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MORE ABOUT η*-NORMAL SPACES

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ABSTRACT

In this paper, we study η-normal spaces and the relationships among normal, η*-normal, quasi normal, softly normal, almost normal, π-normal and mildly normal spaces are investigated. We also obtain some characterizations of η*-normal spaces. Moreover, we also introduce some closed and continuous functions such as almost J-closed, η*-continuous and almost η*-continuous functions and investigate some properties of η*-normal spaces in the terms of these functions.*

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1. INTRODUCTION

In 1923, Tietze **[24]** first defined the notion of normal spaces and studied their properties. In 1925, Urysohn **[25]** studied normal spaces and gave a major result is called Urysohn's Lemma. In 1937, M. Stone **[23]** introduced the notion of regular open sets. In 1970, Levine **[11]** introduced the notion of generalized closed sets and studied their properties of g-closed sets in topological spaces. In 1968, the notion of quasi normal space was introduced by Zaitsev [**27**]. In 1970, Singal and Arya [**21**] introduced the concept of almost normal spaces which is a weaker form of normal space. In 1973, Singal and Singal [**20**] introduced the notion of mildly normal spaces which are weaker forms of quasi normal and π-normal spaces. In 1990, Lal and Rahman [**10**] further studied notions of quasi normal and mildly normal spaces. In 1996, Noiri [**15**] further studied the notion of mildly normal spaces and obtained some new characterizations. In 2000, Dontchev and Noiri [**4**] introduced the notion of πg-closed sets as a weak form of g-closed sets due to Levine [**11**] and by using πg-closed sets, obtained a new characterization and some preservation theorems of quasi normal spaces. In 2007, Kalantan [**6**] introduced a weaker version of normality called π-normality and proved that π-normality lies between normality and almost normality. In 2015, M. C. Sharma and H. Kumar [**18**] introduced the concept of softly normal spaces and obtained their properties. In 2018, H. Kumar and M. C. Sharma [**7**] introduced the concept of softly regular spaces and obtained their characterizations. Recently, H. Kumar and J. Kumar [**8**] introduced the concept of η*-regular and η*-normal spaces and obtained some basic properties of η*-regular and η*-normal spaces.

2. PRELIMINARIES

In what follows, spaces always mean topological spaces on which no separation axioms are assumed unless explicitly stated and f : (X, ℑ) → (Y, σ) (or simply f : X → Y) denotes a function f of a space (X, ℑ) into a space (Y, σ). Let A be a subset of a space X. The closure and the interior of A are denoted by cl(A) and int(A), respectively. A subset A is said to be **regular open** [**9**, **23**] (resp. **regular closed [9, 23]**) if A ⊂ int(cl(A)) (resp. A ⊂ cl(int(A)). The finite union of regular open sets is said to be **π-open [27]**. The complement of a π-open set is said to be **π-closed.**

2.1 Definition: A subset A of a topological space (X, ℑ) is said to be
(i) g-closed [11] if cl(A) ⊂ U whenever A ⊂ U and U ∈ ℑ.
(ii) πg-closed [4] if cl(A) ⊂ U whenever A ⊂ U and U is π-open in X.
(**iii**) **regular generalized closed** (briefly r**g-closed**) [**16]** if cl(A) ⊂ U whenever A ⊂ U and U is regular-open in X.

The complement of a g-closed (resp. πg-closed, rg-closed) set is said to be **g-open** (resp**. πg-open, rg-open**).
The **generalized closure** of A is defined as the intersection of all g-closed sets in X containing A and is denoted by **cl*(A)**. The **generalized interior** of A is defined as the union of all g-open sets in X contained in A and is denoted by **int*(A)**.
2.2 Definition. The **δ-interior** of a subset A of a space X is the union of all regular open sets of X contained in A and is denoted by **δ-int(A)**. The subset A is called **δ-open** [**26**] if δ-int(A) = A. i.e. a set A is called δ-open if it is the union of regular open sets and the

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complement of a δ-open set is called **δ-closed**. Alternatively, a set A ⊂ X is δ-closed if A = δ-cl(A), where δ-cl(A) is the intersection of all regular closed sets of (X, ℑ) containing A.

2.3 Definition. A subset A of a topological space (X, ℑ) is called **regular*-open** [**17**] (or r*-open) if A = int(cl*(A)). The complement of a regular*-open set is called **regular*-closed**. The union of all regular*-open sets of X contained in A is called **regular*-interior** of A and is denoted by **r*-int(A)**. The intersection of all regular*-closed sets of X containing A is called **regular*-closure of A** and is denoted by **r*-cl(A)**.

2.4 Definition. A subset A of a topological space (X, ℑ) is called **η*-open** [**12**] set if it is the union of regular*-open sets (r*-open sets).The complement of a η*-open set is called **η*-closed**. A subset A of a topological space (X, ℑ) is called **η*-Interior** of A if the union of all η*-open sets of X contained in A and is denoted by **η*-int(A)**. The intersection of all η*-closed sets of X containing A is called the **η*-closure** of A and is denoted by **η*-cl(A)**.

2.5 Definition: A subset A of a topological space (X, ℑ) is said to be **J-closed [13]** if cl(A) ⊂ U whenever A ⊂ U and U is η*-open. The complement of the J-closed set is called **J-open** set. The collection of all J-open (resp. J-closed) sets is denoted by J-O(X) (resp. J-C(X)).

2.6 Remark.
(i) regular open ⇒ π-open ⇒ δ-open ⇒ η*-open ⇒ open ⇒ g-open

2.7 Remark. For every subset U of X,
(i) g-cl(U) ⊂ cl(U)) ⊂ η*-cl(U) ⊂ δ-cl(U) ⊂ π-cl(U) ⊂ r-cl(U).

2.8 Remark. We have the following implications for the properties of subsets:
closed ⇒ g-closed ⇒ J-closed ⇒ πg-closed ⇒ rg-closed

Where none of the implications is reversible as can be seen from the following examples:
2.9 Example. Let X = {a, b, c} and ℑ = {ϕ, X, {a}, {b}, {a, b}}. Then
(i) regular closed sets are : ϕ, X, {a, c}, {b, c}.
(ii) π-closed sets are : ϕ, X, {c}, {a, c}, {b, c}.
(iii) regular*-closed sets are : ϕ, X, {a, c}, {b, c}.
(iv) η*-closed sets are : ϕ, X, {c}, {a, c}, {b, c}.
(v) closed sets are : ϕ, X, {c}, {a, c}, {b, c}.
(vi) g-closed sets are : ϕ, X, {c}, {a, c}, {b, c}.
(vii) πg-closed sets are : ϕ, X, {c}, {a, c}, {b, c}.
(viii) rg-closed sets are : ϕ, X, {c}, {a, b}, {a, c}, {b, c}.
(ix) J-closed sets are : ϕ, X, {c}, {a, c}, {b, c}.

3. η*-NORMAL SPACES

3.1 Definition. A topological space (X, ℑ) is called
1. π-normal [**6**] if for any two disjoint closed subsets A and B of X, one of which is π-closed, there exist two disjoint open sets U and V of X such that A ⊂ U and B ⊂ V.
2. almost normal [**21**] if for any two disjoint-closed subsets *A* and *B* of *X*, one of which is closed-domain, can be separated.
3. quasi normal [**27**] if for every pair of disjoint π-closed subsets A and B of X, there exist disjoint open sets U and V of X such that A ⊂ U and B ⊂ V.
4. mildly normal [**20**] if for any pair of disjoint closed-domain subsets A and B of X can be separated.
5. softly normal [**18**] if for any two disjoint closed subsets A and B of X, one of which is π–closed and other is regularly closed, there exist two disjoint open sets U and V of X such that A ⊂ U and B ⊂ V.
6. η*-normal [**8**] if for every pair of disjoint η*-closed subsets A and B of X, there exist disjoint open sets U and V of X such that A ⊂ U and B ⊂ V.

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3.2 Remark. By the definitions stated above, we have the following diagram:

normal $\Rightarrow$ η^*-normal $\Rightarrow$ quasi normal $\Rightarrow$ softly normal $\Rightarrow$ mildly normal

Where none of the implications is reversible

(**i**) η^*-normal and almost normal spaces are independent of each other.

(**ii**) η^*-normal and π-normal spaces are independent of each other.

4. CHARACTERIZATIONS OF η^*-NORMAL SPACES

4.1 Theorem. Let $(X, \Im)$ be a topological space X. Then the following statements are equivalent:

(a) X is η^*-normal.

(b) For every pair of η^*-open subsets U and V of X whose union is X, there exist closed subsets G and H of X such that $G \subset U$, $H \subset V$ and $G \cup H = X$.

(c) For any η^*-closed set A and every η^*-open set B in X such that $A \subset B$, there exists an open subset U of X such that $A \subset U \subset cl(U) \subset B$.

(d) For every pair of disjoint η^*-closed subsets A and B of X, there exist open subsets U and V of X such that $A \subset U$, $B \subset V$ and $cl(U) \cap cl(V) = \phi$.

Proof. (a) $\Rightarrow$ (b), (b) $\Rightarrow$ (c), (c) $\Rightarrow$ (d) and (d) $\Rightarrow$ (a).

(a) $\Rightarrow$ (b). Let U and V be any η^*-open subsets of a η^*-normal space X such that $U \cup V = X$. Then, $X - U$ and $X - V$ are disjoint η^*-closed subsets of X. By η^*-normality of X, there exist disjoint open subsets U_1 and V_1 of X such that $X - U \subset U_1$ and $X - V \subset V_1$. Let $G = X - U_1$ and $H = X - V_1$. Then, G and H are closed subsets of X such that $G \subset U$, $H \subset V$ and $G \cup H = X$.

(b) $\Rightarrow$ (c). Let A be a η^*-closed and B is a η^*-open subsets of X such that $A \subset B$. Then, $X - A$ and B are η^*-open subsets of X such that $(X - A) \cup B = X$. Then, by part (b), there exist closed sets G and H of X such that $G \subset (X - A)$, $H \subset B$ and $G \cup H = X$. Then, $A \subset (X - G)$, $(X - B) \subset (X - H)$ and $(X - G) \cap (X - H) = \phi$. Let $U = X - G$ and $V = (X - H)$. Then U and V are disjoint open sets such that $A \subset U \subset X - V \subset B$. Since $X - V$ is closed, then we have $cl(U) \subset (X - V)$. Thus, $A \subset U \subset cl(U) \subset B$.

(c) $\Rightarrow$ (d). Let A and B be any disjoint η^*-closed subset of X. Then $A \subset X - B$, where $X - B$ is η^*-open. By the part (c), there exists an open subset U of X such that $A \subset U \subset cl(U) \subset X - B$. Let $V = X - cl(U)$. Then, V is an open subset of X. Thus, we obtain $A \subset U$, $B \subset V$ and $cl(U) \cap cl(V) = \phi$.

(d) $\Rightarrow$ (a). Let A and B be any pair of disjoint η^*-closed subsets of X. By assumption, there exist open sets U containing A and V containing B such that $A \subset U$, $B \subset V$ and $cl(U) \cap cl(V) = \phi$. We have $U \cap V = \phi$ and thus $(X,\Im)$ is η^*-normal.

4.2 Proposition. Let $f : X \to Y$ be a function, then:

(a) The image of open subset under an open continuous function is open.

(b) The inverse image of open (resp. closed) subset under an open continuous function is open (resp. closed) subset.

(c) The image of closed subset under an open and a closed continuous surjective function is open.

4.3 Theorem. The image of a η^*-normal space under an open continuous injective function is a η^*-normal.

Proof. Let X be a η^*-normal space and let $f : X \to Y$ be an open continuous injective function. We need to show that f(X) is a η^*-normal space. Let A and B be any two disjoint η^*-closed sets in f(X). Since the inverse image of a η^*-closed set under an open continuous function is a η^*-closed. Then, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint η^*-closed sets in X. By η^*-normality of X, there exist open subsets U and V of X such that $f^{-1}(A) \subset U$, $f^{-1}(B) \subset V$ and $U \cap V = \phi$. Since f is an open continuous injective function, we have $A \subset f(U)$, $B \subset f(V)$ and $f(U) \cap f(V) = \phi$. By **Proposition 4.2**, we obtain f(U) and f(V) are disjoint open sets in f(X) such that $A \subset f(U)$ and $B \subset f(V)$. Hence f(X) is η^*-normal.

From the above theorem, we have the following corollary.

4.4 Corollary. η^*-normality is a topological property.

The following lemma helps us to show that η^*-normality is a hereditary with respect to closed domain subspaces.

4.5 Lemma. Let M be a closed domain subspace of a space X. If A is a open set in X, then $A \cap M$ is open set in M.

4.6 Theorem. η^*-normality is a hereditary with respect to closed domain subspaces.

Proof. Let M be a closed domain subspace of a η^*-normal space X. Let A and B be any disjoint η^*-closed sets in M. Since M is a closed domain subspace of X, then we have A and B be any disjoint η^*-closed sets of X. By η^*-normal of X, there exist disjoint open

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subsets U and V of X such that $A \subset U$ and $B \subset V$. By the **Lemma 4.5,** we obtain $U \cap M$ and $V \cap M$ are disjoint open sets in M such that $A \subset U \cap M$ and $B \subset V \cap M$. Hence, M is η*-normal subspace.
Since every closed and open (clopen) subset is a closed domain, then we have the following corollary.

4.7 Corollary. η*-normality is a hereditary with respect to clopen subspaces.
Proof. Every clopen set is closed domain.
The following result is useful for giving some other characterizations of η*-normal spaces.

4.8 Lemma. A subset A of a space X is J-open if and only if $F \subset \text{int}(A)$ whenever $F \subset A$ and F is η*-closed.

4.8 Theorem. Let $(X, \Im)$ be a topological space. Then the following statements are equivalent:
(a) X is η*-normal.
(b) For any disjoint η*-closed sets H and K, there exist disjoint g-open sets U and V such that $H \subset U$ and $K \subset V$.
(c) For any disjoint η*-closed sets H and K, there exist disjoint J-open sets U and V such that $H \subset U$ and $K \subset V$.
(d) For any η*-closed set H and any η*-open set V containing H, there exists a g-open set U of X such that $H \subset U \subset \text{cl}(U) \subset V$.
(e) For any η*-closed set H and any η*-open set V containing H, there exists a J-open set U of X such that $H \subset U \subset \text{cl}(U) \subset V$.

Proof. (a) ⇒ (b), (b) ⇒ (c), (c) ⇒ (d), (d) ⇒ (e) and (e) ⇒ (a).
(a) ⇒ (b). Let X be η*-normal space. Let H, K be disjoint η*-closed sets of X. By assumption, there exist disjoint open sets U, V such that $H \subset U$ and $K \subset V$. Since every open set is g-open, U and V are g-open sets such that $H \subset U$ and $K \subset V$.
(b) ⇒ (c): Let H, K be two disjoint η*-closed sets. By assumption, there exist disjoint g-open sets U and V such that $H \subset U$ and $K \subset V$. Since every g-open set is J-open, so U and V are J-open sets such that $H \subset U$ and $K \subset V$.
(c) ⇒ (d): Let H be any η*-closed set and V be any η*-open set containing H. By assumption, there exist disjoint J-open sets U and W such that $H \subset U$ and $X - V \subset W$. By **Lemma 4.8**, we get $X - V \subset \text{int}(W)$ and $\text{cl}(U) \cap \text{int}(W) = \phi$. Hence $H \subset U \subset \text{cl}(U) \subset X - \text{int}(W) \subset V$.
(d) ⇒ (e): Let H be any η*-closed set and V be any η*-open set containing H. By assumption, there exists g-open set U of X such that $H \subset U \subset \text{cl}(U) \subset V$. Since, every g-open set is J-open, there exists J-open sets U of X such that $H \subset U \subset \text{cl}(U) \subset V$.
(e) ⇒ (a): Let H, K be any two disjoint η*-closed sets of X. Then $H \subset X - K$ and $X - K$ is η*-open. By assumption, there exists J-open set G of X such that $H \subset G \subset \text{cl}(G) \subset X - K$. Put $U = \text{int}(G)$, $V = X - \text{cl}(G)$. Then U and V are disjoint open sets of X such that $H \subset U$ and $K \subset V$. Hence X is η*-normal.

5. SOME FUNCTIONS

5.1 Definition. A function $f : X \to Y$ is said to be
(a) completely continuous [1] (resp. R-map [3]) if $f^{-1}(V)$ is regularly open in X for every open (resp. regularly open) set V of Y.
(b) almost continuous [19] if $f^{-1}(V)$ is open in X for every regular open set V of Y.
(c) η*-continuous (resp. **almost η*-continuous**) if $f^{-1}(F)$ is η*-closed in X for every closed (resp. regular closed) set F of Y.
(d) g-continuous [2] (resp. **almost g-continuous [5]**) if $f^{-1}(F)$ is g-closed in X for every closed (resp. regular closed) set F of Y.
(e) J-continuous [14] (resp. **almost J-continuous**) if $f^{-1}(F)$ is J-closed in X for every closed (resp. regular closed) set F of Y.
(f) π-continuous [4] (resp. **almost π-continuous [4]**) if $f^{-1}(F)$ is **π**-closed in X for every closed (resp. regular closed) set F of Y.

From the definitions stated above, we obtain the following diagram:

completely continuous	⇒	R-map
⇓		⇓
π-continuous	⇒	almost π-continuous
⇓		⇓
η*-continuous	⇒	almost η*-continuous
⇓		⇓
continuous	⇒	almost continuous
⇓		⇓
g-continuous	⇒	almost g-continuous
⇓		⇓

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J-continuous	$\Rightarrow$	almost J-continuous
$\Downarrow$		$\Downarrow$
πg-continuous	$\Rightarrow$	almost πg-continuous

None of the implications in above Diagram is reversible as shown by the following examples:

5.2 Example. Let X = {x, y, z}, $\Im$ = {ϕ, X, {x}}, Y = {a, b} and ρ = {ϕ, Y, {a}}. Define f : (X, $\Im$) $\rightarrow$ (Y, ρ) as follows: f(x) = f(z) = b and f(y) = a. Then f is g-continuous as well as J-continuous. But it is not continuous.

5.3 Example. Let X = Y = {x, y, z}, $\Im$ = {ϕ, X, {x}, {z}, {x, z}} and ρ = {ϕ, X, {x}, {x, y}}. Let f : (X, $\Im$) $\rightarrow$ (X, ρ) defined as follows: f(x) = f(z) = x and f(y) = y. Then f is g-continuous as well as J-continuous.

5.4 Example. Let X = Y = {x, y, z}, $\Im$ = {ϕ, X, {x}, {z}, {x, z}} and ρ = {ϕ, Y, {x}}. Define f : (X, $\Im$) $\rightarrow$ (Y, ρ) as follows: f(x) = f(z) = x and f(y) = y. Then, f is g-continuous as well as J-continuous.

5.5 Example. Let X = {x, y, z}, $\Im$ = {ϕ, X, {x}, {x, y}} and ρ = {ϕ, X, {x}, {x, z}}. Then, the identity function f : (X, $\Im$) $\rightarrow$ (X, ρ) is R-map as well as almost π-continuous. But it is not continuous.

5.6 Example. Let X = {x, y, z}, $\Im$ = {ϕ, X, {x}, {y}, {x, y}, {x, z},} and ρ = {ϕ, X, {x}, {y}, {x, y}}. Then, the identity function f : (X, $\Im$) $\rightarrow$ (X, ρ) is continuous as well as J-continuous.

5.7 Example. Let X = {x, y, z, w} and $\Im$ = {ϕ, X, {x}, {y}, {x, y}, {x, y, z}, {x, y, w}}. Let f : (X, $\Im$) $\rightarrow$ (X, $\Im$) be a function defined by: f(x) = z, f(y) = x, f(z) = y and f(w) = z. Then f is almost πg-continuous but it is not almost g-continuous.

5.8 Example. Let X = {x, y, z, w} and $\Im$ = {ϕ, X, {x}, {y}, {x, y}, {x, y, z}, {x, y, w}}. Let f : (X, $\Im$) $\rightarrow$ (X, $\Im$) be a function defined by: f(x) = x, f(y) = w, f(z) = y and f(w) = x. Then f is πg-continuous but it is not g-continuous.

5.9 Example. Let X = {x, y, z, w} and $\Im$ = {ϕ, X, {x}, {y, z}, {x, y, z},}. Then, the identity function f : X $\rightarrow$ X is πg-continuous.

5.10 Example. Let X = {x, y, z} and $\Im$ = {ϕ, X, {x}, {y}, {x, y}}. Let f : (X, $\Im$) $\rightarrow$ (X, $\Im$) be a function defined by: f(x) = f(y) = x and f(z) = z. Then f is π-continuous as well as η*-continuous but it is not R-map.

5.11 Definition. A function f : X $\rightarrow$ Y is said to be
(a) **J-closed** (resp. **g-closed [4]**, **πg-closed** [**4**], **rg-closed** [**15**]) if f(F) is J-closed (resp. g-closed, πg-closed, rg-closed) in Y for every closed set F of X.
(b) **rc-preserving** [**15**] (resp. **almost closed** [**19**], **almost g-closed [4]**, **almost J-closed, almost πg-closed** [**4**], **almost rg-closed** [**15**]) if f(F) is regular closed (resp. closed, g-closed, J-closed, πg-closed, rg-closed) in Y for every F $\in$ RC(X).
From the definitions stated above, we obtain the following diagram:

closed	$\Rightarrow$	almost closed
$\Downarrow$		$\Downarrow$
g-closed	$\Rightarrow$	almost g-closed
$\Downarrow$		$\Downarrow$
J-closed	$\Rightarrow$	almost J-closed
$\Downarrow$		$\Downarrow$
πg-closed	$\Rightarrow$	almost πg-closed
$\Downarrow$		$\Downarrow$
rg-closed	$\Rightarrow$	almost rg-closed

None of the implications in above Diagram is reversible as shown by the following examples:

5.12 Example. Let X = {x, y, z}, $\Im$ = {ϕ, X, {x}, {y}, {x, y}} and ρ = {ϕ, X, {x}, {x, y}, {x, z}}. Then, the identity function f : (X, $\Im$) $\rightarrow$ (X, ρ) is πg-closed as well as rg-closed. But it is not almost g-closed. Note that {x, z} $\in$ RC(X, $\Im$) but f({x, z}) is not g-closed in (X, ρ). Moreover, f^{-1} : (X, ρ) $\rightarrow$ (X, $\Im$) is almost closed but it is not rg-closed. Clearly {y} is closed in (X, ρ) but f^{-1} : ({y}) is not

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rg-closed in (X, ℑ).

6. PRESERVATION THEOREMS FOR η*-NORMAL SPACES

6.1 Theorem. If f : X → Y is an almost η*-continuous and J-closed function, then f(A) is J-closed in Y for every J-closed set A of X.
Proof. Let A be any J-closed set of X and V be any η*-open set of Y containing f(A). Since f is almost η*-continuous, $f^{-1}(V)$ is η*-open in X and $A \subset f^{-1}(V)$. Therefore, we have $cl(A) \subset f^{-1}(V)$ and hence $f(cl(A)) \subset V$. Since f is J-closed, f(cl(A)) is J-closed in Y and hence we obtain $cl(f(A)) \subset cl(f(cl(A))) \subset V$. Hence f(A) is J-closed in Y.

6.2 Theorem. A surjection f : X → Y is almost J-closed if and only if for each subset S of Y and each U ∈ RO(X) containing $f^{-1}(S)$, there exists a J-open set V of Y such that S ⊂ V and $f^{-1}(V) \subset U$.

Proof. Necessity. Suppose that f is almost J-closed. Let S be a subset of Y and U ∈ RO(X) containing $f^{-1}(S)$. If V = Y − f(X − U), then V is a J-open set of Y such that S ⊂ V and $f^{-1}(V) \subset U$.

Sufficiency. Let F be any regularly closed set of X. Then $f^{-1}(Y - f(F)) \subset (X - F)$ and (X − F) ∈ RO(X). There exists a J-open set V of Y such that Y − f(F) ⊂ V and $f^{-1}(V) \subset (X - F)$. Therefore, we have f(F) ⊃ (Y − V) and $F \subset X - f^{-1}(V) \subset f^{-1}(Y - V)$. Hence we obtain f(F) = Y − V and f(F) is J-closed in Y, which shows that f is almost J-closed.

6.3 Theorem. If f : X → Y is an almost J-continuous, rc-preserving injection and Y is η*-normal then X is η*-normal.
Proof. Let A and B be any disjoint η*-closed sets of X. Since f is an rc-preserving injection, f(A) and f(B) are disjoint η*-closed sets of Y. Since Y is η*-normal, there exist disjoint open sets U and V of Y such that f(A) ⊂ U and f(B) ⊂ V.
Now if G = int(cl(U)) and H = int(cl(V)). Then G and H are regularly open sets such that f(A) ⊂ G and f(B) ⊂ H. Since f is almost J-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint J-open sets containing A and B which shows that X is η*-normal.

6.4 Theorem. If f : X → Y is a η*-continuous, almost g-closed surjection and X is a η*-normal space then Y is normal .
Proof. Let A and B be any two disjoint closed sets of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint η*-closed sets of X. Since X is η*-normal, there exist disjoint open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$.
Let G = int(cl(U)) and H = int(cl(V)). Then G and H are disjoint regularly open sets of X such that $f^{-1}(A) \subset G$ and $f^{-1}(B) \subset H$. Now, we set K = Y − f(X − G) and L = Y − f(X − H). Then K and L are g-open sets of Y such that A ⊂ K, B ⊂ L, $f^{-1}(K) \subset G$ and $f^{-1}(L) \subset H$. Since G and H are disjoint, K and L are disjoint. Since K and L are g-open and we obtain A ⊂ int(K), B ⊂ int(L) and int(K) ∩ int(L) = ϕ. Therefore, Y is normal.

6.5 Theorem. Let f : X → Y be an almost η*-continuous and almost J-closed surjection. If X is η*-normal space then Y is η*-normal.

Proof. Let A and B be any disjoint η*-closed sets of Y. Since f is almost η*-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint η*-closed sets of X. Since X is η*-normal, there exist disjoint open sets U and V of X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$.
Put G = int(cl(U)) and H = int(cl(V)). Then G and H are disjoint regularly open sets of X such that $f^{-1}(A) \subset G$ and $f^{-1}(B) \subset H$. By

Theorem 6.2, there exist J-open sets K and L of Y such that A ⊂ K, B ⊂ L, $f^{-1}(K) \subset G$ and $f^{-1}(L) \subset H$. Since G and H are disjoint, so are K and L by **Lemma 4.8,** A ⊂ int(K), B ⊂ int(L) and int(K) ∩ int(L) = ϕ. Therefore, Y is η*-normal.

6.6 Corollary. If f : X → Y is an almost continuous and almost closed surjection and X is a normal space, then Y is η*-normal.
Proof. Since every almost closed function is almost J-closed by **Theorem 6.5,** Y is η*-normal.

Conclusion. In this paper, we study a new class of generalized normal space is called η^*-normal space which is weaker than normal space. The relationships among normal η*-normal, quasi normal, softly normal, almost normal, π-normal and mildly normal spaces are investigated. Some of basic properties and characterizations of η^*-normal spaces are obtained. This idea can be extended to topological ordered, bitopological, bitopological ordered and fuzzy topological spaces etc. We also define separation axioms by using η^*-open sets such as

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Definition. A space X is said to be:
(i) η^*-T_0 if for each pair of distinct points x and y in X, there exists an η*-open set G containing x but not y or an η*-open set H containing y but not x.
(ii) η^*-T_1 if for each pair of distinct points x, y in X, there exist an η*-open set G containing x but not y and an η*-open set H containing y but not x.
(iii) η^*-T_2 if for each pair of distinct points x, y of X, there exist two of disjoint η*-open sets U and V containing x and y respectively.
Conflict of Interest: We certify that this work is original, have never been published before, and is not being considered for publication anywhere else at this time. This publication is free from any conflicts of interest. As the Corresponding Author, I certify that each of the listed Authors has read the paper and given their approval for publication.

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