SJIF Impact Factor: 7.001| ISI I.F.Value:1.241| Journal DOI: 10.36713/epra2016 ISSN: 2455-7838(Online)
EPRA International Journal of Research and Development (IJRD)
Volume: 5 | Issue: 12 | December 2020 - Peer Reviewed Journal

HEAT CONDUCTED THROUGH FINS OF VARYING CROSS-SECTIONS VIA ROHIT TRANSFORM

Neeraj Pandita
Assistant Professor, Department of Mechanical Engineering, Yogananda College of Engineering and Technology, Jammu

Rohit Gupta
Lecturer of Physics, Department of Applied Sciences, Yogananda College of Engineering and Technology, Jammu

ABSTRACT

The conduction of heat takes place through the fins or spines from particle to particle due to temperature gradient in the direction of decreasing temperature. Heat is not lost equally by each element of the fin but is lost mostly near the base of the fin. Thus there would be wastage of the material if a uniform fin is used. These aspects demand the construction of the fins of varying cross-sections like triangular fin, hyperbolic fin and the parabolic fin. The triangular and parabolic fins of varying cross-sections are usually analyzed by ordinary calculus approach. The paper analyzes triangular and parabolic fins of varying cross-sections to find the rate of conduction of heat through them via the new integral transform called Rohit Transform.

INDEX TERMS: *Triangular fin, Parabolic fin, Rohit Transform, Temperature Distribution, Rate of Conduction of Heat.*

I. INTRODUCTION

The conduction of heat takes place through the fins or spines from particle to particle due to temperature gradient in the direction of decreasing temperature [1, 2]. Fins or spines are the extended surfaces which are mostly used in the devices which exchange heat [3-8] like computer central processing unit, power plants, radiators, heat sinks, etc. The triangular and parabolic fins of varying cross-sections are usually analyzed by ordinary calculus approach [4-7]. The paper analyzes triangular and parabolic fins of varying cross-sections to find the rate of conduction of heat through them via the new integral transform called Rohit Transform. This Transform has been put forward by the author Rohit Gupta in recent years [8] and is not widely known. The Rohit Transform has been applied in science and engineering to solve most of the initial value problems in science and engineering [9-16]. The Rohit Transform comes out to be very effective tool to find the temperature distribution along a triangular fin, and a parabolic fin and hence the rate of conduction of heat through them.

Basics of Rohit Transform:

The Rohit Transform [10, 11, 12] of g(y), $y \geq 0$ is defined as

$R\{g(y)\} = r^3 \int_0^\infty e^{-ry}\, g(y)dy = G(r)$, provided that the integral is convergent, where r may be a real or complex parameter.

The Rohit Transform of some derivatives [7, 13, 14, 15] of g(y) is given by

$R\,\{g'(y)\} = rR\{g(y)\} - r^3 g(0)$

Or

$R\,\{g'(y)\} = rG(r) - r^3 g(0)$,

$R\{g''(y)\} = r^2 G(r) - r^4 g(0) - r^3 g'(0)$,

$$R\{yg(y)\} = \frac{3}{r}R\{g(y)\} - \frac{d}{dr}R\{g(y)\},$$

SJIF Impact Factor: 7.001| ISI I.F.Value:1.241| Journal DOI: 10.36713/epra2016 ISSN: 2455-7838(Online)
EPRA International Journal of Research and Development (IJRD)
Volume: 5 | Issue: 12 | December 2020 - Peer Reviewed Journal

$$R\{yg'(y)\} = 2R\{g(y)\} - r\frac{d}{dr}R\{g(y)\},$$

$$R\{yg''(y)\} = rR\{g(y)\} + r^3 g(0) - r^2\frac{d}{dr}R\{g(y)\}$$

II. MATERIAL AND METHOD

Case I: Triangular Fin

The differential equation for analyzing a triangular fin [5, 8] (assuming that heat flow pertains to one - dimensional conduction of heat) is given by

$$\theta''(x) + \frac{1}{x}\theta'(x) - \frac{D^2}{x}\theta(x) = 0 \ldots\ldots (1),$$

$where\ D = \sqrt{\frac{2hL}{tk}}$, L is the length of the fin between the base at x = L and the tip at x = 0, t is the thickness of the fin which increases uniformly from zero at the tip to t at the base, k is thermal conductivity, h is the coefficient of transfer of heat by convection, $\theta(x) = T(x) - T_s$, T_s is the temperature of the environment of the fin and T_0 is the temperature at the base x = 0 of the fin.

Multiplying both sides of (1) by x, we get

$$x\,\theta''(x) + \theta'(x) - D^2\theta(x) = 0 \ldots\ldots (2)$$

The Rohit Transform [10, 11, 12, 13] of (2) gives

$$\left[-p^2\frac{d}{dp}\theta(p) + p^3\,\theta(0) + p\,\theta(p)\right] + p\,\theta(p)] - p^3\,\theta(0) - D^2\theta(p) = 0 \ldots\ldots (3)$$

Put $\theta(0) = b\ and\ \theta'(0) = a$, and simplifying and rearranging (3), we get

$$\frac{\theta'(p)}{\theta(p)} = [\frac{2}{p} - \frac{D^2}{p^2}] \ldots\ldots (4)$$

Integrating both sides of (4) w.r.t. p and simplifying, we get

$$\log_e\theta(p) = [2\log_e p + D^2\frac{1}{p} + \log_e c] \ldots.. (5)$$

Simplifying (5), we get

$$\theta(p) = cp^2 e^{(D^2\frac{1}{p})}$$

Expanding the exponential term, we get

$$\theta(p) = cp^2\left[1 + D^2\frac{1}{p} + \frac{\left(D^2\frac{1}{p}\right)^2}{2!} + \frac{\left(D^2\frac{1}{p}\right)^3}{3!} + \frac{\left(D^2\frac{1}{p}\right)^4}{4!} \ldots\ldots\right]$$

or

$$\theta(p) = c\left[p^2 + D^2 p + \frac{D^4}{2!} + \frac{D^6}{3!}\frac{1}{p} + \frac{D^8}{4!}\frac{1}{p^2} \ldots\ldots\right] \ldots\ldots (6)$$

The inverse Rohit Transform [8] of (6) provides

$$\theta(x) = c\left[1 + D^2 x + \frac{D^4x^2}{2!\,2!} + \frac{D^6x^3}{3!\,3!} + \frac{D^8x^4}{4!\,4!} \ldots\right]$$

or

$$\theta(x) = c\begin{bmatrix}1 + \frac{1}{4}\left(2D\sqrt{x}\right)^2 + \frac{1}{2!2!}\left(\frac{2D\sqrt{x}}{2}\right)^4 \\ + \frac{1}{3!3!}(\frac{2D\sqrt{x}}{2})^6 + \frac{1}{4!4!}(\frac{2D\sqrt{x}}{2})^8 \ldots\end{bmatrix} \ldots.. (7)$$

The modified Bessel function [17] of the first kind of order n and its first order derivative are given by

$$I_n(z) = \sum_{r=0}^{\infty}\frac{1}{r!(n+r)!}(\frac{z}{2})^{n+2r} \ldots\ldots (8)$$

Also $\frac{d}{dx}\left(I_n(z)\right) = I_{n+1}(z)\frac{d}{dx}(z) \ldots. (9)$

Put z = $2D\sqrt{x}$ and n = 0, we get

$$I_0\left(2D\sqrt{x}\right) = \sum_{r=0}^{\infty}\frac{1}{r!\,r!}(\frac{2D\sqrt{x}}{2})^{2r}$$

Or

$$I_0\left(2D\sqrt{x}\right) = 1 + \left(\frac{2D\sqrt{x}}{2}\right)^2 + \frac{1}{2!2!}(\frac{2D\sqrt{x}}{2})^4 + \frac{1}{3!3!}(\frac{2D\sqrt{x}}{2})^6 + \frac{1}{4!4!}(\frac{2D\sqrt{x}}{2})^8 \ldots\ldots$$

Hence (7) can be rewritten as

$$\theta(x) = cI_0\left(2D\sqrt{x}\right) \ldots. (10)$$

To find the constant c, at x = L, $\Theta(L) = \Theta_0$, therefore, c = $\frac{\Theta_0}{I_0(2D\sqrt{L})}$

Hence (10) can be written as

$$\Theta(x) = \frac{\Theta_0}{I_0(2D\sqrt{L})}I_0\left(2D\sqrt{x}\right) \ldots (11)$$

The equation (11) gives the temperature distribution along the length of the triangular fin.

SJIF Impact Factor: 7.001| ISI I.F.Value:1.241| Journal DOI: 10.36713/epra2016 ISSN: 2455-7838(Online)
EPRA International Journal of Research and Development (IJRD)
Volume: 5 | Issue: 12 | December 2020 - Peer Reviewed Journal

The heat conducted through the triangular is given by the Fourier's Law [18, 19, 20] of heat conduction as

$$H = kA\left(\Theta'(x)\right)_{x=L} = kbt\left(\Theta'(x)\right)_{x=L}$$

Using (11), we get

$$H = kbt\,\frac{\Theta_0}{I_0(2D\sqrt{L})} I_1(2D\sqrt{L})\ \left(\frac{d}{dx}(2D\sqrt{x})\right)_{x=L}$$

On simplifying, we get

$$H = kbtD\,\frac{\Theta_0}{\sqrt{L}\,I_0(2D\sqrt{L})} I_1(2D\sqrt{L}) \ \ldots. (12)$$

Put the value of D, we get

$$H = b\sqrt{2hkt}\frac{\Theta_0}{I_0(2D\sqrt{l})} I_1(2D\sqrt{L}) \ldots\ldots (13)$$

This equation (13) gives the expression for the rate of conduction of heat through the triangular fin.

Case II: Parabolic fin

The differential equation for analyzing a parabolic fin [5, 9] (assuming that heat flow pertains to one dimensional conduction of heat) is given by

$$x^2\theta''(x) + 2x\theta'(x) - M^2l^2\theta(x) = 0 \ldots\ldots (14),$$

$where\ M = \sqrt{\frac{2h}{tk}}$, l is the length of the fin between the base at x = l and the tip at x = 0, t is the thickness of the fin which increases uniformly from zero at the tip to t at the base, k is thermal conductivity, h is the coefficient of transfer of heat by convection, $\theta(x) = T(x) - T_s$, T_s is the temperature of the environment of the fin and T_0 is the temperature at the base x = 0 of the fin.

Substituting $x = e^z$, the equation (14) can be rewritten into a form:

$$\theta''(x) + \theta'(x) - M^2l^2\theta(x) = 0 \ldots\ldots (15) \quad \theta' \equiv \frac{d}{dz}$$

The Rohit Transform [13, 14, 15, 16] of (15) gives

$$[p^2\theta(p) - p^4\theta(0) - p^3\theta'(0)] + p\,\theta(p) - p^3\,\theta(0) - M^2l^2\theta(p) = 0\ldots.. (16)$$

Put $\theta(0) = P\ and\ \theta'(0) = Q$, and simplifying and rearranging (16), we get

$$\theta(p) = \frac{p^3(p+1)P + p^3Q}{p^2 + p - M^2l^2}$$

Or

$$\theta(p) = \frac{p^3[pP + (P+Q)]}{(p-c_1)(p+c_2)} \ldots (17)$$

$where\ c_1 = \frac{-1+(1+4M^2l^2)^{1/2}}{2}\ and\ c_1 = \frac{-1-(1+4M^2l^2)^{1/2}}{2}.$

This equation (17) can be rewritten as

$$\theta(p) = \frac{c_1P + P + Q}{c_1 + c_2}\frac{p^3}{(p-c_1)} + \frac{c_2P + P + Q}{c_2 - c_1}\frac{p^3}{(p+c_2)} \ldots. (18)$$

The inverse Rohit Transform [8] of (18) provides

$$\Theta(x) = \frac{c_1P + P + Q}{c_1 + c_2}e^{c_1z} + \frac{c_2P + P + Q}{c_2 - c_1}e^{-c_2z}$$

Or

$$\Theta(x) = \frac{c_1P+P+Q}{c_1+c_2}x^{c_1} + \frac{c_2P+P+Q}{c_2-c_1}x^{-c_2} \ldots. (19)$$

As $\Theta(0)\ is\ finite$ [4 − 7], $therefore,$ $the\ term\ \frac{c_2P+P+Q}{c_2-c_1}x^{-c_2}\ is\ equated\ to\ zero$

$i.e. \frac{c_2P+P+Q}{c_2-c_1}x^{-c_2} = 0, which\ gives\ c_2P + P + Q = 0$ or $Q = -(c_2P + P)$.

From (19), we have

$$\Theta(x) = \frac{c_1-c_2}{c_1+c_2}P\,x^{c_1} \ldots. (20)$$

To find the constant P, at x = l, $\Theta(l) = \Theta_0$, therefore from (20), $P = \frac{c_1+c_2}{c_1-c_2}\Theta_0 l^{-c_1}$

Hence (20) can be rewritten as

$$\Theta(x) = \Theta_0 l^{-c_1}x^{c_1}$$

Or

$$\Theta(x) = \Theta_0(x/l)^{c_1}\ldots (21)$$

The equation (21) gives the temperature distribution along the length of the parabolic fin.

The heat conducted through the parabolic fin is given by the Fourier's Law of heat conduction [19, 20] as

$$H = kA\left(\Theta'(x)\right)_{x=l} = kbt\left(\Theta'(x)\right)_{x=l}$$

Using equation (21), we get

$H = \text{kbt}\, \Theta_0 c_1/l$

Or

$H = \text{kbt}\, \Theta_0 \frac{-1+(1+4M^2l^2)^{1/2}}{2l}$ ….. (22)

This equation (22) gives the expression for the rate of conduction of heat through the parabolic fin.

III. RESULT AND CONCLUSION

We have found find the temperature distribution along the lengths of the triangular fin as well as parabolic fin and hence the rate of conduction of heat through them via Rohit Transform means. It is found that with the increase in the length of the triangular fin or parabolic fin, temperature increases and hence the rate of conduction of heat at any cross-section of the triangular fin or parabolic fin increases. The method has come out to be a very effective tool to find temperature distribution along the lengths of the triangular fin as well as parabolic fin and hence the rate of conduction of heat through them.

REFERENCES

1. *Rohit Gupta, Amit Pal Singh, Dinesh Verma, Flow of Heat through A Plane Wall, And Through A Finite Fin Insulated At the Tip, International Journal of Scientific & Technology Research, Vol. 8, Issue 10, Oct. 2019, pp. 125-128.*
2. *Rohit Gupta, Rahul Gupta, Dinesh Verma, Laplace Transform Approach for the Heat Dissipation from an Infinite Fin Surface, Global Journal Of Engineering Science And Researches, 6(2), February 2019, pp. 96-101.*
3. *Rohit Gupta, Rahul Gupta, Heat Dissipation From The Finite Fin Surface Losing Heat At The Tip, International Journal of Research and Analytical Reviews, Vol. 5, Issue 3, Sep. 2018, pp. 138-143.*
4. *J.P. Holman, Heat transfer. 10th Edition. Publisher: Tata McGraw-Hill Education Pvt. Ltd, 2016.*
5. *Gayatree Behura, Banamali Dalai, Analysis of heat transfer for the varying surface fin, international journal of scientific and engineering research, volume 9, issue 4, April-2018.*
6. *D.S. Kumar, Heat and mass transfer, (Seventh revised edition), Publisher: S K Kataria and Sons, 2013.*
7. *P.K. Nag, Heat and mass transfer. 3rd Edition. Publisher: Tata McGraw-Hill Education Pvt. Ltd., 2011.*
8. *Rohit Gupta, Neeraj Pandita, Dinesh Verma, Conduction of heat through the thin and straight triangular fin, ASIO Journal of Engineering & Technological Perspective Research (ASIO-JETPR), Volume 5, Issue 1, 2020, pp. 01-03.*
9. *Rohit Gupta, Neeraj Pandita, Rahul Gupta, Heat conducted through a parabolic fin via Means of Elzaki transform, Journal of Engineering Sciences, Vol. 11, Issue 1, Jan. 2020, pp. 533-535.*
10. *Rohit Gupta, On Novel Integral Transform: Rohit Transform and Its Application to Boundary Value Problems, "ASIO Journal of Chemistry, Physics, Mathematics and Applied Sciences", 4(1), 2020, pp. 08-13.*
11. *Rohit Gupta, Rahul Gupta, Dinesh Verma, Solving Schrodinger equation for a quantum mechanical particle by a new integral transform: Rohit Transform, "ASIO Journal of Chemistry, Physics, Mathematics & Applied Sciences", 2020, 4(1), pp. 32-36.*
12. *Rohit Gupta, Rahul Gupta, Analysis of RLC circuits with exponential excitation sources by a new integral transform: Rohit Transform, "ASIO Journal of Engineering & Technological Perspective Research", 2020, 5(1), pp.22-24.*
13. *Rohit Gupta, Yuvraj Singh, Dinesh Verma, Response of a basic series inverter by the application of convolution theorem, "ASIO Journal of Engineering & Technological Perspective Research", 5(1), 2020, pp. 14-17.*
14. *Anamika, Rohit Gupta, Analysis Of Basic Series Inverter Via The Application Of Rohit Transform, International Journal of Advance Research and Innovative Ideas in Education, Vol-6, Issue-6, 2020, pp. 868-873.*
15. *Loveneesh Talwar, Rohit Gupta, Analysis of Electric Network Circuits with Sinusoidal Potential Sources via Rohit Transform, International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering (IJAREEIE), Volume 9, Issue 11, November 2020, pp. 3929-3023.*
16. *Neeraj Pandita, and Rohit Gupta. Analysis Of Uniform Infinite Fin Via Means Of Rohit Transform, International Journal Of Advance Research And Innovative Ideas In Education, Volume 6, Issue 6, 2020, pp. 1033-1036.*
17. *N.P. Bali and Dr. Manish Goyal, A Text Book of Engineering Mathematics, 9th edition, 2014.*
18. *Rohit Gupta, Rahul Gupta, Matrix method approach for the temperature distribution and heat flow along a conducting bar connected between two heat sources, Journal of Emerging Technologies and Innovative Research, Vol. 5 Issue 9, Sep. 2018, pp. 210-214.*
19. *Rohit Gupta, Dinesh Verma, Heat emitted from a uniform rod with ends maintained at unequal*

temperatures, "ASIO Journal of Chemistry, Physics, Mathematics & Applied Sciences (ASIO-JCPMAS)", Volume 4, Issue 1, 2020, pp. 01-03.

20. *Rahul Gupta and Rohit Gupta, Laplace Transform method for obtaining the temperature distribution and the heat flow along a uniform conducting rod connected between two thermal reservoirs maintained at different temperatures, Pramana Research Journal, Volume 8, Issue 9, 2018, pp. 47-54.*