



# STUDY OF VELOCITY DISTRIBUTION OVER A NON-LINEAR STRETCHING IN PRESENCE OF VISCO-ELASTIC LIQUIDS

**Dr. Naveen Kumar N P**

Department of Mathematics, Government First Grade College, Malleshwaram 18<sup>th</sup> Cross, Bangalore-560012

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## ABSTRACT

The study deals with the investigation of velocity of a boundary layer flow of a visco-elastic liquid over a non-linear stretching sheet. For analyzing velocity profiles have been employed for reducing the nonlinear model equation to a system of ordinary differential equations by employing analytical method velocity distribution are studied.

It is found that with the increase of magnetic field intensity the fluid velocity at a particular point of the sheet the fluid velocity decreases.

**KEY WORDS:** *Visco-elastic, Newtonian fluid, viscosity*

## INTRODUCTION

In reality most of liquids are non-Newtonian in nature, which are abundantly used in many industrial applications, such as in the manufacture of plastic films and artificial fibers, aerodynamic extrusion of plastic sheets, cooling of metallic sheets in a cooling bath, crystal growing, liquid film condensation process, continuous polymer sheet extrusion, heat treated materials traveling between a feed roll, wind up roll or on a conveyer belt, geothermal reservoirs and petroleum industries. In view of this, the study of visco-elastic boundary layer flow problem has been further channelized to non-Newtonian fluid flow Hence in this paper investigate the non-Newtonian visco-elastic boundary layer flow past a stretching sheet and velocity distribution characteristics are examined for two different kinds of function  $\lambda$  visco-elastic parameter and  $v_c$  velocity component.

## MATHEMATICAL FORMULATION

We consider a steady, two-dimensional boundary layer flow of an incompressible liquid subjected to a transverse effect (see the Fig. 2). The liquid is at rest and the motion is affected by pulling the sheet on both ends with equal forces parallel to the sheet and a speed  $u$ , which varies quadratically with distance from the slit as  $u = cx + dx^2$ .

The steady, two-dimensional conservation of mass and the momentum boundary layer equations for the quadratically stretching sheet problem involving non-Newtonian liquids are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \nu \lambda^* \left( u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} \right), \quad (2)$$

Subject to the boundary conditions:

$$\left. \begin{aligned} u &= cx + dx^2 & \text{at } y &= 0, \\ v &= v_c + \delta x & \text{at } y &= 0, \\ u &= 0 & \text{as } y &\rightarrow \infty. \end{aligned} \right\} \quad (3)$$



Here  $u$  and  $v$  are the components of the liquid velocity in the  $x$  and  $y$  directions, respectively,  $\mu$  is the dynamic viscosity and  $\nu$  is the kinematic viscosity. Further, we assume  $d$  and  $\delta$  quite small that facilitates the assumption of a weakly two-dimensional flow and  $\lambda^*$  the visco-elastic parameter.

We now make the equations and boundary conditions dimensionless using the following definition:

$$(X, Y) = \sqrt{\frac{c}{\nu}}(x, y), \quad (U, V, V_c) = \frac{(u, v, v_c)}{\sqrt{c\nu}},$$

$$\beta^* = \frac{d}{c} \sqrt{\frac{\nu}{c}}, \quad \delta^* = \frac{\delta}{2c}.$$
(4)

Equations (1) and (2) take the non-dimensional form as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{5}$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + \lambda_1 \left( U \frac{\partial^3 U}{\partial X \partial Y^2} + V \frac{\partial^3 U}{\partial Y^3} \right),$$
(6)

Where  $\lambda_1 = \frac{\lambda^*}{c}$  (visco-elastic parameter).

Introducing the stream function  $\psi(X, Y)$  as

$$U = \frac{\partial \psi}{\partial Y}, \quad V = -\frac{\partial \psi}{\partial X}, \tag{7}$$

we get from equation (6) the following equation:

$$\frac{\partial^3 \psi}{\partial Y^3} + \frac{\partial \psi}{\partial X} \frac{\partial^2 \psi}{\partial Y^2} - \frac{\partial \psi}{\partial Y} \frac{\partial^2 \psi}{\partial X \partial Y} + \lambda_1 \left( \frac{\partial \psi}{\partial Y} \frac{\partial^4 \psi}{\partial X \partial Y^3} - \frac{\partial \psi}{\partial X} \frac{\partial^4 \psi}{\partial Y^4} \right) = 0.$$
(8)

The boundary conditions to be satisfied by  $\psi$  can be obtained from the equations (3), (4) and (7) as:

$$\left. \begin{aligned} \frac{\partial \psi}{\partial Y} &= X + \beta^* X^2 && \text{at } Y = 0, \\ -\frac{\partial \psi}{\partial X} &= 2V_c + 2\delta^* X && \text{at } Y = 0, \\ \frac{\partial \psi}{\partial Y} &= 0 && \text{as } Y \rightarrow \infty. \end{aligned} \right\} \tag{9}$$



In order to convert the partial differential equation (8) in to ordinary differential equations the following similarity transformation are used:

$$\psi = Xf(Y) - \beta^* X^2 f'(Y), \tag{10}$$

Substituting equation (10) in the equation (8), we get the ordinary differential equation in the form:

$$f''' + ff'' - (f')^2 + \lambda_1 (ff''' - f'f''') = 0, \tag{11}$$

Where  $\lambda_1$  is the dimensionless visco-elastic parameter.

The boundary conditions, for solving equation (11) for f, can be obtained from equations (9) in the form:

$$f(0) = -2V_c, \quad f'(0) = 1, \quad f'(\infty) = 0. \tag{12}$$

The solution of equation (11) subject to (12) is

$$f(Y) = \frac{1}{s} (1 - e^{-sY}) - 2V_c, \tag{13}$$

Where 's' satisfies the equation

$$s^3(\lambda_1 V_c) - s^2(1 + \lambda_1) - (sV_c - 1) = 0. \tag{14}$$

In the above equation put  $\lambda_1 = 0$  the equation (14) reduces to  $s^2 + sV_c - 1 = 0$  which is same as the equation we used in the Newtonian liquids.

Substituting equation (10) into equation (7), we can get velocity components U and V as:

$$U = Xf'(Y) - \beta^* X^2 f''(Y), \tag{15}$$

$$V = -f(Y) + 2\beta^* Xf'(Y). \tag{16}$$

Having obtained the velocity distribution we discuss the heat transport in the aforementioned forced convective flow due a stretching sheet.

## RESULTS AND DISCUSSION

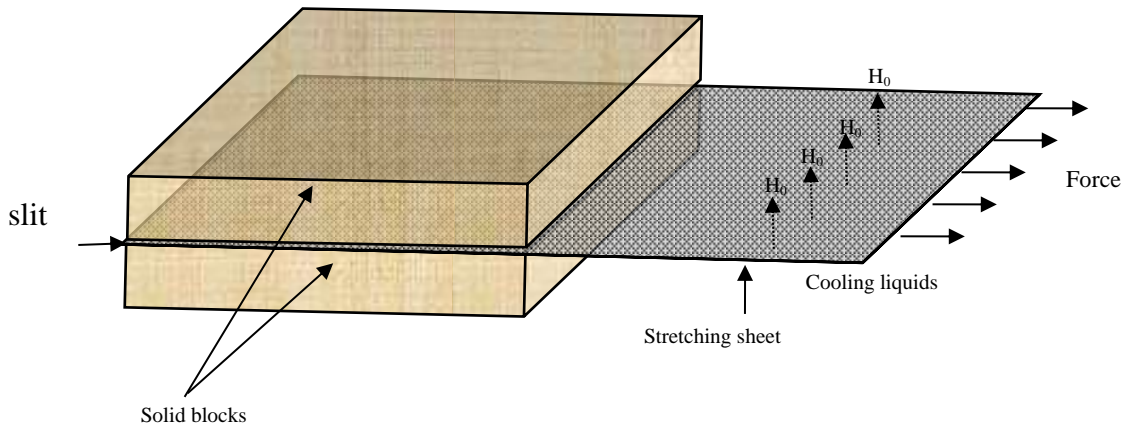
A boundary layer problem for momentum in viscoelastic liquid flow over a stretching sheet is examined in this paper. Parameter s which is function of the  $V_c$  and  $\lambda_1$  (see table 1) contributes to the slope of above exponentially decreasing velocity profiles. Thus 's' is an important parameter in the present study.

It is clear from equation 14 that s, which is function of the  $V_c$  and  $\lambda_1$  see the table 1 contributes to the slope of above exponentially decreasing velocity profiles. Thus 's' is an important parameter in the present study. From Figs. 3 and 4 it is evident that s is an increasing function of  $V_c$  and  $\lambda_1$  thus implying that increasing  $V_c$  and  $\lambda_1$  gives us steeper gradients in the axial and transverse velocity profiles.

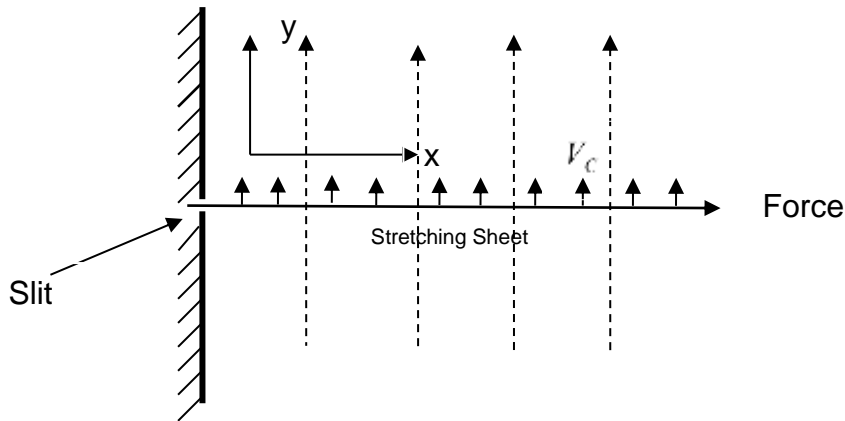
## CONCLUSION

The effect of two different kinds of function  $\lambda$  visco-elastic parameter and  $v_c$  velocity component studied in this paper increasing of the visco-elastic and velocity component which will contributes to the slope of exponentially decreasing velocity distribution in the flow region the strength of viscosity should be as mild as possible which will increase the speed of the flow of the liquid.

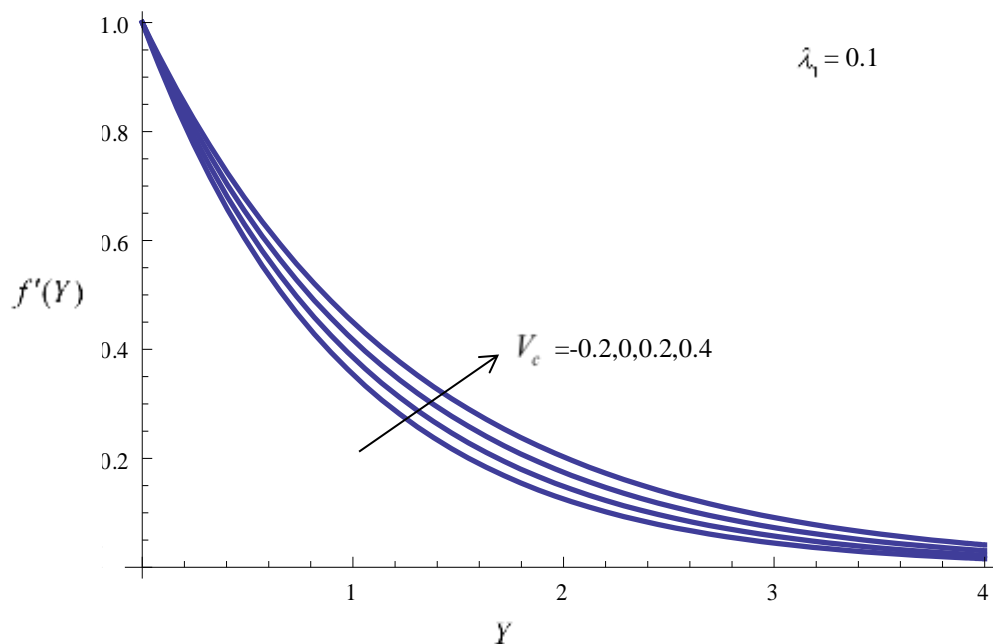
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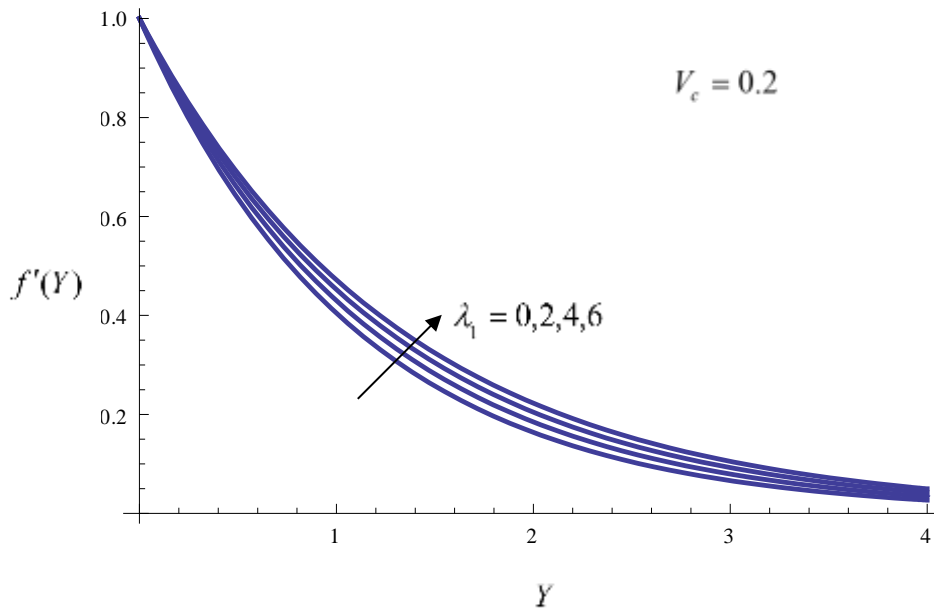


**Fig. 1 : Schematic of the 3-D stretching sheet problem.**



**Fig. 2 : Schematic of the 2-D stretching sheet problem.**





**Fig. 4:** Effect of  $\lambda_1$  on velocity profile  $f'(Y)$ .

**Table 1** Values of  $s$  for different values  $\lambda_1$  and  $V_c$

$\lambda_1$	$V_c$	$s$
0	0.2	0.904988
0.1		0.873176
0.2		0.844208
0.3		0.817719
0.4		0.793399
0.5		0.770987
0	-0.2	1.10499
0.1		1.03802
0.2		0.982586
0.3		0.935613
0.4		0.895085
0.5		0.859613
$V_c$	$\lambda_1$	$s$
-0.4	0.1	1.12539
		1.03802
		0.953463
		0.873176
		0.798309
		0.758983
-0.2	0.3	0.992148
		0.935613
		0.877058
		0.817719
		0.758983
		0.700000



## REFERENCE

1. Sakiadis, B. C. *Boundary-layer behavior on continuous solid surfaces I: The boundary layer on a equations for two dimensional and axisymmetric flow*, *A.I.Ch.E. Journal*, 7, 1961a, 26.
2. Hayat, T., Qasim, M. and Abbas, Z. *Three-dimensional flow of an elastic-viscous fluid with mass transfer*, *Int. Journal of Numerical methods in fluids*, 66, 2011, 194.
3. Abbas, Z., Hayat, T. Sajid, M. and Asghar, S. *Unsteady flow of a second grade fluid film over an unsteady stretching sheet*, *Mathematical and computer modelling*, 48, 2008, 518.
4. Abel, M. S., Mahantesh, M. N. Vajaravelu, K. and Chiu-on Ng. *Heat transfer over a non-linearly stretching sheet with non-uniform heat source and variable wall temperature*, *Int. Journal of Heat and Mass transfer*, 54, 2011, 4960.
5. Hady, F. M., R. A. Mohamed, and Hillal M. ElShehabey. "Thermal Radiation, Heat Source/Sink and Work Done by Deformation Impacts on MHD Viscoelastic Fluid over a Nonlinear Stretching Sheet." *World Journal of Mechanics* 04, 2013, 203.
6. Mishra and Sujata Panda. *Mixed convective radioactive heat transfer in a particle-laden boundary layer fluid over an exponentially stretching permeable surface*. In *AIP Conference Proceedings*, vol. 2435, 2022, 145.
7. A Samanta, *Wave dynamics of a visco-elastic liquid*, *International Journal of Engg.Science*, Vol.193, 2023, 103954.