SJIF Impact Factor: 6.260| ISI I.F.Value:1.241 Volume: 4 | Issue: 9 | September | 2019 ISSN: 2455-7838(Online)
EPRA International Journal of Research and Development (IJRD)
Peer Reviewed Journal

A STUDY ON PRODUCTION OF ELECTRODES USING DIFFERENTIAL EQUATION

Elsy Gnanapushpam A[1]
Assistant Professor, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore, Tamil Nadu, India.

Harini A[2]
UG Scholar, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore,Tamil Nadu, India.

Naveen C[3]
UG Scholar, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore,Tamil Nadu, India.

Dineshkumar K[4]
UG Scholar, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore,Tamil Nadu, India.

Janani R[5]
UG Scholar, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore,Tamil Nadu, India.

ABSTRACT
The main aim of the paper is to use the differential equation to solve the real world problems. Differential equation have a remarkable ability to predict the world around us. They are used in wide variety of disciplines, from biology, economics, physics, chemistry and engineering. Also they are used to calculate the rate of change of time, size, kilograms, current of the electrodes.
KEY WORDS: *Size, Kilograms, Current, Time, Electrodes.*

INTRODUCTION

A differential equation is a mathematical equation that relates some function with its derivatives. In applications, A differential equation, the functions usually represent physical quantities, the derivatives represent their rates of change, and the differential equation defines a relationship between the two.The theory of dynamical systems puts emphasis on qualitative analysis of system described by differential equations, while many numerical methods have been developed to determine solutions with a given degree of accuracy.

CHARACTERISTIC POLYNOMIAL HAS COMPLEX CONJUGATE ROOTS

Suppose that the characteristic polynomial has complex roots a+ib and a-ib, where a and b are real. These are distinct roots, so the general solution can be written:

$$y(t) = C_1 e^{(a+ib)t} + C_2 e^{(a-ib)t}$$

Recall, Euler's identity: $e^{iy} = \cos(y) + isin(y)$

Using this identity, we have: $e^{(a+ib)t} = e^{at} e^{ibt} = e^{at}(\cos(bt) + isin(bt))$

Similarly, we have $e^{(a-ib)t} = e^{at}e^{-ibt} = e^{at}(\cos(bt) - isin(bt))$

Substituting, these two expressions into the general solution we have

$$y(t) = C_1e^{at}(\cos(\mathrm{bt})) + \mathrm{isin}(\mathrm{bt})) + C_2e^{at}(\cos(\mathrm{bt}) - \mathrm{isin}(\mathrm{bt})$$

APPLICATIONS

The study of differential equations is a wide filed in pure and applied mathematics, physics, and engineering etc. Uniqueness of solutions in *pure mathematics*, Rigorous justification of the methods for approximating solutions in *applied mathematics*, Euler – Lagrange equation in classical mechanics, Radioactive decay in *nuclear physics*, Verhulst equation – biological population growth, von Bertalanffy model – biological individual growth in *biology.*

PROBLEM:1

An electrode is produced in 3mins and 6mins respectively, the size of electrodes are 2mm & 5mm. Find the required kilograms of these electrodes.

SOLUTION:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = +5y = 0 - - - (1)$$
$$D^2 + 2D + 5Y = 0 - - - (2)$$
$$\mathrm{m} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

m $=-1 \pm 2i$. Hence the roots are conjugate complex numbers, $a \pm bi$ where $a = -1, b = 2$

Then the general solution of given differential equation is,

$$y = e^{ax}(c_1sinbx + c_2cosbx)$$
$$y = e^{-x}(c_1sin2x + c_2cos2x) - - - -(3)$$
$$\frac{dy}{dx} = e^{-x}(2c_1cos2x - 2c_2sin2x) + (c_1sin2x + 1c_2cos2x)e^{-1x} - 1$$
$$\frac{dy}{dx} = e^{-x}[(-1c_1 - 2c_2)sin2x + (2c_1 - 1c_2)cos2x] - - -(4)$$

Now apply the initial condition to equation(3)

$$3 = e^0(c_1sin0 + c_2cos0)$$
$$c_2 = 3 - - - - - - - - - - - - - -(5)$$

Now apply the initial condition to equation (4) i.e y'(0)=6,x'(0)=0 we get, [sin(0)=0,cos(0)=1]

$$6 = (1c_1 - 2c_2)\sin 0 + (2c_1 - 1c_2)\cos 0$$
$$2c_1 - c_2 = 6 - - - - - - - - - - - - (6)$$
$$2c_1 - 3 = 6$$
$$c_1 = 4.5 \text{ (or) } c_1 = 4 - - - - - - - - - - (7)$$
$$y = e^{-x}[c_1sin2x + c_2cos2x]$$
$$y = e^{-x}[4sin2x + 3cos2x]$$

Hence one electrode of 4kgs is produced in 6mins and another electrode of 3kgs in 3mins.

PROBLEM:2

Using the Frobenius method obtain the recurrence and find the series is either in an increasing order or in a decreasing order in the general solution of the given D.E. Use the time value of an electrode produces i.e.4 mins &2 mins respectively. [Hints: use the chemical values of the electrode in an obtained recurrence relation]

SOLUTION:

$$4\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0 - - - - - - - - - - (*)$$

By assuming the general equation of Frobenius equation,

$$y = a_0x^m + a_1x^{m+1} + \cdots + a_nx^{m+n} + - - - - - - - -(1)$$

On differentiating (1) we get,

$$y' = a_0mx^{m-1} + a_1(m+1)x^m + \cdots + a_n(m+n)x^{m+n-1} + - - - - - -(2)$$
$$y'' = a_0m(m-1)x^{m-2} + a_1(m+1)mx^{m-1} + \cdots + a_n(m+n)(m+n-1)x^{m+n-2} ..(3)$$

Multiply (3) by 4x we get,

$$4xy'' = 4x[a_0m(m-1)x^{m-2} + a_1(m+1)mx^{m-1} + \cdots + a_n(m+n)(m+n-1)x^{m+n-2}$$
$$= 4a_0m(m-1)x^{m-1} + 4a_1(m+1)mx^m + \cdots + 4a_n(m+n)(m+n-1)x^{m+n-1} \ldots (4)$$

Multiply (2) by 2 we get,

$$2y' = 2a_0mx^{m-1} + 2a_1(m+1)x^m + \cdots + 2a_n(m+n)x^{m+n-1} \ldots (5)$$
$$y = a_0x^m + a_1x^{m+1} + \cdots + a_nx^{m+n-1} + .. a_nx^{m+n} + \cdots (6)$$

Now add (4),(5),(6) we get,

$$=[2a_1(m+1)[2m+1] + a_0]x^m + \cdots [2a_n(m+n)[2(m+n-1)+1] + a_{n-1}]x^{m+n-1}$$

Coefficient of x^{m-1}:

$$2a_0m[2(m-1)+1]=0$$

m=0,2m-1=0 m=1/2

Coefficient of x^m:

$$2a_1(m+1)(2m+1)+a_0=0$$

$$a_1=\frac{-a_0}{2(m+1)(2m+1)}$$

Coefficient of x^{m+n-1}:

$$2a_n(m+n)[2(m+n-1)+]+a_{n-1=0}$$

$$a_n=\frac{-a_{n-1}}{2(m+n)(2m+2n-1)} \text{ ---- } (7)$$

Equation (7) is called as Recurrence Relation.

CASE:1

$$a_n=\frac{-1}{2n(2n-1)}a_{n-1}$$

$$a_1=\frac{-1}{2(1)}a_0=\frac{-a_0}{2!} \text{ where n=1}$$

$$a_2=\frac{-1}{4(3)}a_1=\frac{a_0}{4!} \text{ where n=2}$$

Here $y_1(x)=a_0x^m+a_1x^{m+1}+a_2x^{m+2}+\cdots$

$$y_1(x)=a_0+a_1x+a_2x^2+\cdots$$

$$y_1(x)=a_0[1-\frac{x}{2!}+\frac{x^2}{4!} \text{ ---} (8)$$

CASE:2

$$a_n=\frac{-a_{n-1}}{(1+2n)2n}$$

$$a_1=\frac{-1}{3(2)}a_0=\frac{-a_0}{3!} \text{ where n=1}$$

$$a_2=\frac{-1}{5(4)}a_1=\frac{a_0}{5!} \text{ where n=2}$$

Here $y_2(x)=a_0x^m+a_1x^{m+1}+a_2x^{m+2}+\cdots$

$$y_2(x)=a_0x^m+a_1x^{m+1}+\cdots$$

$$y_2(x)=a_0\left[x^{\frac{1}{2}}-\frac{x^{\frac{3}{2}}}{3!}\right]-----(9)$$

The general solution of the given D.E is given by,

$$y=c_1y_1+c_2y_2$$

$y=c_1\left[a_0\left(1-\frac{x}{2!}+\frac{x^2}{4!}\ .....\right)\right]+c_2\left[a_0\left(x^{\frac{1}{2}}-\frac{x^{\frac{3}{2}}}{3!}+.....\right)\right]---(10)$Therefore, from equation (10) we can say the series is an increasing order. By using the time value and substituting the chemical value in recurrence we can say that the electrode size is goes on increasing.

PROBLEM:3

An electrode is of 1kgs & 5kgs respectively, hence 4 and 6 are the chemical vales of the electrodes [two different electrodes are used]. Find which electrodes is produced faster than the another?

SOLUTION:

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+6y=0---(*)$$

$$D^2+4D+6Y=0---(1)$$

$$m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$m=-2\pm\sqrt{3}\,i$$

Here the roots are conjugate complex numbers $a\pm bi$, where $a=-2, b=\sqrt{3}$

The general equation solution the given D.E is,

$$y=e^{ax}(c_1sinbx+c_2cosbx)$$

$$y=e^{-2x}(c_1sin\sqrt{3}x+c_2cos\sqrt{3}x)----(2)$$

$$\frac{dy}{dx}=e^{-2x}(\sqrt{3}c_1cos\sqrt{3}x-\sqrt{3}c_2sin\sqrt{3}x)+(c_1sin\sqrt{3}x+c_2cos\sqrt{3}x)e^{-2x}.-2$$

$$\frac{dy}{dx}=e^{-2x}[(-2c_1-\sqrt{3}c_2)sin\sqrt{3}x+(\sqrt{3}c_1-2c_2)cos\sqrt{3}x)----(3)$$

Now apply the initial condition to equation(2)

$$1 = e^{-2(0)}(c_1 \sin 0 + c_2 \cos 0)$$
$$c_2 = 1 - - - - - - - - - - - (4)$$

Now apply the initial condition to equation (3)

$$5 = (-2c_1 - \sqrt{3}c_2)\sin 0 + (\sqrt{3}c_1 - 2c_2)\cos 0$$
$$\sqrt{3}c_1 - 2c_2 = 5 - - - - - - (5)$$
$$c_1 = 4.04 \ \ or\ c_1 = 4 - - - - - (6)$$
$$y = e^{-2x}(4\sin\sqrt{3}x + 1\cos\sqrt{3}x)$$

Hence we can say that mild steel electrodes is produced faster than cast iron electrodes.

CONCLUSION

By using the differential equation we found the general equation by using it we can find any required data such as time, size, current values, kilograms of the electrodes. In addition to that, using frobenius method we can also say that production of electrodes are in increasing or decreasing order. Variety of electrode problems can be solved using this type of differential equation.

REFERENCES

1. *Eric Wolfgang Weisstein, Ordinary differential equation order.*
2. *Paul Blanthard, Robert L. Devaney, Vlenn R. Hall, Differential Equation (preliminary edition), PWS publishing Boston,(1996).*
3. *Ince, E.L., Ordinary Differential Equations, Dover Publications,(1956).*
4. *Cole. R, Theory of Ordinary Differential Equation (Appleton – Century- Crofts, New York) (1968).*
5. *Shepley L. Ross, Text book on Differential equations, Third edition, Published by John Wiley & Sons, Inc.*
6. *S. Narayanan T. K. Manickavachagom Pillay, Textbook on Differential Equations and its Applications, Published by V. Subramanian.*
7. *http://mathworld.wolfram.com/OrdinaryDifferentialEquationOrder.html*
8. *https://www.math.ust.hk/differentialequation*
9. *www.sosmath.com>diffeq>second*