



STUDY OF VELOCITY DISTRIBUTIONS FOR A HYDROMAGNETIC FLOW OVER A NON-LINEAR STRETCHING SHEET

Naveen Kumar N P^a, Dinesh P A^b, Dinesh Kumar S T^c

^a Department of Mathematics, Government First Grade College, Malleshwaram 18th Cross, Bangalore-560012. India.

^b Department of Mathematics, MSRIT, Bangalore. Karnataka.

^c Department of Mathematics, Government Science College, Chitradurga, Karnataka.

ABSTRACT

The study deals with the investigation of velocity of a boundary layer flow of a Newtonian liquid over an non-linear stretching sheet in presence of magnetic field. For analyzing velocity profiles have been employed for reducing the nonlinear model equation to a system of ordinary differential equations by employing analytical method velocity distribution are studied. It is found that with the increase of magnetic field intensity the fluid velocity at a particular point of the sheet the fluid velocity decreases.

KEY WORDS : Stretching sheet, Magnetic field, Stream function

INTRODUCTION

Polymer extrusion processes involve the stretching of thin films by equal and opposite forces, the entire system housed in cooling liquids. The rate of cooling and the nature of the coolant determine the property of the thin film. In this paper we study the non-linear flow of a Newtonian liquid due to a sheet that is stretched between two solid blocks (Fig. 1) and velocity distribution in the boundary layer flow of the stretching sheet.

MATHEMATICAL FORMULATION

We consider a steady two-dimensional boundary layer flow of an incompressible liquid subjected to a transverse magnetic field (see the Fig. 1). The liquid is at rest and the motion is effected by pulling the sheet on both ends with equal forces parallel to the sheet and a speed u , which varies quadratically with distance from the slit as $u = cx + dx^2$. The flow field is subjected to a transverse uniform magnetic field H_0 is imposed in the vertical direction y-axis. It is assumed that magnetic field is negligibly small.

The steady two-dimensional conservation of mass and the momentum boundary layer equations for the quadratically stretching sheet problem involving Newtonian liquids with transverse magnetic field are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \mu_m^2 \sigma H_0^2 u, \quad (2)$$

where x and y represent horizontal and transverse directions respectively and u, v are components of the liquid velocity in x and y directions, μ is the dynamic viscosity, H_0 is the magnetic field and ν is the kinematic viscosity.

The boundary conditions are considered as :

$$\left. \begin{aligned} u &= cx + dx^2 & \text{at } y &= 0, \\ v &= v_c + \delta x & \text{at } y &= 0, \\ u &= 0 & \text{as } y &\rightarrow \infty. \end{aligned} \right\} \quad (3)$$



We assume c , d and δ is quite small that facilitates the assumption of a weakly two-dimensional flow. The constant v_c represents suction velocity across the stretching sheet when $v_c < 0$, it is blowing velocity when $v_c > 0$ and it represents impermeability of the wall when $v_c = 0$.

We now make the equations and boundary conditions dimensionless using the following

$$(X, Y) = \sqrt{\frac{c}{\nu}}(x, y), \quad (U, V, V_c) = \frac{(u, v, v_c)}{\sqrt{c\nu}},$$

$$\beta^* = \frac{d}{c} \sqrt{\frac{\nu}{c}}, \quad \delta^* = \frac{\delta}{2c},$$
(4)

substituting dimensionless quantity (4) in the equations (1) and (2) and it takes the non-dimensional form as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad (5)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} - QU, \quad (6)$$

where $Q = \frac{\mu_m^2 \sigma H_0^2}{c}$ is a Chandrasekhar number ($M_n = \sqrt{Q}$ is called Hartmann number),

Now we introducing the stream function $\psi(X, Y)$ as

$$U = \frac{\partial \psi}{\partial Y}, \quad V = -\frac{\partial \psi}{\partial X}, \quad (7)$$

which satisfies the continuity equation (5), by substituting (7) in (6) we get following partial differential equation

$$\frac{\partial^3 \psi}{\partial Y^3} + \frac{\partial \psi}{\partial X} \frac{\partial^2 \psi}{\partial Y^2} - \frac{\partial \psi}{\partial Y} \frac{\partial^2 \psi}{\partial X \partial Y} - Q \frac{\partial \psi}{\partial Y} = 0. \quad (8)$$

Similarly, substituting (7) in the boundary condition (3) using dimensionless quantities (4) which obtained in the following form :

$$\left. \begin{aligned} \frac{\partial \psi}{\partial Y} &= X + \beta^* X^2 & \text{at } Y = 0, \\ -\frac{\partial \psi}{\partial X} &= 2V_c + 2\delta^* X & \text{at } Y = 0, \\ \frac{\partial \psi}{\partial Y} &= 0 & \text{as } Y \rightarrow \infty. \end{aligned} \right\} \quad (9)$$

The solution to equation (8) subject to equations (9) may be taken as

$$\psi = Xf(Y) - \beta^* X^2 f'(Y). \quad (10)$$

Substituting equation (10) into equation (8) obtain the following ordinary differential equation

$$f''' + ff'' - (f')^2 - Qf' = 0. \quad (11)$$

The boundary conditions for solving equation (11) for dimensionless stream function f , can be obtained from equations (9) in the form



$$f(0) = -2V_c, \quad f'(0) = 1, \quad f'(\infty) = 0. \quad (12)$$

The solution of equations (11) subject to (12) is

$$f(Y) = \frac{1}{s}(1 - e^{-sY}) - 2V_c, \quad (13)$$

where 's' is given by

$$s = -V_c + \sqrt{V_c^2 + (1+Q)}. \quad (14)$$

Substituting equation (10) into equation (7), we can get velocity components U and V as

$$U = Xf'(Y) - \beta^* X^2 f''(Y), \quad (15)$$

$$V = -f(Y) + 2\beta^* Xf'(Y). \quad (16)$$

Above equation is shows that velocity distribution convective flow due a stretching sheet in presence of a transverse magnetic field.

RESULTS AND DISCUSSION

In this problem we investigate the velocity distribution in a Newtonian liquid over a stretching sheet in the presence of a transverse magnetic field. Similarity solution is obtained for the velocity distribution. It is clear from equation (14) that s , which is function of the V_c and Chandrasekhar number Q contributes to the slope of the exponentially decreasing velocity profiles. Thus 's' is an important parameter in the present study. From the fig 3 it is apparent that the transverse velocity profile decays faster than the axial velocity profile for increasing V_c . The effect of magnetic field is to provide rigidity to the electrically conducting liquid.

CONCLUSION

The effect of magnetic field is to provide rigidity to the electrically conducting liquid and the magnetic field is perpendicular to the velocity field.

The effect of Chandrasekhar number Q is to increase the velocity distribution in the flow region the strength of external magnetic field should be as mild as possible for effective cooling of the stretching sheet.

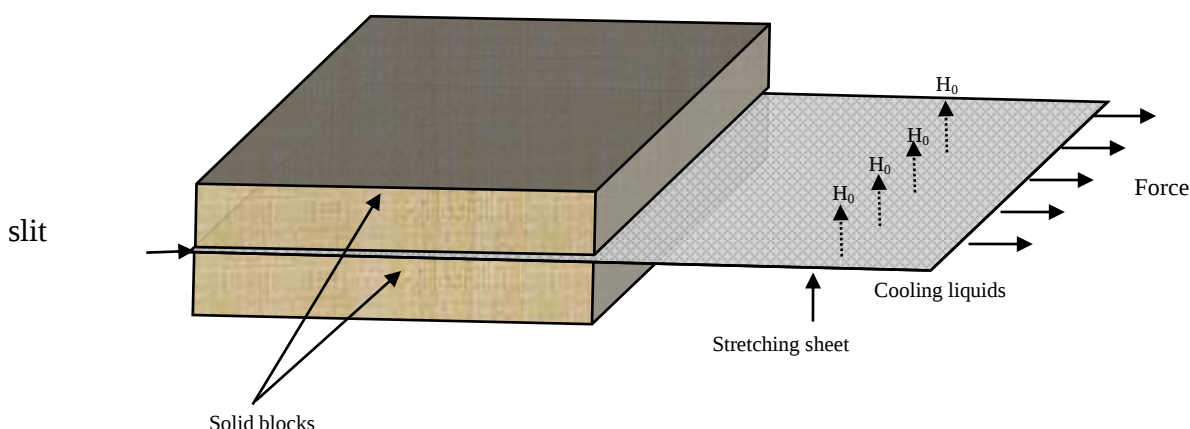


Fig. 1 : Schematic of the 3-D stretching sheet problem.

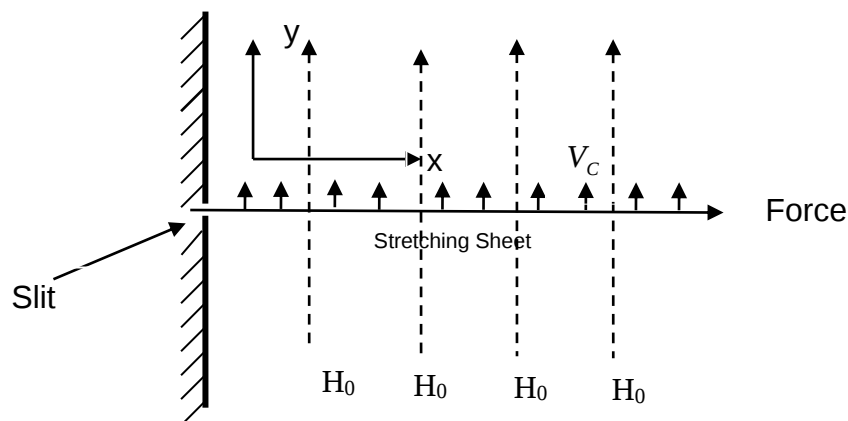


Fig. 2 : Schematic of the 2-D stretching sheet problem.

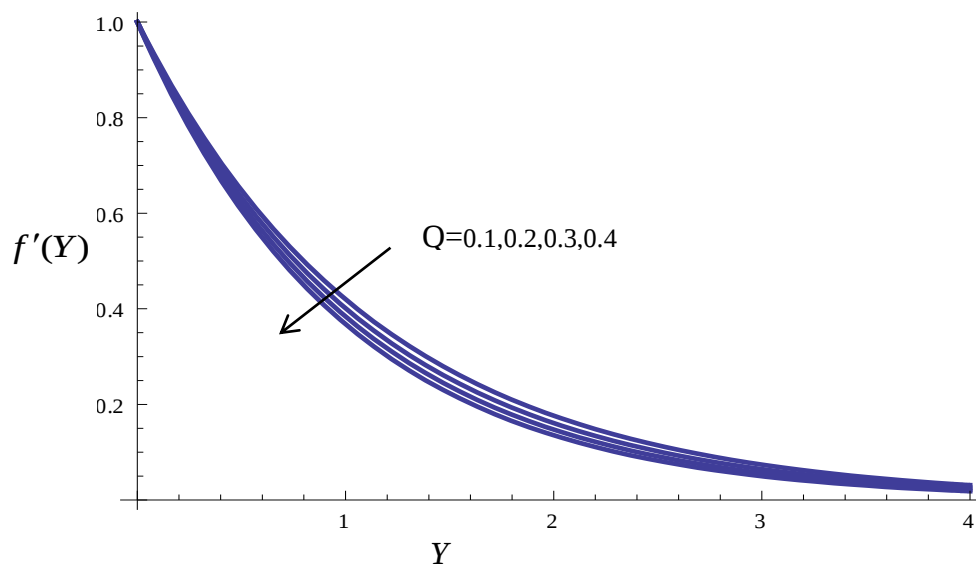


Fig. 3 : Effect of Q on velocity profile $f'(Y)$.

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