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CUBE MULTIPLICATIVE LABELING FOR SOME GRAPHS

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ABSTRACT

Let G be (p,q) graph. G is said to be a Cube multiplicative labeling if there exists a bijection $f: V(G) \rightarrow \{1,2,\ldots,p\}$ such that the induced function $f: E(G) \rightarrow N$ given by $f^*(uv) = [f(u)]^3.[f(v)]^3$ for every $uv \in E(G)$ are all distinct. A graph which admits Cube multiplicative labeling is called Cube multiplicative graph. In this paper we prove Star graph, path, triangular snake, cycle, quadrilateral snake are Cube multiplicative graphs.

KEYWORDS: *Cube multiplicative labeling, Star graph, Path, Triangular snake, Cycle, Quadrilateral snake.*

1. INTRODUCTION

In this paper we deal only finite, simple, connected and undirected graphs obtained through graph operations. Graph labeling was first introduced in the mid sixties. A labeling of a graph G is an assignment of labels to vertices Vast amount of literature is available on different types of graph labeling and more than 1000 research papers have been published so far in past three decades. The origin of labeling can be attributed to Rosa. A useful survey on graph labelling by J.A.Gallian(2015) can be found in [1]. In most applications labels are positive or nonnegative integers. Beinke and Hedge call a graph with p vertices strongly multiplicative if the vertices of G can be labeled with distinct integers 1,2,...,p such that the labels induced on the edges by the product of the end vertices are distinct J. Shiama [5] proved that Path is a Square difference graph. Triangular snakes [2] was proved by S.Murugesan, J.Shiama in Square difference 3-equitable labeling of some graphs. Quadrilateral snakes [4] was proved by S.S.Sandhya, E.Ebin Raja Merly and B.Shiny in Super Geometric Mean Labeling On Double Triangular Snakes. M.Muthusamy, K.C.Raajasekar, J.Baskar Babujee [3] proved that cycles are Strongly multiplicative graphs. R.Sridevi, S.NavanethaKrishnan, K.Nagaraj, A.Nagaraj proved that Star graphs are Odd-Even Graceful. A new concept of Cube multiplicative labeling is defined here. Let G be (p,q) graph. G is said to be a Cube multiplicative labeling if there exists a bijection $f: V(G) \rightarrow \{1,2,\ldots,p\}$such that the induced function $f: E(G) \rightarrow N$ given by $f^*(uv) = [f(u)]^3.[f(v)]^3$ for every

$uv \in E(G)$ are all distinct. A graph which admits Cube multiplicative labeling is called Cube multiplicative graph.

2.DEFINITIONS

2.1 Cube multiplicative graph:-

Let G be (p, q) graph. G is said to be a Cube multiplicative labeling if there exists a bijection $f: V(G) \rightarrow \{1,2, \ldots, p\}$ such that the induced function $f: E(G) \rightarrow N$ given by $f^*(uv) = [f(u)]^3 . [f(v)]^3$ for every $uv \in E(G)$ are all distinct. A graph which admits Cube multiplicative labeling is called Cube multiplicative graph.

2.2 Cycle:-

A closed walk in which no vertex except the end vertices is repeated is called a cycle or a circuit.

2.3 Path:-

A path is a simple graph whose vertices can be ordered so that two vertices are adjacent if and only if they are consecutive in the list.

2.4 Triangular Snake:-

A Triangular Snake T_n is obtained from a path $u_{1,}u_{2,\ldots,}u_n$ by joining u_i and u_{i+1} to a new vertex v_i, $1 \leq i \leq n-1$. That is every edge of a Path is replaced by a triangle C_3.

2.5 Star:-

A Complete bipartite graph $K_{1,n}$ or S_n is called a Star and it has n+1 vertices and n edges.

2.6 Quadrilateral snake:-

The Quadrilateral snake Q_n is obtained from a Path $u_1, u_2, \ldots, u_n$ by joining u_i, u_{i+1} to new vertices v_i, w_i respectively and then joining v_i and w_i. That is every edge of Path is replaced by a Cycle C_4.

3. RESULTS

3.1 Theorem:-

Star graphs $K_{1,n}$ are Cube multiplicative graphs.

Proof:

Let $v_{1,}v_{2,\ldots,}v_n$ be the pendant vertices of the star graph $K_{1,n}$ and c be the center vertex. Let the vertex set be $V(G) = n+1$ and the edge set be $E(G) = n$. The mapping $f: V(G) \rightarrow \{1,2, \ldots, n+1\}$ is defined by $f(u_i) = i+1$; $1 \leq i \leq n$ and $f(c) = 1$. The induced function defined by $f^*(uv) = [f(u)]^3 . [f(v)]^3$ is an increasing function. The values of the edges are distinct. Hence the Star graphs are Cube multiplicative graphs.

3.1.1 Example:

Star graph $K_{1,8}$ is a Cube multiplicative graph.

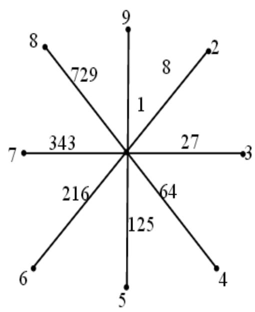

Fig 1

3.2 Theorem:

The Path P_n is a Cube multiplicative graph.

Proof:

Let the graph G be a path P_n. Let $V(G) = n$ and $E(G) = n - 1$. The mapping $f: V(G) \to \{1,2,\dots,p\}$ is defined by $f(u_i) = i$; $1 \le i \le n$ and the induced function $f^*: E(G) \to N$ defined by $f^*(uv) = [f(u)]^3.[f(v)]^3$. The edges of the path have distinct values and hence a one to one mapping. Hence this graph is a Cube multiplicative graph.

3.2.1 Example:

The Path P_3 is a Cube multiplicative graph.

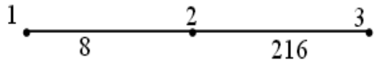

Fig.2

3.3 Theorem:

The Cycle C_n is a Cube multiplicative graph.

Proof:

Let $V(G) = \{v_{1,}\, v_2, \dots, v_n\}$ be the vertex set and $E(G) = E_1 \cup E_2 \cup \dots \cup E_n$ be the edge set where $E_1 = \{v_i\, v_{i+1}\}; 1 \le i \le n$ and $E_2 = \{v_n v_1\}$ of the graph C_n. Define a bijection $f: V(G) \to \{1,2,\dots,n\}$ such that

Case 1:

If n is odd

$$f(v_i) = \begin{cases} 2i - 1 \;\; for\; 1 \le i \le \dfrac{n+1}{2} \\ 2(n - i + 1) \;\; for\; \left[\dfrac{n+1}{2}\right] + 1 \le i \le n \end{cases}$$

Case 2:

If n is even

$$f(v_i) = \begin{cases} 2i - 1 \;\; for\; 1 \le i \le \frac{n}{2} \\ 2(n - i + 1)\; for\; [\frac{n}{2} + 1] \le i \le n \end{cases}$$

3.3.1 Example:

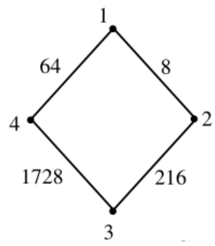

Fig.3

3.4 Theorem:

Triangular snakes are Cube multiplicative graphs

Proof:

Consider the graph T_2 with vertices labeled as shown in the Fig 4. Here $|V(G)| = 2n + 1$ and $|E(G)| = 3n$.

We define a function $f: V(G) \to \{1,2,\dots,2n + 1\}$ by

$f(u_i) = 2i - 1;\; i = 1,2, \dots n + 1$

$f(v_i) = 2i;\; i = 1,2, \dots n$

And the induced function $f: E(G) \to N$ defined by $f^*(uv) = [f(u)]^3 . [f(v)]^3$ for every $uv \in E(G)$ are all distinct. Hence T_2 is a Cube multiplicative graph.

3.4.1 Example:

The graph T_2 is a Cube multiplicative graph.

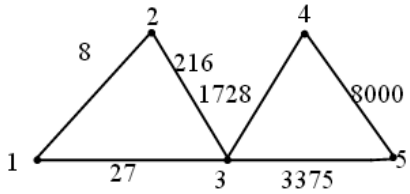

Fig.4

3.5 Theorem:

Quadrilateral Snakes are Cube multiplicative graphs.

Proof:

Consider Q_n with vertices labeled. Here $|V(G)| = 3n + 1$ and $|E(G)| = 4n$. We define a function $f: V(G) \to \{1,2,\dots,3n+1\}$ by

$f(u_i) = 3i - 2;\ i = 1,2,\dots,n+1$

$f(v_i) = 3i;\ i = 1,2,\dots,n$

$f(w_i) = 3i - 1;\ i = 1,2,\dots,n$

And the induced function $f: E(G) \to N$ defined by $f^*(uv) = [f(u)]^3 . [f(v)]^3$ for every $uv \in E(G$ are all distinct. Hence Q_n are Cube multiplicative graphs.

3.5.1. Example

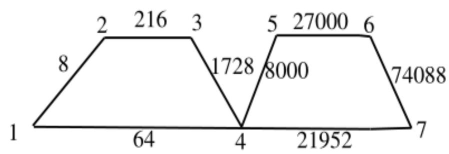

Fig.5

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