SJIF Impact Factor (2024): 8.675| ISI I.F. Value: 1.241| Journal DOI: 10.36713/epra2016 ISSN: 2455-7838(Online)
EPRA International Journal of Research and Development (IJRD)
Volume: 9 | Issue: 7 | July 2024 - Peer Reviewed Journal

ON Q*g-CLOSED SETS

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ABSTRACT
*The author introduced the notion of Q*g-closed sets in the paper entitled "Q*g-closed sets in topological space" by P. Padma and S. Uday Kumar [5]. However, there is a false theorem, namely Theorem 3.4. The correct statement of Theorem 3.4 is mentioned in this paper with correct proof and gave a counter-example in support of this theorem.*
KEYWORDS: *regular open, π-open, g-closed, Q*-closed and Q*g-closed sets. 2020 AMS Classification: 54A05.*

1. INTRODUCTION
In 1958, K. Kuratowski [**2**] introduced and investigated the notion of regular open sets and obtained their properties. In 1968, V. Zaitsev [**7**] introduced and studied the concept of π-open sets and obtained their basic properties. In 1970, Levine [**3**] initiated the investigation of g-closed sets in topological spaces, since then many modifications of g-closed sets were defined and investigated by a large number of topologists. In 1993, N. Palaniappan and K. C. Rao [**6**] introduced the concept of rg-closed sets and obtained some properties of rg-closed sets in topological spaces. In 2000, Dontchev and Noiri [**1**] studied the concept of πg-closed sets and obtained some basic properties. In 2010, M. Murugalingam and N. Lalitha [**4**] introduced and studied the concept of Q*-open sets and obtained some properties of Q*-open sets in topological spaces. In 2015, P. Padma and S. Udaya Kumar [**5**] introduced the notion of Q*g-closed sets in topological spaces and obtained some properties of Q*g-closed sets.

In this paper, we study the notion of Q*g-closed sets in the paper entitled "Q*g-closed sets in topological space" by P. Padma and S. Uday Kumar [**5**], published in Int. J. of Adv. Res. in Engg. and Appl. Sci. However, there is a false theorem, namely Theorem 3.4. The correct statement of Theorem 3.4 is mentioned in this paper with correct proof and gave a counter-example in support of this theorem.

2. PRELIMINARIES
Throughout the present paper, spaces always mean topological spaces on which no separation axioms are assumed unless explicitly stated and $f : (X, \Im) \to (Y, \sigma)$ (or simply $f : X \to Y$) denotes a function f of a space $(X, \Im)$ into a space (Y, σ). Let A be a subset of a space X. The closure and the interior of A are denoted by cl(A) and int(A), respectively. A subset A of a space X is said to be **regularly open** or **open domain** [**2**] if it is the interior of its own closure or, equivalently, if it is the interior of some closed set. A complement of an open domain subset of X is called **closed domain** (or A subset A is said to be **regular open** [**2**] (resp. **regular closed**) if $A \subset int(cl(A))$ (resp. $A \subset cl(int(A))$). The finite union of regular open sets is said to be π-**open [7]**. The complement of a π-open set is said to be **π-closed.**

regular open → π-open → open

Where none of the implications is reversible [**1**].

2.1 Definition. A subset A of a topological space $(X, \Im)$ is said to be
(1) **g-closed** [**3**] if $cl(A) \subset U$ whenever $A \subset U$ and $U \in \Im$.
(2) π**g-closed** [**1**] if $cl(A) \subset U$ whenever $A \subset U$ and U is π-open.
(3) **rg-closed** [**6**] if $cl(A) \subset U$ whenever $A \subset U$ and U is regular open.

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In the definition of g-closed set, we use U as an open set, in the definition of πg-closed set, we use U as a π-open set and in the definition of rg-closed set, we use U as a regular open set.

The complement of a g-closed (resp. π-closed, rg-closed) set is said to be **g-open** (πg-open, rg-open**).**

2.2 Remark. We summarize the fundamental relationships between several types of generalized closed sets in the following diagram. None of the implications is reversible [**1**].

closed → g-closed → πg-closed → rg-closed

3. Q*g-CLOSED SETS

3.1 Definition. A subset A of a topological space X is said to be Q***g-closed** [**5**] if cl(A) ⊂ U whenever A ⊂ U and U is Q*-open.

3.2 Remark. Every Q*-open set is open.

In the definition of Q*g-closed set, we use U as a Q*-open set but in the definition of g-closed set, we use U as an open set. So the correct statement of the **Theorem 3.4** is as follows:

3.3 Theorem. Every g-closed set is Q*g-closed set.
Proof. Let A be g-closed set in X and A ⊂ U where U is Q*-open set. Since every Q*-open set is open and A is g-closed set, cl(A) ⊂ U. Hence A is Q*g-closed set.

Converse of the above theorem is not true and is shown by the following example:

2.3 Example. Let X = {a, b, c} and ℑ = {ϕ, X, {a}, {b}, {a, b}}. Then
(i) closed sets are : ϕ, X, {c}, {a, c}, {b, c}.
(i) Q*-closed sets are : ϕ, {c}.
(i) g-closed sets are : ϕ, X, {c}, {a, c}, {b, c}.
(i) Q*g-closed sets are : ϕ, X, {b}, {c}, {a, c}, {b, c}.

In the above example, every g-closed set is Q*g-closed but converse is not true. In the above example, the set A = {b} is Q*g-closed but not g-closed.

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