



A STUDY ON WELDING ELECTRODES PROCESS USING DIFFERENTIAL EQUATION

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ABSTRACT

The Main aim of this paper is to show how differential equations is used in real life. Differential equations by their applications can predict the happenings around us. They are employed in various disciplines such as Biology, Physics ,Chemistry ,Engineering. They are used to calculate the rate of change of time, size, kilogram, current of the electrodes. This paper particularly explores the use of differential equations in electrodes.

KEY WORDS: *Time, size, current, electrodes.*

INTRODUCTION

A Differential equation is a mathematical equation that relates some function with its derivatives. In applications, the functions usually represent physical quantities, the derivatives represent their rates of change, and the differential equation defines a relationship between the two. because such relations are extremely common.

Example: $\frac{dy}{dx} + x^2y = x$

PROBLEMS

A Electrodes contains a size 4mm, 6mm and kilograms 2kgs,5kgs. Find the required time of these electrodes.

SOLUTION:

$$+a \frac{dy}{dx} + a'y = 0 \dots\dots\dots(*)$$

$$\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 6y = 0 \dots\dots\dots(1)$$

$$D^2 + 4D + 6y = 0 \dots\dots\dots(2)$$

$$a=1 \quad b=4 \quad c=6$$

$$M = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$M = -2 \pm \sqrt{2}i$$

Here the roots are conjugate complex numbers $a \pm bi$ where $a = -2$ $b = \sqrt{2}$

Then the general solution of given differential equation is,

$$y = e^{-2x} (c_1 \sin \sqrt{2}x + c_2 \cos \sqrt{2}x) \dots\dots\dots(3)$$

By differentiating equation (3) we get,

$$\frac{dy}{dx} = e^{-2x} [(2c_1 - \sqrt{2}c_2) \sin \sqrt{2}x + (\sqrt{2}c_1 - 2c_2) \cos \sqrt{2}x] \dots\dots\dots(4)$$

Apply initial condition,

$$(i.e) \quad y(0) = 2 \quad x(0) = 0$$

We get,

$$2 = e^0 [c_1 \sin 0 + c_2 \cos 0]$$

$$c_2 = 2 \dots\dots\dots(5)$$

Now again apply initial condition

(i.e) $y'(0)=5$ $x'(0)=0$

$$5 = [(2 c_1 - \sqrt{2} c_2) \sin(0) + (\sqrt{2} c_1 - 2 c_2) \cos(0)]$$

$$\sqrt{2} c_1 - 2 c_2 = 5 \dots\dots\dots(6)$$

$$\sqrt{2} c_1 = 4 + 5$$

$$c_1 = \frac{9}{\sqrt{2}} \quad c_1 = 6$$

$$y = e^{-2x} [6 \sin \sqrt{2}x + 2 \cos \sqrt{2}x]$$

Hence, c_1 and c_2 are constant values so we can say that the company can produce one electrode of a size 4mm in 2 minutes another electrode of size 6mm in 6 minutes.

PROBLEM 2:

Using Different size value of an electrode 3mm and 5mm. Find which of the electrode is in more weight?

SOLUTION:

Assume the differential equation,

$$\frac{d^2y}{dx^2} + b \frac{dy}{dx} + b'y = 0 \dots\dots\dots(*)$$

$$\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 4y = 0 \dots\dots\dots(1)$$

$$D^2 + 2D + 4y = 0 \dots\dots\dots(2)$$

From the above equation (2) $a=1$ $b=2$ $c=4$

$$M = -1 \pm \sqrt{3}i$$

Here the roots are conjugate complex number $a \pm bi$ where $a=1$ $b=\sqrt{3}$ then the general solution of given differential equation is,

$$y = e^{-x} (c_1 \sin \sqrt{3} x + c_2 \cos \sqrt{3} bx)$$

By differentiating equation (3) we get,

$$\frac{dy}{dx} = e^{-x} [(1 c_1 - \sqrt{3} c_2) \sin \sqrt{3} x + (\sqrt{3} c_1 - 1 c_2) \cos \sqrt{3} x] \dots\dots\dots(4)$$

Now apply the initial condition to equation (3)

(i.e) $y(0)=3$ $x(0)=0$

We get,

$$3 = e^0 [c_1 \sin 0 + c_2 \cos 0]$$

$$c_2 = 3 \dots\dots\dots(5)$$

Now again apply initial condition to equation (4) we get,

(i.e) $y'(0)=4$ $x'(0)=0$

$$\sqrt{2} c_1 - 2 c_2 = 4 \dots\dots\dots(6)$$

$$\sqrt{2} c_1 - c_2 = 4$$

$$\sqrt{2} c_1 = 7$$

$$c_1 = \frac{7}{\sqrt{2}} \quad c_1 = 4$$

Substituting these values in equation (3)

$$y = e^{2x} [4 \sin \sqrt{3}x + 3 \cos \sqrt{3}x]$$

Hence, c_1 and c_2 are constant values so we can say that $c_2 > c_1$. so Stainless steel electrode is heavier than hard facing electrodes

PROBLEM 3:

By substituting the size value instead of n and find the series solution in powers of x & say that the series is in increasing order or decreasing order?

SOLUTION:

Assume the differential equation,

$$4 \frac{d^2y}{dx^2} + 9xy = 0. \dots\dots\dots(1)$$

Assume the series,

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_nx_n + a^{n-1}x^{n-1} + \dots a_{n+1}x^{n+1} + a_{n+2}x^{n+2} \dots\dots\dots(2)$$

$$\frac{dy}{dx} = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + \dots + na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + (n+1)a_{n+1}x^n + (n+2)a_{n+2}x^{n+1} + \dots$$

$$\frac{d^2y}{dx^2} = 2a_2 + 6a_3x + 12a_4x^2 + 20a_5x^3 + \dots + n(n-1)a_nx^{n-2} + n(n+1)a_{n+1}x^{n-1} + (n+1)(n+2)a_{n+2}x^n + \dots$$

Equation (1) becomes,

$$4 \frac{d^2y}{dx^2} + 9xy = 0$$

$$8a_2 + 24a_3x + 48a_4x^2 + 80a_5x^3 + \dots + 4n(n-1)a_nx^{n-2} + 4n(n+1)a_{n+1}x^{n-1} + 4(n+1)(n+2)a_{n+2}x^n + \dots$$

$$+ 9a_0x + 9a_1x^2 + 9a_2x^3 + 9a_3x^4 + 9a_4x^5 + 9a_5x^6 + 9a_{n-1}x^n + 9a_nx^{n+1} + \dots + 9a_{n+1}x^{n+2} + 9a_{n+2}x^{n+3} = 0$$

------(3)

Grouping like terms,

$$8a_2 + (24a_3 + 9a_0)x + (48a_4 + 9a_1)x^2 + (80a_5 + 9a_2)x^3 + (4(n+1)(n+2)a_{n+2} + 9a_{n-1})x^n = 0$$

Equating constant terms,

$$8a_2 = 0$$

$$a_2 = 0$$

------(4)

Equating coefficient of x,

$$24a_3 = -9a_0$$

$$a_3 = -\frac{9a_0}{4!}$$

------(5)

Equating coefficient of x²,

$$48a_4 + 9a_1 = 0$$

$$a_4 = -\frac{9a_1}{48}$$

------(6)

Equating coefficient of xⁿ,

$$4(n+1)(n+2)a_{n+2} + 9a_{n-1} = 0$$

$$a_{n+2} = -\frac{9a_{n-1}}{4(n+1)(n+2)}$$

------(7)

The equation (7) is called recurrence relation.

Put n=3 in (7)

$$a_{3+2} = \frac{-9a_{(3)-1}}{4(3+1)(3+2)} \quad \text{since } a_2 = 0$$

$$a_5 = 0$$

------(8)

Put n=4 in (7)

$$a_{4+2} = \frac{-9a_{(4)-1}}{4(4+1)(4+2)} \quad \text{since } a_3 = -\frac{9a_0}{4!}$$

$$a_6 = \frac{9a_0}{2^2 \cdot 6!}$$

------(9)

Put n=5

$$a_{5+2} = \frac{-9a_{(5)-1}}{4(5+1)(5+2)} \quad \text{since } a_4 = -\frac{9a_1}{48}$$

$$a_7 = \frac{5 \cdot 9^2 a_1}{2^3 \cdot 7!}$$

------(10)

Put n=6

$$a_{6+2} = \frac{-9a_{(6)-1}}{4(6+1)(6+2)} \quad \text{since } a_5 = 0$$

$$a_8 = 0$$

------(11)

Put n=7

$$a_{7+2} = \frac{-9a_{(7)-1}}{4(7+1)(7+2)} \quad \text{since } a_6 = \frac{9a_0}{2^2 \cdot 6!}$$

$$a_9 = -7 \cdot \frac{9^3 a_0}{2^4 \cdot 9!}$$

------(12)

Substituting,

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8 + a_9x^9$$

$$y = a_0 + a_1x + (0)x^2 + \left(-\frac{9a_0}{4!}\right)x^3 + \left(-\frac{9a_1}{48}\right)x^4 + (0)x^5 + \left(\frac{9^2 a_0}{2^2 \cdot 6!}\right)x^6 + \left(\frac{5 \cdot 9^2 a_1}{2^3 \cdot 7!}\right)x^7 + (0)x^8 + \left(-\frac{9^3 a_0}{2^4 \cdot 9!}\right)x^9$$

$$y = a_0 \left(1 - \frac{9x^3}{4!} + \frac{9^2 x^6}{4 \cdot 6!} - \frac{7 \cdot 9^3 x^9}{2^4 \cdot 9!}\right) + a_1 \left(x - \frac{9x^4}{2 \cdot 4!} + \frac{5 \cdot 9^2 x^7}{2^3 \cdot 7!}\right)$$

While the value of size changes the kilogram of an electrode goes on increasing....Hence the series solution is in increasing order.

CONCLUSION

Using Differential equations we have found the general solution using this we can find the required solution such as time ,size ,kilogram current values of the electrodes .In Addition to that, Using Series Solution Method we can also say that the production of electrodes are in increasing order or decreasing order. Variety of differential equation problems can be solved using the same concept of differential equation.

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