID LEVEL OF TWO INTERACTING TANKS

Mariano Pérez Camacho[1], Angelica Vianney Chicas Reyes[2], Ricardo Pérez Camacho[3]
[1]Departamento de Ingeniería Química, UNAM, Facultad de Estudios Superiores, Zaragoza, Ciudad de México C.P. 09230
[2]Departamento de Ingeniería Química, UNAM, Facultad de Química, Ciudad de México C.P. 04510
[3]Departamento de Ingeniería Química,UNAM, Facultad de Química, Ciudad de México C.P. 04510

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SUMMARY

This article presents the experimental and theoretical results of the discharge and filling of water between two torispherical tanks connected in series at different heights, the model that was used was developed with equations reported in the literature for geometries of torispherical tanks and it was necessary to develop two more equations in the laboratory to calculate the discharge and fill volumetric flows for the tanks used. The unloading and filling model was obtained with transitory mass balances resulting in two simultaneous linear differential equations where the unloading and filling heights are related as a function of the operation time. The model was solved numerically, and the experimental results were compared against those of the proposed model, finally the transfer function for this system of interacting tanks was found.

***KEYWORDS:** Transient mass balances in two atmospheric interacting tanks at different heights and equal torispherical geometries. The discharge and fill model of two interacting tanks as a function of time. The volumes of partially filled torispherical tanks. The transfer function of a pulse in the discharge pipe that affects the volumetric flow within the discharge tank.*

1. INTRODUCTION

The purpose of this article is to support students in the experimental field in the transfer functions of dynamics and control systems. Our study contemplates two first-order coupled linear differential equations that result from a mass balance in the discharge of water between two interacting atmospheric tanks with a difference in height Z between the bases of the tanks and with equal torispherical geometries and propose a model theoretical where the time constant and the linear resistance during operation can be known, to suppose that when the water filling volume of 611.5 liters is reached in the lower tank, a centrifugal pump located at a time t = 0 is put to work in the discharge line of the upper tank, causing the volumetric flow to increase by 30%, going from 65.4 L/min to (65.4 x 1.3) = 85.02 L/min for a time of 0.1 min, and graph the response of the level of the water filling tank using the unit step function shifted "h" units of time to the right.

2. THEORETICAL FOUNDATIONS

2.1 The Transfer Function [1, 5, 6]

The Transfer Function is a mathematical expression that characterizes the "Input – Output" relationships of time-invariant linear systems. It is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (supplied function), under the assumption of zero (0) time initial conditions.

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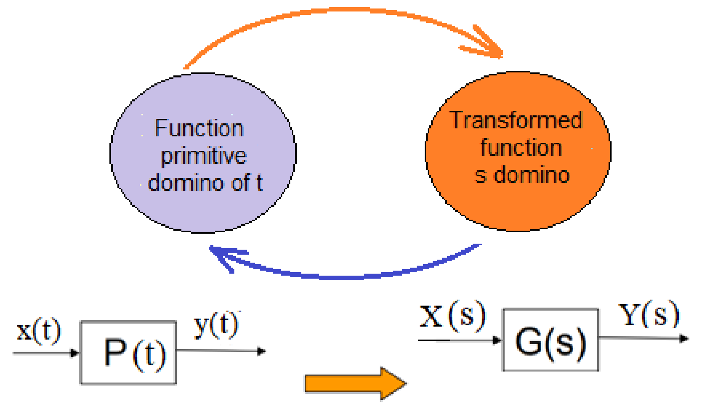

$$\frac{dy(t)}{dt} = b \cdot x(t) - a \cdot y(t) \qquad \frac{Y(s)}{x(s)} = G(s) = \frac{b}{s+a}$$

- To obtain the transfer function G(s), introduce the deviation variables into the linear differential equation:

$Y = y - y_s$; where y_s is the variable in the steady state

$X = x - x_S$; where x_S e is the variable in the steady state

$$\frac{d(y - y_S)}{dt} = b(x - x_S) - a(y - y_s)$$

$$\frac{d(Y)}{dt} = b(\mathrm{X}) - a(Y)$$

- Apply the Laplace transform to the previous differential equation to obtain the transfer function, remember that the transform of the derivative of a function is

$$\pounds\left[C_A'(z)\right] = s\,f(s) - C_A(Z)\Big|_{Z=0}$$

$$\therefore$$

$$s\,Y(s) = b\,X(s) - a\,Y(s)$$

$$s\,Y(s) + a\,Y(s) = b\,X(s)$$

$$Y(s)(s+a) = b\;X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{b}{s+a} \qquad (1)$$

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A transfer function is a mathematical model that, through a ratio, relates the response of a system (or output signal) to an input signal (or power supply).

The transfer function is a linear mathematical function that uses the Laplace transform and allows to represent the dynamic and stationary behavior of any system. However, we will detail this concept.

We know that when we are in front of a process, whatever it is, this process will generally have actuators and sensors. The actuators will make the variables (pressure, temperature, liquid level, humidity, speed, etc.) begin to vary over time, while the sensors are in charge of measuring and showing how these variables are changing over time.

Obviamente nosotros vamos a querer controlar estas variables del proceso, porque simplemente no vamos a dejar que estas variables evolucionen con el tiempo de manera que avancen al azar**.** Por decir algo, si tenemos un horno donde estamos cocinando galletas; no vamos a dejar que la variable temperatura suba a valores muy elevados, porque el resultado sería tener galletas totalmente quemadas. Es por eso que debemos controlar la temperatura para que esta se mantenga sobre una determinada zona y nos permita obtener el producto deseado.

2.2 First order series systems [1, 5, 6]

On many occasions, a process connected in series can be represented by first-order systems, that is, they are those that can be raised and solved by first-order differential equations. To illustrate this type of system, consider the liquid level of two tanks where the outflow of the first tank is the inflow for the second tank. Two possible arrangements are shown in Figs. 1. In Fig.1-a, the outlet of tank 1 discharges directly to the atmosphere at the inlet of tank 2 and the flow through depends only on. The variation of in tank 2 does not affect the transient response that occurs in tank 1. This type of system is referred to as a non-interacting system. In contrast, the system of Fig. 1.b is shown, which is qualified as interacting because the flow through depends on the difference between and.

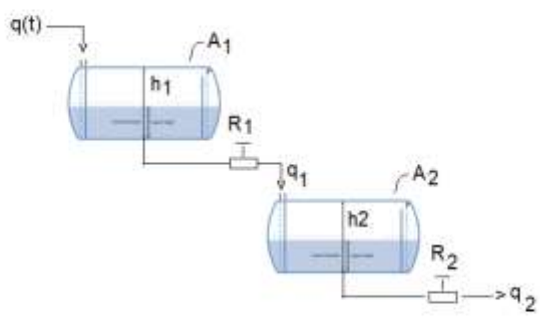

Fig. 1. A. Two Non-Interacting Tank Liquid Level System

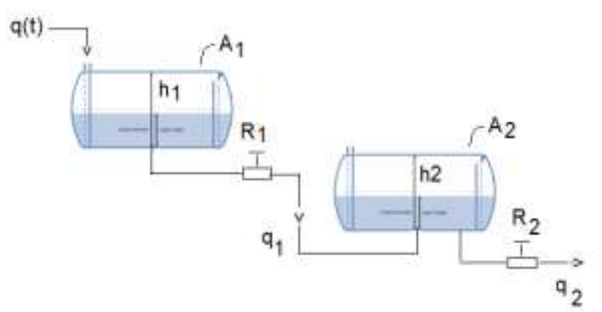

Fig. 1.b. Interacting Two Tank Liquid Level System

First-order processes are characterized by:

1. Its ability to store matter and/or energy and/or momentum
2. Its resistance is associated with the flow of mass, energy, or momentum
3. In general, the term R is a conversion factor that relates h(t) to q(t). This resistance is associated with pumps, valves, pipes and hydraulic accessories in general.

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For Fig.1.a $\begin{cases} q(t) = \dfrac{h_1}{R_1} \\ q(2) = \dfrac{h_2}{R_2} \end{cases}$; For Fig. 1.b, we have $q(1) = \dfrac{h_1 - h_2}{R_1}$ (2.1)

On other occasions, first-order physical systems can also be presented as the one shown in the following Fig. 2

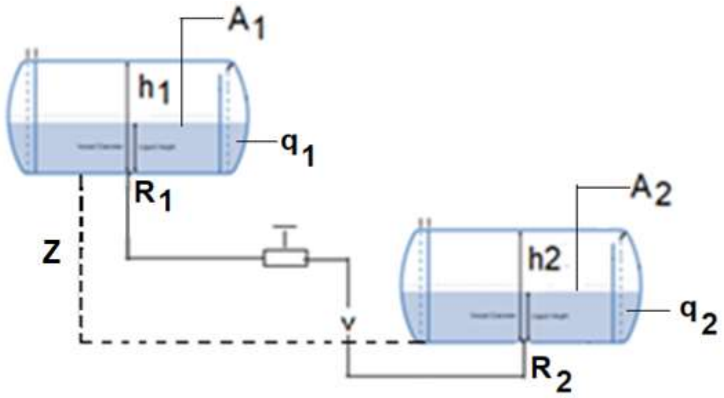


Fig. 2. Liquid discharge system between two interacting tanks without feeding and without discharge with a height difference Z, the emptying of the tank is by gravity.

In this system, the resistances R1 and are located at the bases of the tanks and the volumetric flows in the discharge and filling of water inside each of the tanks, that is:

$$\begin{cases} q_1 = \dfrac{(h_1 + Z) - h_2}{R_1}\ ;\ where\ q_1 = f\,(h_1) & (2.2-a) \\ q_2 = \dfrac{(h_1 + Z) - h_2}{R_2}\ ;\ where\ q_2 = f\,(h_2) & (2.2-b) \end{cases}$$

In each of these equations the height Z between the bases of the tanks has been considered.

In the Appendix (7.1) the equations of the volume, the wetted area and the loading and unloading expenses of the partially filled torispherical tanks [2, 3, 4] are described to calculate the system of two tanks which are between the equipment of the Chemical Engineering Laboratory.

To analyze this interacting system, the transitory mass balances for each tank can be written:

Mass flow in – Mass flow out = Rate of mass accumulation in the tank

1.- For the tank (1), there is no incoming mass flow

$$q(t)\,\rho - q_1(t)\,\rho = \frac{d\,(V_1\,\rho)}{d\,t}$$

$$-\rho q_1(t) = \frac{d(\rho V_1)}{dt}$$

$$-q_1 = \frac{d(A_1 h_1)}{dt} \qquad (3)$$

It can be seen in Fig. 2 that it is a variable that is not a function of discharge time but a function of the height of the liquid level. Substituting equation (2.2-a) in equation (3) we obtain:

$$-\frac{1}{R_1}[h_1 + Z) - h_2] = A_1 \frac{d(h_1)}{dt} \qquad (4)$$

2.- For tank (2), there is no outlet mass Flow

$$q_1(t)\rho - q_2(t)\rho = \frac{d(V_2\rho)}{dt}$$

$$\rho q_1(t) = \frac{d(\rho V_2)}{dt}$$

$$q_1 = \frac{d(A_2 h_2)}{dt} \qquad (5)$$

It can be seen in Fig. 2 that, is also a function of the height of the liquid level in the tank

Substituting equation (2.2-b) in equation (5) we obtain:

$$\frac{1}{R_2}[h_1 + Z) - h_2] = A_2 \frac{d(h_2)}{dt} \qquad (6)$$

2.3 The transfer function for the system of two interacting tanks of the Chemical Engineering Laboratory $\frac{H_2(s)}{Q_1(s)}$

The following procedure is used for systems of linear differential equations.

1.- Apply the deviation variables to equations (2.2 -a), (4) and (6).
A deviance variable considers the interacting variables in a process initially they are in the steady state x(s). This means that before a zero time there are no changes in these variables. But at zero time, the inputs to the process represented by x(t) change suddenly in time deviating to the transitory regime. Therefore, to the variables in the regime transitory, the variables in the permanent regime will be subtracted to quantify the values of the changes with reference to the permanent regime, to this difference of These variables are known as deviance variables.

2.- The resulting equations are transformed to the Laplace field

3.- Finally, combine them together to eliminate unwanted variables.

(2.2-a) $$q_1 = \frac{(h_1 + Z) - h_2}{R_1}$$

(4) $$-q_1 = A_1 \frac{dh_1}{dt}$$

(6) $$q_1 = A_2 \frac{dh_2}{dt}$$

Introducing the deviation variables in the previous equations and clarifying that the subscript "s" has been used to indicate the values of the variables in the steady state

De (4) $-(q_1 - q_{1S}) = A_1 \frac{d(h_1 - h_{1S})}{dt}$; donde $Q_1 = q_1 - q_{1S}$; $H_1 = h_1 - h_{1S}$

$\therefore$

$$Q_1 = -A_1 \frac{dH_1}{dt} \qquad (7)$$

De (6) $(q_1 - q_{1S}) = A_2 \frac{d(h_2 - h_{2S})}{dt}$; donde $Q_1 = q_1 - q_{1S}$; $H_2 = h_2 - h_{2S}$

$\therefore$

$$Q_1 = A_2 \frac{dH_2}{dt} \qquad (8)$$

De (2.2-a) $(q_1 - q_{1S}) = \frac{1}{R_1}\left[[(h_1 + Z) - (h_{1S} + Z)] - (h_2 - h_{2S})\right]$

reducing terms

$(q_1 - q_{1S}) = \frac{1}{R_1}\left[(h_1 - h_{1S}) - (h_2 - h_{2S})\right]$; where $H_1 = h_1 - h_{1S}$, $H_2 = h_2 - h_{2S}$ and

$Q_1 = q_1 - q_{1S}$

$\therefore$

$$Q_1 = \frac{1}{R_1}\left(H_1 - H_2\right) \quad (9)$$

Transforming equations (7), (8) and (9) to the Laplace field

$$Q_1(s) = -A_1\, s\, H_1(s) \quad (10)$$

$$Q_1(s) = A_2\, s\, H_2(s) \quad (11)$$

$$Q_1(s) = \frac{H_1(s)}{R_1} - \frac{H_2(s)}{R_1} \quad (12)$$

Substituting equation (11) in (12)

$$A_2\, s\, H_2(s) = \frac{H_1(s)}{R_1} - \frac{H_2(s)}{R_1} \quad (13)$$

From equation (10) the following is obtained

$$H_1(s) = \frac{-Q_1(s)}{A_1 \times s} \quad (14)$$

Substituting equation (14) into equation (13)

$$A_2 \times s \times H_2(s) = \frac{-Q_1(s)}{R_1 \times A_1 \times s} - \frac{H_2(s)}{R_1}$$

$$A_2 \times s \times H_2(s) + \frac{H_2(s)}{R_1} = \frac{-Q_1(s)}{R_1 \times A_1 \times s}$$

Factoring the above equation, we get

$$H_2(s)\left(A_2 \times s + \frac{1}{R_1}\right) R_1 \times A_1 \times s = -Q_1(s)$$

Dividing the equation by $Q_1(s)$

$$\frac{H_2(s)}{Q_1(s)}\left(A_2 \times s + \frac{1}{R_1}\right) R_1 \times A_1 \times s = -1$$

$$\frac{H_2(s)}{Q_1(s)} = -\frac{1}{\left(A_2 \times s^2 + \frac{1}{R_1}\right) R_1 \times A_1 \times s}$$

$$\frac{H_2(s)}{Q_1(s)} = -\frac{1}{A_2 \times s^2 \times R_1 \times A_1 \times s + A_1 \times s}$$

$$Yeah\ \tau_1 = A_1 R_1 \qquad (15)$$

where

τ_1 = System time constant

R_1 = Linear resistance at the base of the discharge tank.

Simplifying you get

$$\frac{H_2(s)}{Q_1(s)} = -\frac{1}{A_2 \times s^2 \times \tau_1 \times s + A_1 \times s}$$

$$\frac{H_2(s)}{Q_1(s)} = -\frac{1}{s \times (A_2 \times s \times \tau_1 + A_1)} \qquad (16)$$

Now consider that you want to use the response of a first-order system as the unit step function. Therefore, if the volumetric flow rate changes according to a unit step translated “h” units to the right as shown by the following equation

$$Q(t) = \frac{1}{h}\left[u(t) - u(t-h)\right]$$

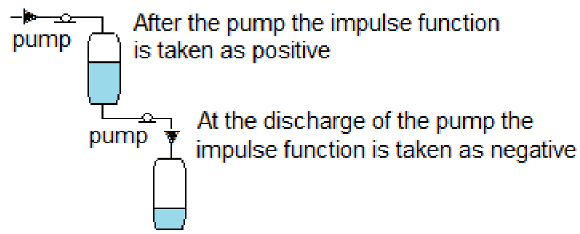
where $u(t)$ is the unit step function and $u(t-h)$ is the step function translated h units. Therefore, the input to the system will be expressed as the difference of step functions. The inverse function for the Laplace field is:

$$Q(\mathrm{s}) = \frac{1}{h} \times \frac{1 - e^{-h \times s}}{s}$$

Yeah $Q_1(\mathrm{s}) = \frac{1}{h \times s} - \frac{e^{-h \times s}}{h \times s}$ and it is substituted in equation (16) the transfer function of a unit step translated “h” units to the right will be obtained.

If the step function is applied using the pump before the liquid enters the upper tank inlet, it will be considered as positive.

If the step function is applied using the pump after the liquid outlet from the upper tank, it will be considered as negative.



$$-H_2(s)=\left\{-\frac{1}{s\times\left(A_2\times s\times\tau_1+A_1\right)}\left[\frac{1}{h\times s}-\frac{e^{-h\times s}}{h\times s}\right]\right\}$$

$$-H_2(s)=\left\{-\frac{1}{h\times s^2\times\left(A_2\times s\times\tau_1+A_1\right)}+\frac{e^{-h\times s}}{h\times s^2\times\left(A_2\times s\times\tau_1+A_1\right)}\right\}$$

$$H_2(t)=-\left\{\begin{array}{l}-\frac{1}{h}\left(\frac{t}{A_1}-\frac{A_2\,\tau_1}{A_1^2}+\frac{A_2\,e^{\frac{-A_1 t}{A_2\,\tau_1}}\,\tau_1}{A_1^2}\right)+\frac{1}{h}\left(\frac{-h+t}{A_1}-\frac{A_2\,\tau_1}{A_1^2}+\frac{A_2\,e^{\frac{-A_1(-h+t)}{A_2\,\tau_1}}\,\tau_1}{A_1^2}\right)\times\\ HeavisideTheta\left[-h+t\right]\end{array}\right\}$$

...........(17)

The function HeavisideTheta [-h + t] is worth 1 if the argument is > 0 and is worth 0 if the argument is < 0

3. EXPERIMENTAL DATA OF THE SYSTEM OF INTERACTING TANKS FROM THE LABORATORY OF CHEMICAL ENGINEERING

TABLE 1. Data for filling the lower tank

a.- The reading of these quantities is from bottom to top according to the metallic scale from Figure (3)

b.- The values reported in column (2) were taken using a small scale metallic.

Reading number	Volumes filling on the metal scale Fig.14 (L)	Heights partial fill between brands (cm)	Continuous heights tank filler (cm)	Discrete fill times (min)	Continuous fill times (min)
	(1)	(2)	(3)	(4)	(5)
23	0	0 a 100 = 9.5	92.1	5´ 13” = 5.217	57.651
22	100	100 a 200 = 5.5	82.6	3´ 37” = 3.617	52.434
21	200	200 a 300 = 4.5	77.1	3´ 9” = 3.15	48.817
20	300	300 a 400 = 4.5	72.6	3´ 11” = 3.183	45.667
19	400	400 a 500 = 4.0	68.1	2´ 43” = 2.717	42.484
18	500	500 a 600 = 3.4	64.1	3´ 5” = 3.083	39.767
17	600	600 a 700 = 3.5	60.7	2´ 55” = 2.917	36.684
16	700	700 a 800 = 3.0	57.2	2´ 19” = 2.317	33.767
15	800	800 a 900 = 3.4	54.2	2´ 51” = 2.85	31.45

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14	900	900 a 1000 = 4.0	50.8	2´ 54” = 2.9	28.6
13	1000	1000 a 1100 = 4.0	46.8	2´ 9” = 2.15	25.7
12	1100	1100 a 1200 = 4.0	42.8	2´ 37” = 2.617	23.55
11	1200	1200 a 1300 = 4.0	38.8	3´ 10” = 3.167	20.933
10	1300	1300 a 1400 = 3.7	34.8	2´ 35” = 2.583	17.766
9	1400	1400 a 1500 = 3.5	31.1	2´ 50” = 2.833	15.183
8	1500	1500 a 1600 = 3.6	27.6	2´ 10” = 2.167	12.35
7	1600	1600 a 1700 = 3.6	24.0	1´ 53” = 1.883	10.183
6	1700	1700 a 1800 = 3.8	20.4	2´ 2” = 2.033	8.3
5	1800	1800 a 1900 = 3.8	16.6	2´ 1” = 2.017	6.267
4	1900	1900 a 2000 = 4.5	8.3 + 4.5 = 12.8	1´ 51 “= 1.85	2.4 + 1.85 = 4.25
3	2000	2000 a 2100 = 5.8	2.5 + 5.8 = 8.3	1´ 35” = 1.583	0.817 + 1.583 = 2.4
2	2100	2100 a 2145 = 2.5	0 + 2.5 = 2.5	49” = 0.817	0 + 0.817 = 0.817
1	2145 tank bottom	0 tank bottom	0 tank bottom	0 tank bottom	0 tank bottom

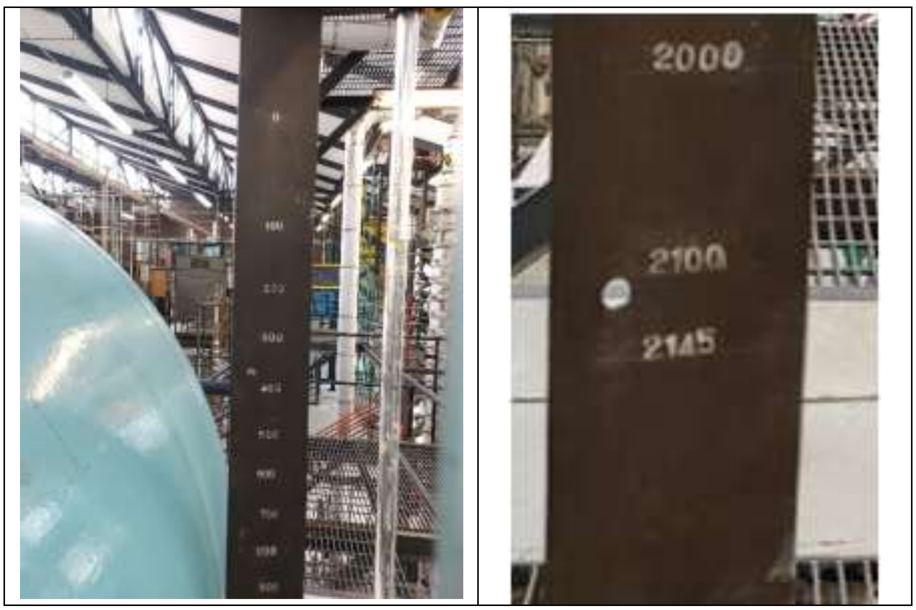

Fig. 3. The counting of the filling time of the bottom tank begins with the value of 2145 liters on the metal scale

To start the readings, it is necessary to open the discharge valves between tanks except the one that feeds the lower tank. It begins by opening this last valve, starting the stopwatch to record the filling times starting at zero minutes, which corresponds to the value of 2145 liters on the metal scale of the empty lower tank. See Fig. 3

The data from TABLE 1 is now presented from top to bottom in TABLE 2. The count starts at zero (0) liters at the bottom of the tank and ends at (2145) liters when the tank is full.

TABLE 2. Data for filling the lower tank

The data is now presented from top to bottom for the fill of tank 2

Volumes filling on the metal scale Fig.12 (L)	Discrete volumes filling (L)	Continuous volumes filling (L))	Heights continue filling (cm)	Continuous fill times (min)
	(6)	**(7)**	**(8)** column (3) inverted	**(9)** column (5) inverted
It's the bottom of the tank 0	It's the bottom of the tank 0	It's the bottom of the tank 0	It's the bottom of the tank 0	It's the bottom of the tank 0
100	2145 – 2100 = 45	0 + 45 = 45	2.5	0.817
200	2100 – 2000 =100	45 + 100 = 145	8.3	2.4
300	2000 – 1900 100	145 + 100 = 245	12.8	4.25
400	1900 – 1800 =100	245 + 100 = 345	16.6	6.267
500	1800 – 1700 =100	345 + 100 = 445	20.4	8.3
600	1700 – 1600 =100	445 + 100 = 545	24	10.183
700	1600 – 1500 =100	545 + 100 = 645	27.6	12.35
800	1500 – 1400 =100	645 + 100 = 745	31.1	15.183
900	1400 – 1300 =100	745 + 100 = 845	34.8	17.766
1000	1300 – 1200 =100	845 + 100 = 945	38.8	20.933
1100	1200 – 1100 =100	945 + 100 = 1045	42.8	23.55
1200	1100 – 1000 =100	1045 + 100 = 1145	46.8	25.7
1300	1000 – 900 = 100	1145 + 100 = 1245	50.8	28.6
1400	900 – 800 = 100	1245 + 100 = 1345	54.2	31.45
1500	800 – 700 = 100	1345 + 100 = 1445	57.2	33.767
1600	700 – 600 = 100	1445 + 100 = 1545	60.7	36.684
1700	600 – 500 = 100	1545 + 100 = 1645	64.1	39.767
1800	500 – 400 = 100	1645 + 100 = 1745	68.1	42.484
1900	400 – 300 = 100	1745 + 100 = 1845	72.6	45.667
2000	300 – 200 = 100	1845 + 100 = 1945	77.1	48.817
2100	200 – 100 = 100	1945 + 100 = 2045	82.6	52.434
2145	100 – 0 = 100	2045 + 100 = 2145	92.1	57.651

TABLE 3. Experimental data for the discharge of the upper tank

Data is presented from top to bottom

Reading number	**volumes download marked on the metal scale (L)**	**Continuous volumes of discharge, are equal to the continuous fill volumes (L))**	**Heights discreet between brands (cm)**	**Continuous heights download (cm)**	**Continuous tenses download (min) query column (5)**
	(10)	**(11)**	**(12)**	**(13)**	**(14)**
1	full tank 2145	full tank 2145	full tank 0 a 100 = 9.5	full tank 92.1	full tank 0
2	2100	2045	100 a 200 = 5.5	82.6	0 + 0.817 = 0.817
3	2000	1945	200 a 300 = 4.5	77.1	0.817 + 1.583 = 2.4
4	1900	1845	300 a 400 = 4.5	72.6	2.4 + 1.85 = 4.25
5	1800	1745	400 a 500 = 4.0	68.1	6.267

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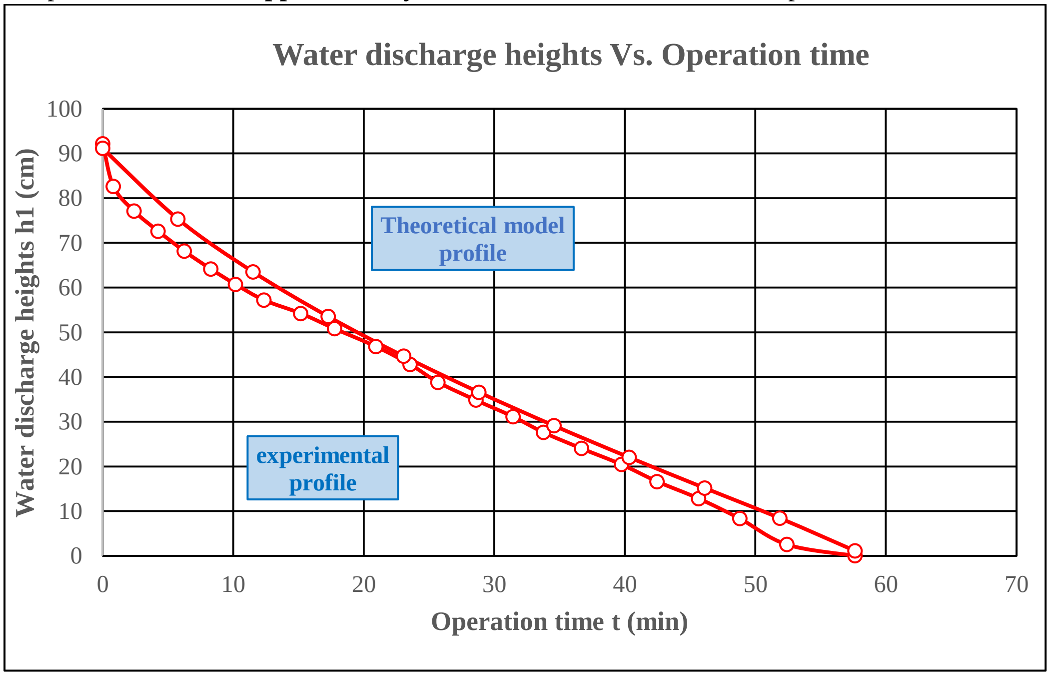
6	1700	1645	500 a 600 = 3.4	64.1	8.3
7	1600	1545	600 a 700 = 3.5	60.7	10.183
8	1500	1445	700 a 800 = 3.0	57.2	12.35
9	1400	1345	800 a 900 = 3.4	54.2	15.183
10	1300	1245	900 a 1000 = 4.0	50.8	17.766
11	1200	1145	1000 a 1100 = 4.0	46.8	20.933
12	1100	1045	1100 a 1200 = 4.0	42.8	23.55
13	1000	945	1200 a 1300 = 4.0	38.8	25.7
14	900	845	1300 a 1400 = 3.7	34.8	28.6
15	800	745	1400 a 1500 = 3.5	31.1	31.45
16	700	645	1500 a 1600 = 3.6	27.6	33.767
17	600	545	1600 a 1700 = 3.6	24.0	36.684
18	500	445	1700 a 1800 = 3.8	20.4	39.767
19	400	345	1800 a 1900 = 3.8	16.6	42.484
20	300	245	1900 a 2000 = 4.5	8.3 + 4.5 = 12.8	45.667
21	200	145	2000 a 2100 = 5.8	2.5 + 5.8 = 8.3	48.817
22	100	45	2100 a 2145 = 2.5	0 + 2.5 = 2.5	52.434
23	0	0	0	0	57.651

4. QUESTIONNAIRE AND ANSWERS

4.1 Present in Graph 1 the profile of the descending heights h1 of the water discharge of the model and the profile of the descending heights h1 experimental, both Vs. the operation time t. In Graph 2 the profiles of the rising filling heights h2 of the model and experimental Vs. the operation time t. In **Appendix 7.2** you will find the data to build these profiles.



Graph 1. Comparative profile of the water discharge heights of the model and the profile of the discharge heights of the experimental data

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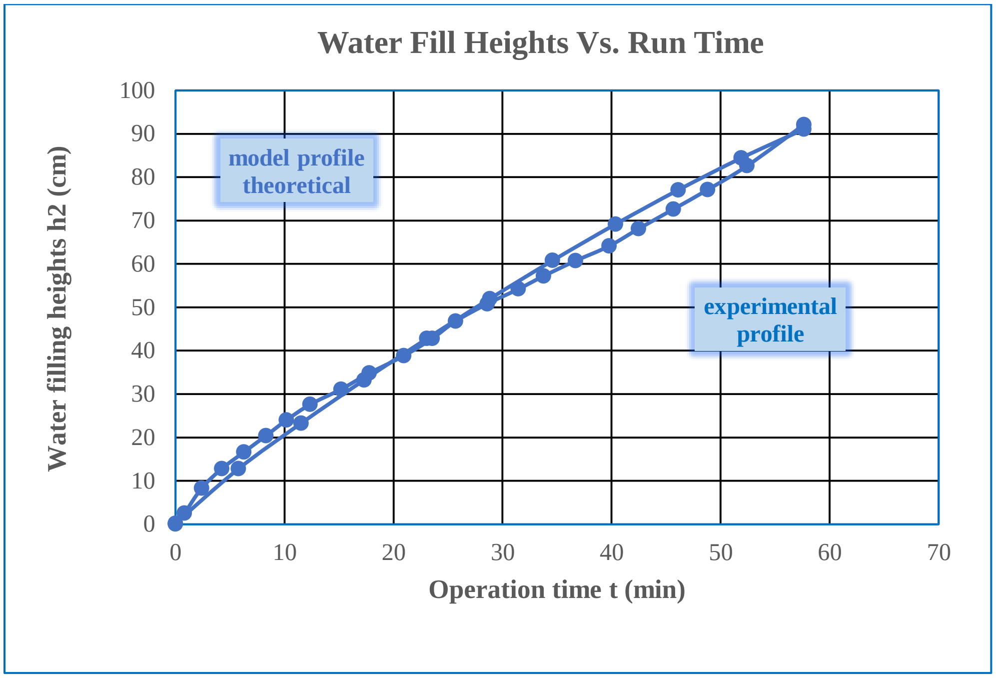

Graph 2. Comparative profile of the water filling heights of the model and the profile of the filling heights of the experimental data

4.2 Use the results of the interacting tank unloading model program reported in Table 4 that solves the theoretical model of interacting tank unloading and filling to find the answer to the following statement:

Suppose that when the water fill volume of 611.5 liters is reached in the lower tank, a centrifugal pump located in the discharge line of the upper tank is started to work at a time t = 0, causing the volumetric flow to increase by 30 %, going from 65.4 L/min to (65.4 x 1.3) = 85.02 L/min for a time of 0.1 min. Plot the response of the fill tank level using the unit step function shifted "h" units of time to the right.

Data

Figure (4) shows the representative diagram of this problem, the unitary pulse function moved "h" units to the right

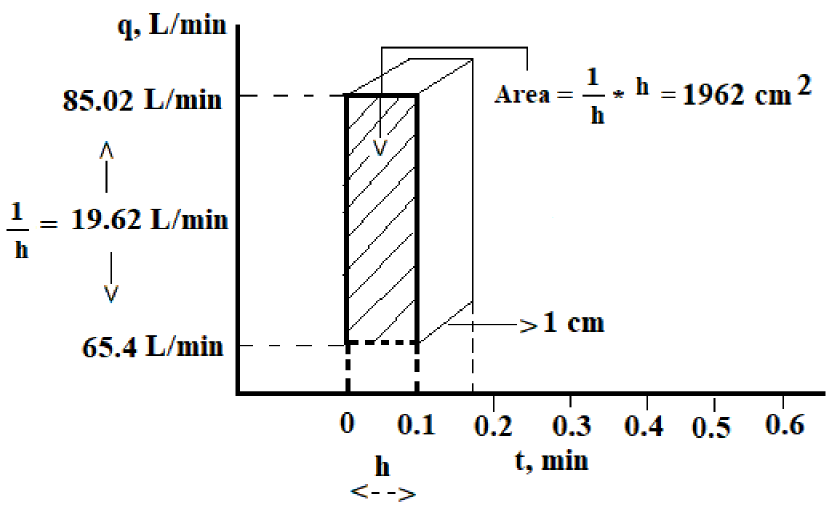

Fig. 4 Input pulse function to the water discharge line of tank 1

For a first-order system, the time constant τ for the step function it is expressed in the following terms.

$$\tau = A \times R \tag{18}$$

Were

$$A = Area\ o\ magnitude = \frac{1}{h} \times h,\ shown\ on\ Fig.\ 4 \tag{19}$$

$$R = \frac{h}{Q} = \frac{\text{Flow Driving Force (Hydralic Head)}}{\text{Volumetric flow}} \tag{20}$$

TABLE 4. Results obtained by the interacting tank discharge model, consult the Wolfram Mathematica program reported in Appendix 7.3

Filling tank 2					
Fill volume of tank 2 (cm3)	**Fill volume of tank 2 (liters)**	**Volumetric flow fill to tank 2 (cm^3/s)**	**Volumetric flow fill to tank 2 (L/min)**	**Linear resistance for tank 1 R1 cm/(cm^3/s)**	**Linear resistance for tank 1 R1 cm/(cm^3/min)**
0	0	0	0	0	0
155205	155.205	675.43	40.526	0.073665	0.001228
371456	371.456	915.137	54.908	0.0934666	0.001558
611551	**611.551**	1091.47	**65.488**	0.108094	**0.001802**
858428	858.428	1232.44	73.946	0.119427	0.00199
1099370	1099.0	1348.16	80.89	0.18543	0.003091
1323460	1323.0	1443.21	86.593	0.136408	0.002273
1521140	1521.0	1519.85	91.191	0.144382	0.002406
1683860	1684.0	1579.09	94.745	0.155396	0.00259
1802980	1803.0	1620.69	97.241	0.179971	0.003
1862820	1863.0	1641.09	98.465	0.4108	0.006847

.

Continuous areas download A1 (cm2)	**Continuous areas filling A2 (cm2)**	**operation time t (s)**	**operation time t (min)**
0	0	0	0
69885.1	21322.1	345.9	5.765
60050.5	31078.7	691.8	11.53
52850.1	**38779.8**	1037.7	17.295
46779.6	45500.9	1383.6	23.06
41188.7	51792.9	1729.5	28.825
35692.4	58035.4	2075.4	34.59
29999.4	64511.1	2421.3	40.355
23775.9	71504.4	2767.2	46.12
16362.6	79579.3	3113.1	51.885
3977.72	93691.3	3459	57.65

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R_1 = 0.001802 cm/(cm3/min)

$\tau_1 = A \times R_1$ = 1962 cm^2 $\times$ 0.001802 cm/(cm3/min) = 3.535 min

h = 0.1 min

1/h = 19.62 L/min = 19620 cm^3 / min

A_1 = 52850.1 cm^2

A_2 = 38779.8 cm^2

Substituting the values in equation (17), column (4) of Table 5 is obtained, which allows the construction of Graph (3) “H2(t) Vs. t” that shows the profile of the transfer function.

NOTE. - The HeavisideTheta function [-h + t] is equal to 1 if the argument is > 0 and equals 0 if the argument is < 0.

Table 5. Results of the transfer function $\frac{H_2(s)}{Q_1(s)}$

Time scale where the pulse function starts t (min)	Argument of the Heaviside Function Theta [-h+t]	Value of the Heaviside function Theta [-h+t]	Value of the function of H2(t) (cm)
(1)	(2)	(3)	(4)
0	-0.1	0	0
0.2	0.1	1	0.07084
0.3	0.2	1	0.06956
0.4	0.3	1	0.06834
0.5	0.4	1	0.06716
0.6	0.5	1	0.06602
0.7	0.6	1	0.06493
4	3.9	1	0.04491
8	7.9	1	0.03879
12	11.9	1	0.03748
16	15.9	1	0.0372
20	19.9	1	0.03714
28	27.9	1	0.03712

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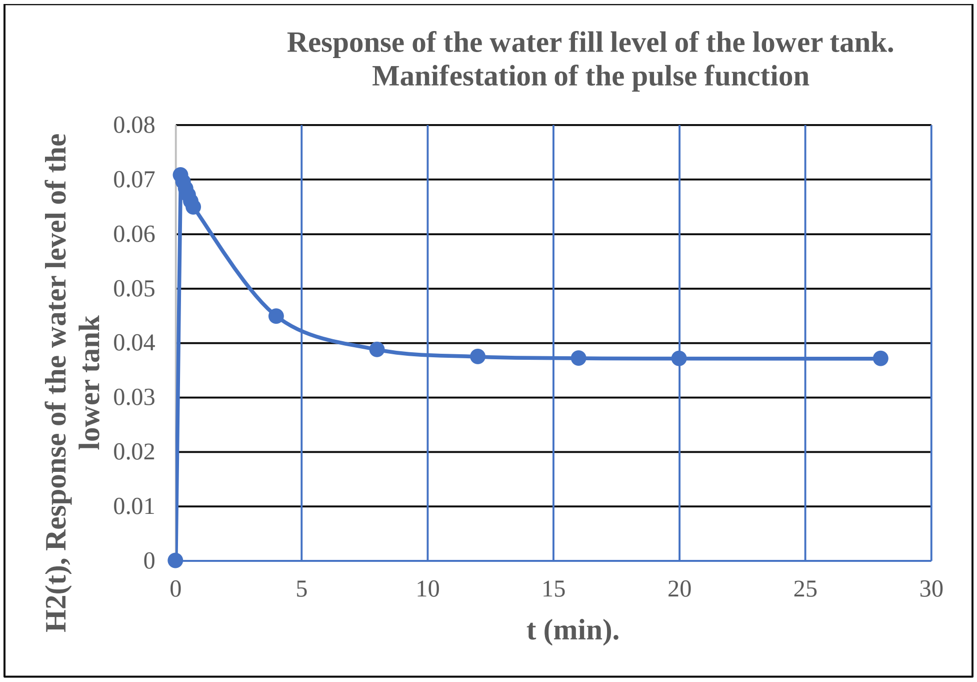

Graph 3. Profile of the transfer function on the surface of the tank lower when the water discharge is affected with the pulse function

Note that the water level rises rapidly for 0.1 min when the additional flow from the pump is fed to the lower tank; the water level then drops exponentially and after approximately 10 minutes the natural discharge of the water filling due to gravity is recovered.

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6. NOMENCLATURE

A = Variable cross-sectional area of the variable water surface in the tanks: cm^2
AW = Wetted area of the cylindrical section of a torispherical tank, Wetted area of a torispherical head: cm^2
B = Hydraulic variable from equation (16)
C = Hydraulic variable of equation (20)
H = Deviation variable for the heights of the liquid level in the tank: cm
h = Variable height of the liquid level in the tank: cm
L = Length of the cylindrical section of the torispherical tank: cm
q = Volumetric flow rate: $cm^{3/}$min
Q = Deviation variable for volumetric flows: cm^3/min
Q1 = Volumetric discharge flow rate of tank one, equation (22): cm^3/min
Q2 = Volumetric flow rate for filling tank two, equation (23): cm^3/min
R = Resistance to volumetric flow: cm / (cm^3/min)
t = Operation time: min

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Z = Difference in heights from the base of tank 1 to the base of tank 2, Fig. 7: cm
ρ = Density of water: g/cm^3

7. APPENDIX

7.1. Geometric description of the volume and wetted area of torispherical tanks in the Chemical Engineering Laboratory of the Faculty of Chemistry of the Universidad Nacional Autónoma de México UNAM. **[2, 3, 4]**.

Fig.5. Torispherical tanks of the Chemical Engineering Laboratory

Fig. 6. Sliding train of centrifugal pumps

The tank unloading diagram

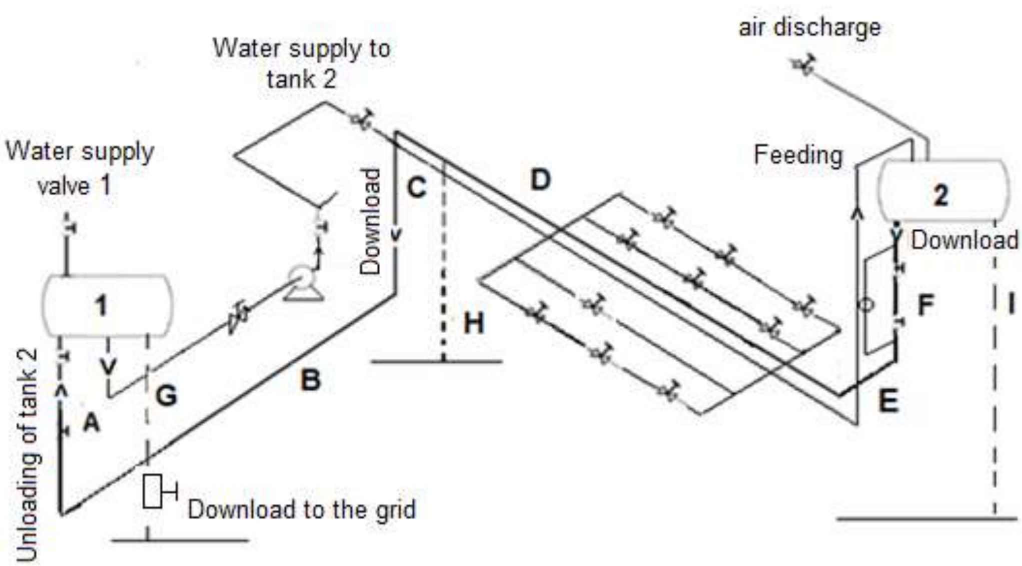

Fig. 7. Simplified piping diagram of the two-tank system of the Chemical Engineering Laboratory

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Segment A	88 cm length, φ = 1 ½”, schedule 40, commercial iron	Ball valve and globe valve
Segment B	360 cm length, φ = 1 ½”, schedule 40, commercial iron	Two elbows at 90°
Segment C	88 cm length, φ = 1 ½”, schedule 40, commercial iron	Two elbows at 90°
Segment D	1168.5 cm length, φ = 1 ½”, schedule 40, commercial iron	Two elbows at 90°
Segment E	123 cm length, φ = 1 ½”, schedule 40, copper	Two copper elbows at 90°
Segment F	134 cm length, φ = 1 ½”, schedule 40, copper	Two ball valves, one 90° copper elbow
Total length of straight tube	TL = 1941.5 cm	
Height G	71 cm	
Height H	88 cm	
Height I	222 cm	
External diameter of tanks	Do = 92.1 cm	
Internal diameter of tanks	Di = 91.1 cm	
Height difference from the bottom of tank 2 to the dome of tank 1	(I – G) + Di = 134 + 91.1 = 225.1 cm	
Height difference from the bottom of tank 2 to the bottom of tank 1	I – G = 222 cm – 88 cm = 134 cm	

Geometric diagrams of torispherical tanks

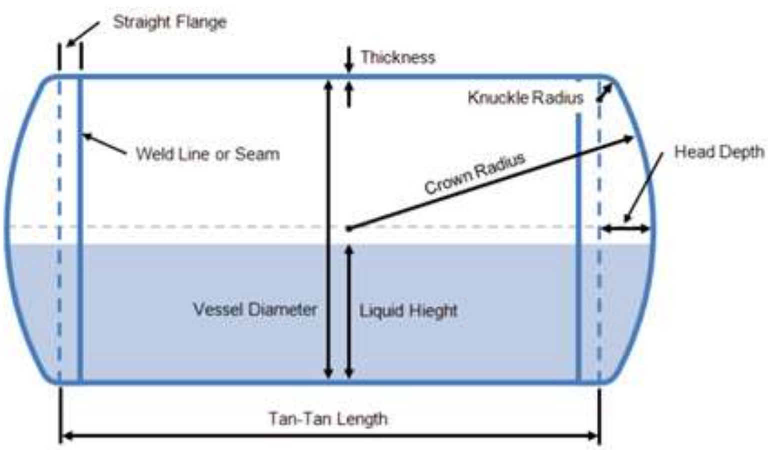

Fig. 8. Diagram of the torispherical tank of the Chemical Engineering Laboratory, the two tanks have the same dimensions

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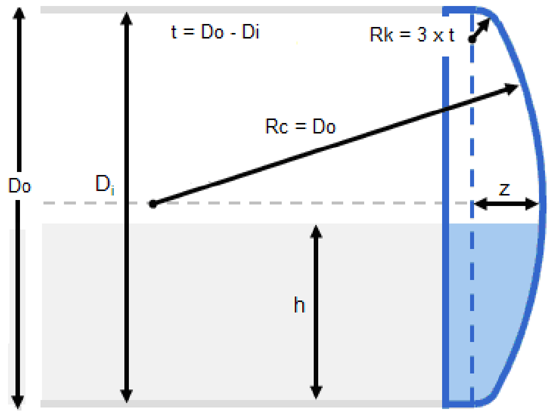

Fig. 9. Diagram of the geometric variables of the torispherical head

Dimensiones de los tanques

- External circumference = 289.34 cm
- Do; External diameter = 92.1 cm
- tt; Plate thickness = 1 cm
- Di; Internal diameter = 91.1 cm
- Rc = Do; Inside Crown radius = 92.1 cm
- Rk = 3 (tt); Inside knuckle = 3 (1) = 3 cm
- L = Length of the cylindrical section of the tank = 271 cm
- h = liquid level height = [cm]
- z = Internal depth of torispherical head = [cm]

$$z = Rc - \sqrt{\left(Rc - R_k\right)^2 - \left(\frac{D_O}{2} - tt - R_k\right)^2} \quad (21)$$

Header Wet Area

$$A_W = \frac{\pi D_i^2}{8}\left[\left(\frac{h}{D_i} - 0.5\right)B + 1 + \frac{1}{4\varepsilon}\ln\left(\frac{4\varepsilon\left(\frac{h}{D_i} - 0.5\right) + B}{2 - \sqrt{3}}\right)\right] \quad (22)$$

$$B = \sqrt{1 + 12\left(\frac{h}{Di} - 0.5\right)^2} \quad (23)$$

$$\varepsilon = \sqrt{1 - \frac{4z^2}{D_i^2}} \quad (24)$$

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Cylindrical section wetted area

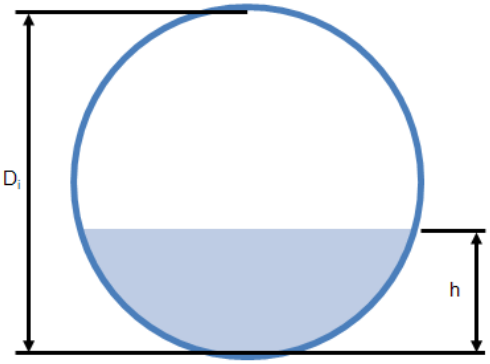

Fig. 10. Geometric variables of the wetted cylindrical section

$$A_W = L\, D_i \cos^{-1}\left(1-2\frac{h}{D_i}\right) \tag{25}$$

Partial volume of torispherical heads

$$V_p = D_i^3\ C\ \frac{\pi}{12}\left[3\left(\frac{h}{D_i}\right)^2 - 2\left(\frac{h}{D_i}\right)^3\right] \tag{26}$$

Where C is proposed by the ASME code through the following expression

$$C = 0.30939 + 1.7197\ \frac{R_k - 0.06\, D_0}{D_i} - 0.16116\ \frac{tt}{D_0} + 0.98997\left(\frac{tt}{D_0}\right)^2 \tag{27}$$

Partial volume of the cylindrical section

$$V_p = L\, D_i^2\left(\frac{1}{4}\cos^{-1}\left(1-2\frac{h}{D_i}\right) - \left(\frac{1}{2}-\frac{h}{D_i}\right)\sqrt{\frac{h}{Di}-\left(\frac{h}{D_i}\right)^2}\right) \tag{28}$$

Rapidez de flujo volumétrico de descarga del tanque superior (2) y de carga del tanque inferior (1)

$$q_2 = \frac{2\left(D_i^3\ C\ \frac{\pi}{12}\left[3\left(\frac{h_2}{D_i}\right)^2 - 2\left(\frac{h_2}{D_i}\right)^3\right]\right) + L\, D_i^2\left(\frac{1}{4}\cos^{-1}\left(1-2\frac{h_2}{D_i}\right) - \left(\frac{1}{2}-\frac{h_2}{D_i}\right)\sqrt{\frac{h_2}{Di}-\left(\frac{h_2}{D_i}\right)^2}\right)}{\frac{\sqrt{8}\, L}{1.777\ C_d\ A_o\sqrt{g}}\left(D_1^{3/2} - \left(D_i - h_2\right)^{3/2}\right)}\quad\left[\frac{cm^3}{s}\right]$$

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(29)

$$q_1 = \frac{2\left(D_i^3\, C\, \frac{\pi}{12}\left[3\left(\frac{h_1}{D_i}\right)^2 - 2\left(\frac{h_1}{D_i}\right)^3\right]\right) + L\, D_i^2\left(\frac{1}{4}\cos^{-1}\left(1-2\frac{h_1}{D_i}\right) - \left(\frac{1}{2}-\frac{h_1}{D_i}\right)\sqrt{\frac{h_1}{Di}-\left(\frac{h_1}{D_i}\right)^2}\right)}{\frac{\sqrt{8}\,L}{1.777\, C_d\, A_o\sqrt{g}}\left(D_1^{3/2} - \left(D_i - h_1\right)^{3/2}\right)\left(0.02\left(D_i - \frac{3}{5}\frac{h_1^{1.04}}{0.87}\right)\right)} \quad \left[\frac{cm^3}{s}\right]$$

(30)

Appendix 7.2. Data of the theoretical and experimental heights to construct the graphs of discharge and filling of water.

Experimental data unloading tank 2

Operation Times (min)	Discharge heights (cm)
(9)	**(13)**
0	92.1
0.817	82.6
2.4	77.1
4.25	72.6
6.267	68.1
8.3	64.1
10.183	60.7
12.35	57.2
15.183	54.2
17.766	50.8
20.933	46.8
23.55	42.8
25.7	38.8
28.6	34.8
31.45	31.1
33.767	27.6
36.684	24.0
39.767	20.4
42.484	16.6
45.667	12.8
48.817	8.3
52.434	2.5
57.651	0

Theoretical model data, Appendix 7.3 unloading tank 2

Operation Times (s)	Operation Times (min)	Discharge heights (cm)
0	0	92.1
345.9	5.765	75.3017
691.8	11.53	63.4711
1037.7	17.295	53.4867
1383.6	23.06	44.6318
1729.5	28.825	36.5751
2075.4	34.59	29.0791
2421.3	40.355	21.9888
2767.2	46.12	15.1596
3113.1	51.885	8.40809
3459	57.65	1.08702

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Experimental data, filling tank 1		Theoretical model data, Appendix 7.3 filling tank 1		
Operation Times (min)	Fill heights (cm)	Operation Times (s)	Operation Times (min)	Fill heigts (cm)
It's the bottom of the tank (9)	It's the bottom of the tank (8)			
0	0	0	0	0.2
0.817	2.5	345.9	5.765	12.7516
2.4	8.3	691.8	11.53	23.2739
4.25	12.8	1037.7	17.295	33.2218
6.267	16.6	1383.6	23.06	42.7715
8.3	20.4	1729.5	28.825	51.9599
10.183	24	2075.4	34.59	60.7665
12.35	27.6	2421.3	40.355	69.1464
15.183	31.1	2767.2	46.12	77.0458
17.766	34.8	3113.1	51.885	84.3969
20.933	38.8	3459	57.65	91.0477
23.55	42.8			
25.7	46.8			
28.6	50.8			
31.45	54.2			
33.767	57.2			
36.684	60.7			
39.767	64.1			
42.484	68.1			
45.667	72.6			
48.817	77.1			
52.434	82.6			
57.651	92.1			

Appendix 7.3. The Wolfram Mathematica program
Nomenclature used in the program

A0 = 13.14.cm2: Internal area of the outlet and inlet connection on the bottoms of the tanks

A_W = Total wetted area of the torispherical tank, the cylindrical section plus the area of the two heads: cm^2

B = Hydraulic variable of the equation (16)

C = Variable for torispherical heads defined by the ASME Code

Cd =0.8: Coefficient of charge and discharge at the inputs and outputs at the base of the tanks

Di = 91.1: Internal diameter of torispherical tanks

D 0 = 92.1 cm ; Diámetro externo de los tanques toriesféricos

g = 980.7 cm / s^2 : acceleration of gravity

L =270 cm: Length of the cylindrical section of torispherical tanks

RC = D 0 : Inside crown radius

Rk = 3 × tt cm :Inside knuckle radius

tt = D 0 - Di cm : Espesor de la placa de los tanques

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Vcabezal1 = Volume of the head of tank 1 as a function of the height of the water: cm^3 Vcabezal2 = Volume of the head of tank 2 as a function of the height of the water: cm^3

Vcilindro1 = Volume of the cylindrical section for tank 1 as a function of the height of the water: cm^3

Vcilindro2 = Volume of the cylindrical section for tank 2 as a function of the height of the water; cm^3

qmodelo1 = Discharge volumetric flow inside tank 1: $cm^3/\ s$

qmodelo2 = Filling volumetric flow inside tank 2; $cm^3/\ s$

R1 = First order resistance at the outlet of tank 1; cm / (cm^3 / s)

R2 = First-order resistance at the input of tank 2; cm / (cm^3 / s)

z= Depth (z) of the heads: cm

ε = Hydraulic variable in the equation (24)

Wolfram Mathematica Program

```
D0 = 92.1;
Di = 91.1;
Rk = 3 * tt;
Rc = D0;
tt = D0 - Di;
Cd = 0.8;
A0 = 13.14;
g = 980.7;
```

$$Z = Rc - \sqrt{(Rc - Rk)^2 - \left(\frac{D0}{2} - tt - Rk\right)^2};$$

$$\epsilon = \sqrt{1 - \frac{4 * Z^2}{Di^2}};$$

$$B1 = \sqrt{1 + 12 * \left(\frac{h1[t]}{Di} - 0.5\right)^2};$$

$$B2 = \sqrt{1 + 12 * \left(\frac{h2[t]}{Di} - 0.5\right)^2};$$

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$$AW1 = L * Di * \text{ArcCos}\left[1 - 2 * \frac{h1[t]}{Di}\right] + \text{(arco coseno)}$$

$$2 * \left(\frac{\pi * Di^2}{8} * \left(\left(\frac{h1[t]}{Di} - 0.5\right) * B1 + 1 + \right.\right.$$

$$\left.\left.\frac{1}{4 * \epsilon} * \text{Log}\left[\left(\frac{4 * \epsilon * \left(\frac{h1[t]}{Di} - 0.5\right) + B1}{2 - \sqrt{3}}\right)\right]\right)\right); \text{(logaritmo)}$$

$$AW2 = L * Di * \text{ArcCos}\left[1 - 2 * \frac{h2[t]}{Di}\right] + \text{(arco coseno)}$$

$$2 * \left(\frac{\pi * Di^2}{8} * \left(\left(\frac{h2[t]}{Di} - 0.5\right) * B2 + 1 + \right.\right.$$

$$\left.\left.\frac{1}{4 * \epsilon} * \text{Log}\left[\left(\frac{4 * \epsilon * \left(\frac{h2[t]}{Di} - 0.5\right) + B2}{2 - \sqrt{3}}\right)\right]\right)\right); \text{(logaritmo)}$$

$$c = 0.30939 + 1.7197 * \frac{Rk - 0.06 * D0}{Di} - 0.16116 * \frac{tt}{D0} + 0.98997 * \left(\frac{tt}{D0}\right)^2;$$

$$Vcabezal1 = Di^3 * c * \frac{\pi}{12} * \left(3 * \left(\frac{h1[t]}{Di}\right)^2 - 2 * \left(\frac{h1[t]}{Di}\right)^3\right);$$

$$Vcabezal2 = Di^3 * c * \frac{\pi}{12} * \left(3 * \left(\frac{h2[t]}{Di}\right)^2 - 2 * \left(\frac{h2[t]}{Di}\right)^3\right);$$

$$Vcilindro1 = L * Di^2 * \left(\frac{1}{4} * \underbrace{ArcCos}_{\text{arco coseno}}\left[\left(1 - 2 * \frac{h1[t]}{Di}\right)\right] - \left(\frac{1}{2} - \frac{h1[t]}{Di}\right) * \sqrt{\frac{h1[t]}{Di} - \left(\frac{h1[t]}{Di}\right)^2}\right);$$

$$Vcilindro2 = L * Di^2 * \left(\frac{1}{4} * \underbrace{ArcCos}_{\text{arco coseno}}\left[\left(1 - 2 * \frac{h2[t]}{Di}\right)\right] - \left(\frac{1}{2} - \frac{h2[t]}{Di}\right) * \sqrt{\frac{h2[t]}{Di} - \left(\frac{h2[t]}{Di}\right)^2}\right);$$

$$Vmodelo1 = 2 * Vcabezal1 + Vcilindro1;$$

$$Vmodelo2 = 2 * Vcabezal2 + Vcilindro2;$$

$$qmodelo1 = \frac{Vmodelo1}{\frac{\sqrt{8} * L}{1.777 * Cd * A0 * \sqrt{g}} * \left(Di^{\frac{3}{2}} - (Di - h1[t])^{\frac{3}{2}}\right) * \left(0.02 * \left(Di - \frac{3}{5} * \frac{h1[t]^{1.04}}{0.87}\right)\right)};$$

$$qmodelo2 = \frac{Vmodelo2}{\frac{\sqrt{8} * L}{1.777 * Cd * A0 * \sqrt{g}} * \left(Di^{\frac{3}{2}} - (Di - h2[t])^{\frac{3}{2}}\right)};$$

$$R1 = \frac{(h1[t] + 134) - h2[t]}{qmodelo1};$$

$$R2 = \frac{(h1[t] + 134) - h2[t]}{qmodelo2};$$

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```
sol = NDSolve[{-AW1 * h1'[t] == 1/R1 * ((h1[t] + 134) - h2[t]),
      resolvedor diferencial numérico
AW2 * h2'[t] == 1/R2 * ((h1[t] + 134) - h2[t]),
h1[0] == 91.1, h2[0] == 0.2}, {h1[t], h2[t]}, {t, 3459}]
Plot[Evaluate[h1[t] /. sol], {t, 0, 3459},
repr··· evalúa
 AxesLabel → {"t[s]", "h1[cm]"}]
 etiqueta de ejes
TableForm[{{"t"  s,  "h1"  cm}}, TableDepth → 2]|
forma de tabla                  profundidad de tabla
Table[{t, h1[t] /. sol}, {t, 0, 3459, 345.9}] // TableForm
tabla                                           forma de tabla
Plot[Evaluate[h2[t] /. sol], {t, 0, 3459},
repr··· evalúa
 AxesLabel → {"t[s]", "h2[cm]"}]
 etiqueta de ejes
TableForm[{{"t"  (s),  "h2"  (cm)}}, TableDepth → 2]
forma de tabla                     profundidad de tabla
Table[{t, h2[t] /. sol}, {t, 0, 3459, 345.9}] // TableForm
tabla                                           forma de tabla
TableForm[{{"t"  (s),  "Vmodelo2"  (cm^3)}}, TableDepth → 2]
forma de tabla                            profundidad de tabla
Table[{t, Vmodelo2 /. sol}, {t, 0, 3459, 345.9}] // TableForm
tabla                                              forma de tabla
TableForm[{{"t"  (s),  "AW1"  (cm^2)}}, TableDepth → 2]
forma de tabla                       profundidad de tabla
Table[{t, AW1 /. sol}, {t, 0, 3459, 345.9}] // TableForm
tabla                                        forma de tabla
TableForm[{{"t"  (s),  "AW2"  (cm^2)}}, TableDepth → 2]
forma de tabla                       profundidad de tabla
Table[{t, AW2 /. sol}, {t, 0, 3459, 345.9}] // TableForm
tabla                                        forma de tabla
```

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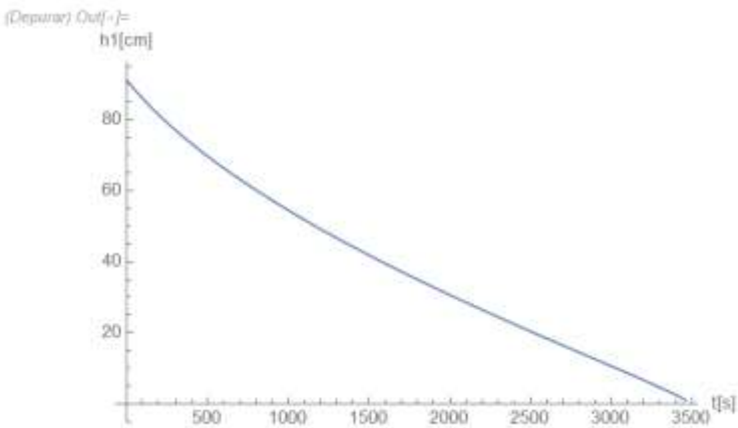
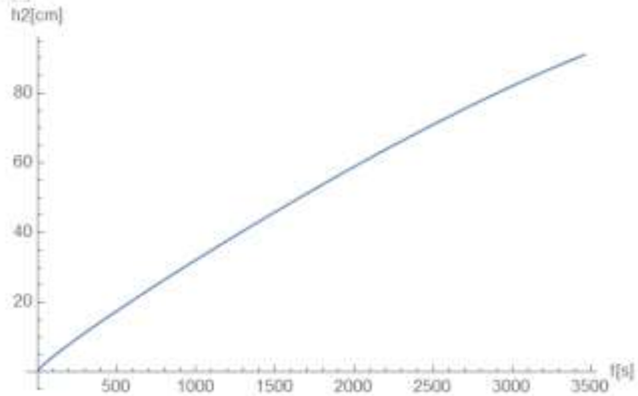
```
TableForm[{{"t"  (s),  "qmodelo2"  (cm³)}}, TableDepth → 2]
forma de tabla                              profundidad de tabla

Table[{t, qmodelo2 /. sol}, {t, 0, 3459, 345.9}] // TableForm
tabla                                               forma de tabla

TableForm[{{"t"  s,  "R1"}}, TableDepth → 2]
forma de tabla                profundidad de tabla

Table[{t, R1 /. sol}, {t, 0, 3459, 345.9}] // TableForm
tabla                                         forma de tabla

{{h1[t] → InterpolatingFunction[ Domain: {{0., 3460.}} Output: scalar ][t],

  h2[t] → InterpolatingFunction[ Domain: {{0., 3460.}} Output: scalar ][t]}}
```



(Depurar) Out[∘]//TableForm=

"t" (s)	"h1" (cm)
0.	91.1
345.9	75.3017
691.8	63.4711
1037.7	53.4867
1383.6	44.6381
1729.5	36.5751
2075.4	29.0791
2421.3	21.9888
2767.2	15.1596
3113.1	8.40809
3459.	1.08702

(Depurar) Out[∘]=



(Depurar) Out[∘]//TableForm=

"t" (s)	"h2" (cm)
0.	0.2
345.9	12.7516
691.8	23.2739
1037.7	33.2218
1383.6	42.7715
1729.5	51.9599
2075.4	60.7665
2421.3	69.1464
2767.2	77.0458
3113.1	84.3969
3459.	91.0477

(Depurar) Out[∘]//TableForm=

"t" (s)	"Vmodelo2" (cm^3)
0.	308.615
345.9	155 205.
691.8	371 456.
1037.7	611 551.
1383.6	858 428.
1729.5	1.09937×10^6
2075.4	1.32346×10^6
2421.3	1.52114×10^6
2767.2	1.68386×10^6
3113.1	1.80298×10^6
3459.	1.86282×10^6

(Depurar) Out[∘]//TableForm=

"t" (s)	"AW1" (cm^2)
0.	94 884.9
345.9	69 855.1
691.8	60 050.5
1037.7	52 850.1
1383.6	46 779.6
1729.5	41 188.7
2075.4	35 692.4
2421.3	29 999.4
2767.2	23 775.9
3113.1	16 362.6
3459.	3977.72

(Depurar) Out[∘]//TableForm=

"t" (s)	"AW2" (cm^2)
0.	536.781
345.9	21 322.1
691.8	31 078.7
1037.7	38 779.8
1383.6	45 500.9
1729.5	51 792.9
2075.4	58 035.4
2421.3	64 511.1
2767.2	71 504.4
3113.1	79 579.3
3459.	93 691.3

(Depurar) Out[∘]//TableForm=

"t" (s)	"qmodelo2" (cm^3)
0.	82.6052
345.9	675.43
691.8	915.137
1037.7	1091.47
1383.6	1232.44
1729.5	1348.16
2075.4	1443.21
2421.3	1519.85
2767.2	1579.09
3113.1	1620.69
3459.	1641.09

(Depurar) Out[ ]//TableForm=

"t" (s)	"R1" $\left(\frac{cm}{cm^3/s}\right)$
0.	0.0434306
345.9	0.073665
691.8	0.0934666
1037.7	0.108094
1383.6	0.119427
1729.5	0.128543
2075.4	0.136408
2421.3	0.144382
2767.2	0.155396
3113.1	0.179971
3459.	0.4108