ISSN (Online): 2455-7838
SJIF Impact Factor (2015): 3.476

EPRA International Journal of

Research & Development

(IJRD)

Volume:1, Issue:9,November 2016

Published By :
EPRA Journals

SJIF Impact Factor: 3.476 ISSN: 2455-7838(Online)
EPRA International Journal of Research and Development (IJRD)
Volume: 1 | Issue : 9 | November | 2016

A STUDY OF M/M/1/N QUEUING SYSTEM FOR IMPATIENT CUSTOMERS WITH BALKING

S. Vijay Prasad[1]
[1] Research Scholar, School of Studies in Mathematics, Vikram University, Ujjain, M. P, 456010, India
V.H. Badshah[2]
[2] Professor & Head, School of Studies in Mathematics, Vikram University, Ujjain, M. P, 456010, India
Tariq Ahmad Koka[3]
[3] Research Scholar, School of Studies in Mathematics, Vikram University, Ujjain, M. P, 456010, India

ABSTRACT

In this paper we study and generalized the M/M/1/N Queuing System for impatient customers with balking and the effect of the probability of balking on the expected system size has been studied, a comparative analysis of some queuing performance measures has been carried out to see the impact of customer balking on the expected system size.

KEY WORDS: *Queuing system, impatient customer, balking.*

1. INTRODUCTION

In queuing theory, customer behavior is considered a very interesting study that influences the nature of the entire system, considering queuing theory is a mathematical study of queues, it is natural for it to be affected by customer behavior at every stage. There is a mathematical study, which, if applied correctly, can cater to the requirements of the customers in a queue and reduce the waiting time. Queue is a very common word and perhaps, every one of us at some point of time must have waited in long queues at airports, post offices, restaurants etc. However, did it ever occur to you why we have queues in the first place when nobody, including service providers, likes them at all? The main reason behind the formation of long queues is the disparity between demand and supply i.e. when the ability to offer a service falls short of the overall demand. And making any effort to offer a service more quickly generally turns out to be an economically impracticable proposition, which eventually creates the need for an optimal method whose formulation entails equal consideration to both economics and the quality of service.

Gross et. al. [3] were generalized that, the customer is said to be impatient if he tends to join the queue only when a short wait is expected and tends to remain in the line if his wait has been sufficiently small. Impatience generally takes three forms. The first is balking, (customers deciding not to join the queue if it is too long), the second reneging, (customers leave the queue if they have waited too long for service) and the third jockeying (customers switch between queues if they think they will get served faster by so doing).

Rakesh and Sharma [10] were generalized that, the single as well as multi server Markovian queuing systems with reneging, balking and retention of reneged customers. They study four queuing models and present their steady-state solutions. Some important measures of performance are derived and finally, some queuing models are derived as particular cases of these models.

The notion of customer impatience appeared in the queuing theory in the work of Haight [4]. He generalized that, the balking for M/M/1 queue in which there is a greatest queue length at which an arrival would not balk. This length was a random variable whose distribution was same for all customers. Haight [5] was generalized that, a queue with reneging in which he studied the problem like how to make rational decision while waiting in the queue, the probable effect of this decision etc. Ancker and Gafarian [1] were generalized that, the M/M/1queuing system with balking and reneging and performed its steady state analysis. Ancker and Gafarian [2] were also proved that, the results for a pure balking system (no reneging) by setting the reneging parameter equal to zero.

Rakesh and Sharma [8] were generalized that, the M/M/1/N queuing model with retention of reneged customers. The steady-state solution has been obtained. Some particular cases of the model have been discussed. This model may be of great importance to the businesses facing the serious problem of customer impatience. The model analysis is limited to finite capacity. The infinite capacity case of the model can also be studied. Further, the model can be solved in transient state to get time-dependent results.

Rakesh and Sharma [7] were generalized that, the Markovian single server finite capacity queuing model with balking and possibilities of retaining reneged customers. The introduction of this concept of customer retention in queuing models has a lot of significance in the revenue generating queuing systems. The steady-state probabilities of system size have been obtained and some measures of performance have also been computed. The effect of parameters like average arrival rate, average service rate etc. have been studied numerically and it has been found that with the increase in probability of retention, the expected system size also increases proportionately and steadily. Some particular cases of the model have been discussed.

Kangzhou et. al. [6] were generalized that, the queuing systems with impatient customers is surveyed in accordance with various dimensions. First, they introduce the impatient behaviors (balking and reneging) and their various rules proposed in literature. Second, analytic solutions, numerical solutions and simulation modeling of the queue with impatient customers are investigated. Third, we propose the optimization both from the perspective of customers and providers. Finally, some research tendencies in the field are included.

Rakesh and Som [9] were generalized that, the concept of reverse balking and reverse reneging is incorporated into an M/M/1/N queuing system. The steady-state analysis of the model is performed and some important measures of performance are derived. Sensitivity analysis of the model is also performed. This model finds its application in investment business facing customer impatience. In future, the multi server case of the model can be studied. Model can be studied in non Markovian environment. They studied steady-state results; the time-dependent studies of the model can also be performed.

In this paper we were study and generalized the M/M/1/N Queuing System with Impatient Customers Balking. The effect of the probability of balking on the expected system size has been studied. The numerical results show that the average system size decreases proportionately and steadily as the probability of balking increases. A comparative analysis of some queuing models has been carried out to see the impact of customer balking on the expected system size.

2. M/M/1/N QUEUING MODEL WITH BALKING

2.1. Description of the Model:-

The model considered in this paper is based on following assumptions:

- Arrivals occur in a Poisson stream one by one with an average arrival rate of λ. The inter-arrival times are independently, identically and exponentially distributed with the parameter λ.
- There is only one server and service times are exponentially distributed with the parameter μ.
- The queue discipline is first-come, first-served (FCFS).
- The capacity of the system is finite (say N).
- On arrival a customer either decides to join the queue with probability p or balks with the probability $q = 1 - p$.

2.2 Steady-State Solution of the M/M/1/N Queuing Model with Balking:-

We consider an M/M/1/N queuing model with balking. It is envisaged that on arrival a customer either decides to join the queue with probability p or balks with the probability $q = 1 - p$.

Define $P_n(t)$ the probability that there are n customers in the system, (that is $n - 1$ in the queue and one in service).

$P_n(t + \delta t) =$ {There are n customers in the system at time$(t + \delta t)$}.

$$P_n(t + \delta t) = [1 - p\lambda\delta t][1 - \mu\delta t]P_n(t) + [\mu\delta t][1 - p\lambda\delta t]P_{n+1}(t) + p\lambda\delta t P_{n-1}(t) + O(\delta t)$$

Finding the difference $P_n(t + \delta t) - P_n(t)$ and dividing both sides by δt and taking limit $\delta t \to 0$, and $O(\delta t)$ Approaches to zero as rapidly as $\delta t \to 0$

Similarly, other equations can be derived.

The differential-difference equations of the model are:

$$\frac{dP_0(t)}{dt} = -p\lambda P_0(t) + \mu P_1(t)$$

$$\frac{dP_n(t)}{dt} = -[p\lambda + \mu]P_n(t) + \mu P_{n+1}(t) + p\lambda P_{n-1}(t); \quad for\ 1 \le n \le N - 1$$

$$\frac{dP_N(t)}{dt} = -\mu P_N(t) + p\lambda P_{N-1}(t) \qquad for\ n = N$$

The steady state equations of M/M/1/N queuing system with balking

(2.2.1) $0 = -p\lambda P_0(t) + \mu P_1(t)$

(2.2.2) $0 = -[p\lambda + \mu]P_n(t) + \mu P_{n+1}(t) + p\lambda P_{n-1}(t);$ $for \quad 1 \leq n \leq N-1$

(2.2.3) $0 = -\mu P_N(t) + p\lambda P_{N-1}(t) \qquad for\; n = N$

From (2.2.1) we get $P_1 = \frac{p\lambda}{\mu} P_0$

Put $n = 1$ to get P_2 in (2.2.2)

$$0 = -[p\lambda + \mu]P_1(t) + [\mu]P_2(t) + \lambda P_0(t)$$

By using $P_1 = \frac{\lambda}{\mu} P_0$ we get P_2 as

$$P_2 = \left(\frac{p\lambda}{\mu}\right)^2 P_0$$

Similarly for $n = 2$ the above procedure give

$$P_3 = \left(\frac{p\lambda}{\mu}\right)^3 P_0$$

Similarly $P_n = \left(\frac{p\lambda}{\mu}\right)^n P_0 \qquad for \quad 1 \leq n \leq N-1$

From (2.2.3) we get P_N as

$$P_N = \frac{p\lambda}{\mu} P_{N-1} \qquad \text{for} \quad n = N$$

$$P_N = \left(\frac{p\lambda}{\mu}\right)^N P_0 \qquad \text{for } n = N$$

For P_0 we use normalization condition

$$\sum_{n=0}^{N} P_n = 1$$

$$P_0 + P_1 + \cdots + P_N = 1$$

$$P_0 + \frac{\lambda}{\mu} P_0 + \left(\frac{p\lambda}{\mu}\right)^2 P_0 + \ldots + \left(\frac{p\lambda}{\mu}\right)^N P_0 = 1$$

$$P_0 = \left[1 + \frac{\lambda}{\mu} + \left(\frac{p\lambda}{\mu}\right)^2 + \ldots + \left(\frac{p\lambda}{\mu}\right)^N\right]^{-1}$$

$$P_0 = \frac{1 - \frac{p\lambda}{\mu}}{1 - \left(\frac{p\lambda}{\mu}\right)^{N+1}}$$

2.3. Performance measures:-

Expected number of customers in the system

$$L_s = \sum_{n=0}^{N} nP_n$$

$$L_s = P_0 \sum_{n=0}^{N} n \left(\frac{p\lambda}{\mu}\right)^n$$

Expected number of customers waiting in the queue

$$L_q = \sum_{n=1}^{N} (n-1)P_n$$

$$L_q = P_0 \sum_{n=1}^{N} (n-1) \left(\frac{p\lambda}{\mu}\right)^n$$

By using little's formula $L = \lambda W$ one can obtain the average waiting time in the system W_s and the average waiting time in the queue W_q

$$W_s = \frac{L_s}{\lambda} \quad \text{and} \quad W_q = \frac{L_q}{\lambda}$$

2.4 Results and Discussions:-

For results and discussions, firstly we consider the system size is $N = 5$, the arrival rate is $\lambda = 3$, the service rate is $\mu = 4$ and different probabilities of the customers either join the queue (p) or not join the queue (q).

Table1. System size when $N = 5, \lambda = 3, \mu = 4,$ and different values of p.

p	q	Ls	Ws
1	0	1.700921	0.566974
0.9	0.1	1.450127	0.483376
0.8	0.2	1.206364	0.402121
0.7	0.3	0.976942	0.325647
0.6	0.4	0.767942	0.255981
0.5	0.5	0.583268	0.194423
0.4	0.6	0.424194	0.141398
0.3	0.7	0.289544	0.096515
0.2	0.8	0.176402	0.058801
0.1	0.9	0.08108	0.027027
0	1	0	0

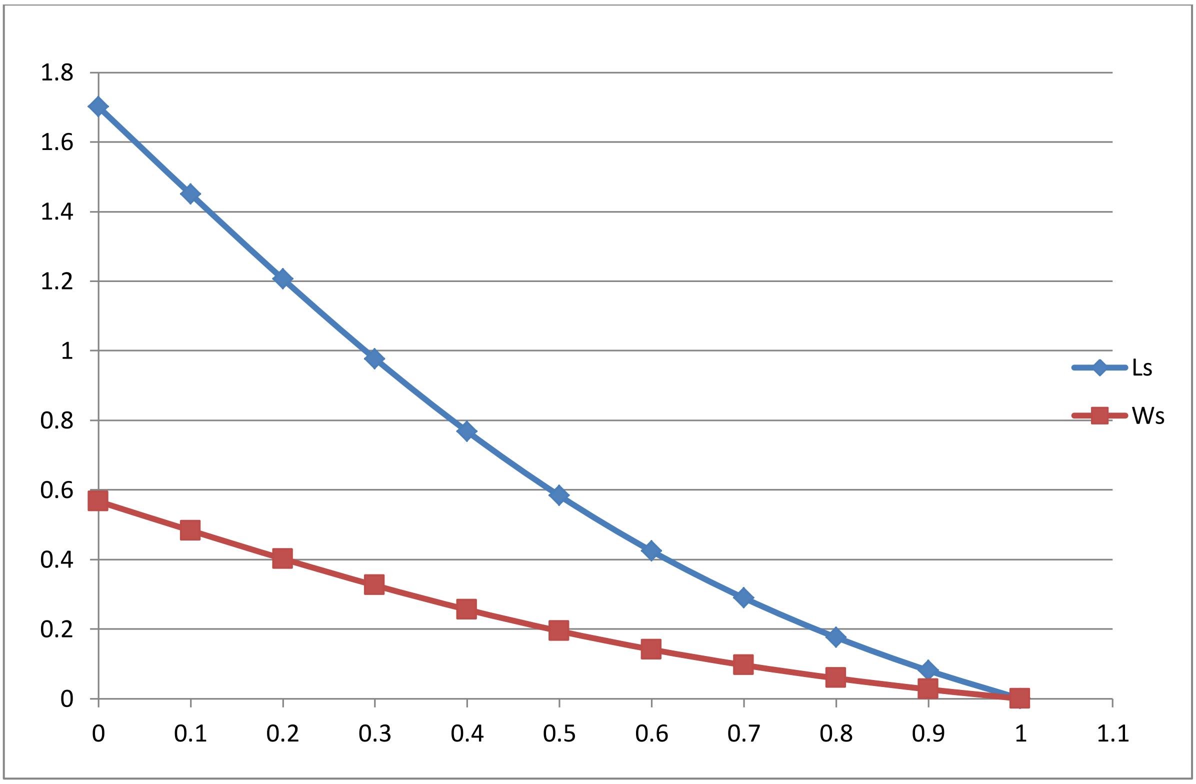

Figure1. System size and waiting time in the system Vs different probabilities of balking (q).

Secondly, we consider the system size is $N = 5$, the service rate is $\mu = 4$, the probabilities of the customers either join the queue (p= 0.4) or not join the queue (q=0.6) and different arrival rate of λ.

Table2. System size when $N = 5$, $\mu = 4$, $q = 0.6$ and different values of λ.

λ	Ls	Ws
3	0.424194	0.141398
3.1	0.443946	0.143208
3.2	0.464139	0.145043
3.3	0.484778	0.146903
3.4	0.505868	0.148785
3.5	0.527412	0.150689
3.6	0.549411	0.152614
3.7	0.571868	0.154559
3.8	0.594783	0.156522
3.9	0.618157	0.158502
4	0.64199	0.160497

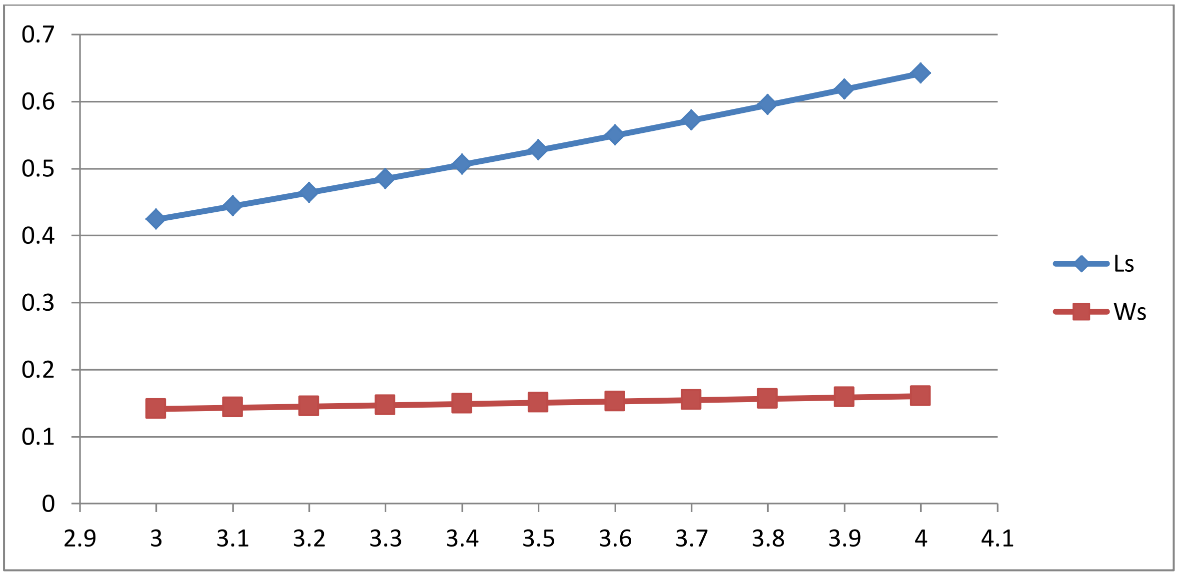

Figure2. System size and waiting time in the system Vs different values of λ.

Thirdly, we consider the system size is $N = 5$, the arrival rate is $\lambda = 3$, the probabilities of the customers either join the queue (p= 0.4) or not join the queue (q=0.6) and different service rate of μ.

Table3. System size for $N = 5$, $\lambda = 3, q = 0.6$ and different values of μ.

μ	Ls	Ws
3	0.64199	0.213997
3.1	0.611324	0.203775
3.2	0.583268	0.194423
3.3	0.557524	0.185841
3.4	0.533835	0.177945
3.5	0.511977	0.170659
3.6	0.491758	0.163919
3.7	0.473009	0.15767
3.8	0.455582	0.151861
3.9	0.439349	0.14645
4	0.424194	0.141398

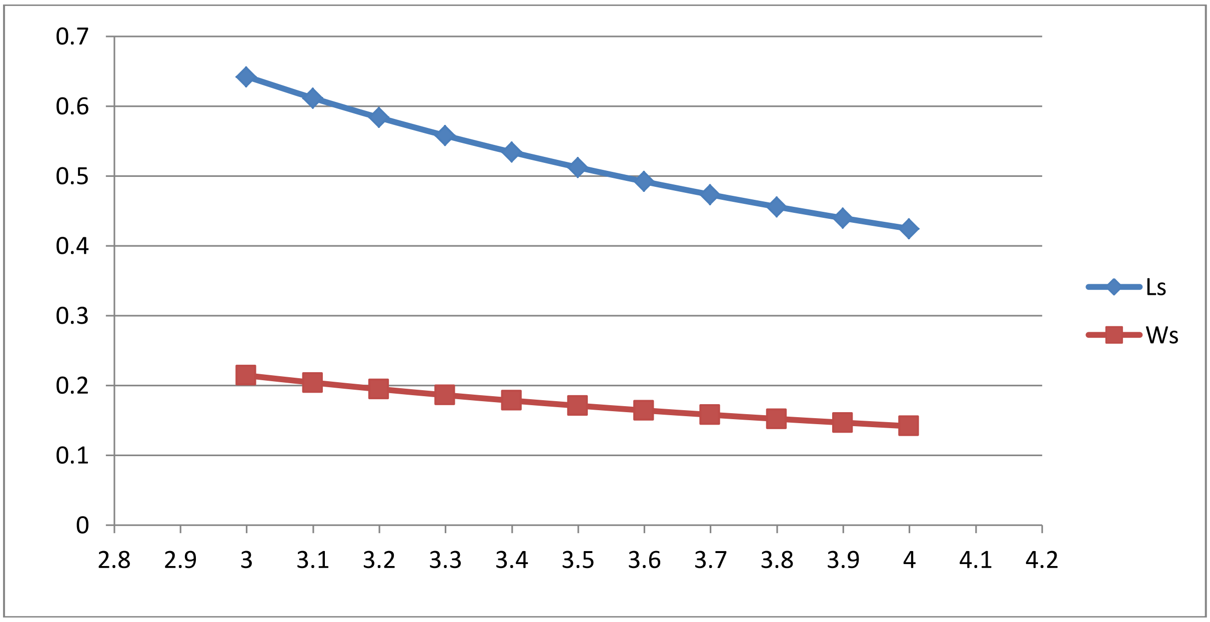

Figure3. System size and waiting time in the system Vs different values of μ.

From figure 1 one can see that a regular decrease in average system size and waiting time in the system with the increase in probability of the balking. From the figures 2 one can see that with the increase in average arrival there is exponential increase in the expected system size and waiting time in the system respectively. From the figures 3 one can see that with the increase in average service rates, there is exponential decrease in the expected system size and waiting time in the system respectively.

3. CONCLUSION

In this paper we have studied and generalized the M/M/1/N Queuing System for Impatient Customers with balking. The effect of the probability of balking on the expected system size and waiting time in the system has been studied. The numerical results show that the average system size and waiting time in the system decreases proportionately and steadily as the probability of balking is increases. The meaning of decrease in expected system size and waiting time in the system is not that balking is good. The customer impatience (due to balking) leads to the loss of potential customers and thereby results into the loss in the total revenue.

REFERENCES

1. **Ancker, Jr. C. J. and Gafarian, A. V.,** *Some Queuing Problems with Balking and Reneging- I, Operations Research, 11(1), (1963), 88-100.*
2. **Ancker, Jr. C. J. and Gafarian, A. V.,***Some Queuing Problems with Balking and Reneging- II, Operations Research, 11 (6), 928-937.*
3. **Gross, D., John, F., Shortle, James, M., Thompson and Harris, C. M.,** *Fundamentals of Queuing Theory, 3rd ed., Wiley, New York, (1998).*
4. **Haight, F. A.,** *Queuing with Balking- I, Biometrika, 44 (3-4), (1957), 360-369.*
5. **Haight, F. A.,** *Queueing with Reneging, Metrika 2(1), (1959), 186-197.*
6. **Kangzhou Wang, Na Li, and Zhibin Jiang,** *Queuing System with Impatient Customers: A Review, Service Operations and Logistics and Informatics, 2010 IEEE International Conference, (2010), 82- 87.*
7. **Rakesh Kumar and Sumeet Kumar Sharma,** *An M/M/1/N Queueing Model with Retention of Reneged Customers and Balking, American Journal of Operational Research, 2(1), (2012), 1-5.*
8. **Rakesh Kumar and Sumeet Kumar Sharma,** *M/M/1/N Queuing System with Retention of Reneged Customers, Pak. Jour. Stat. Oper. Res. Vol. VIII, No.4, (2012), 859-866.*
9. **Rakesh Kumar and Bhupender Kumar Som,** *An M/M/1/N Queuing System with Reverse Balking and Reverse Reneging, Advanced Modeling and Optimization, Vol.16, Number 2, (2014), 339 – 353.*
10. **Rakesh Kumar and Sumeet Kumar Sharma,** *Queuing with Reneging, Balking and Retention of Reneged Customers, International Journal of Mathematical Models and Methods in Applied Sciences, Issue 7, Volume 6, (2012), 819 - 828.*