



SIGNIFICANCE OF DIFFERENTIAL CALCULUS IN THE FIELD OF ENGINEERING

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ABSTRACT

Each day I would go out to measure the amount of water flowing down rivers in the province. We used water flow meters like the one shown in the figure below. We would go across the river in waders and take measurements. We did about 5 to 10 depending on the width of the river. At each point we would measure the depth and the velocity at that point.

After we were done for the day one of us would enter the data into a program to determine the flow. We did this for each river periodically. It was a fantastic job. Especially in the summer. One day the boss found out that I was doing computer science at university and he asked me to write a program to calculate the water flow from the data for their new computer which they had purchased. It had an HP plotter. He also asked me to generate plots of the cross section of the river with the data on it. So I had to figure out how to calculate the flow and how to use the HP plotter language. The point of this article is to explain what I did, that I got it wrong, and how to use Calculus to fix it, and, finally, now that I've done a bit of research, how the Geological Survey does this calculation. I am using this example for my teaching. It's nice. It's got a bit of everything. I think this is a great problem for an integral calculus class using Maple, or class on numerical methods. One useful tool that I've also learned is Maple's canvas facility which I'll use for the sketches in this article.

INTRODUCTION

Some engineers directly use calculus in their daily practice and some use computer programs based on calculus that simplify engineering design. Two methods of calculus, differentiation and integration, are particularly useful in the practice of engineering, and are generally used for optimization and summation, respectively. Many aspects of civil engineering require calculus. Geological work calculation is also done easily with Calculus . Firstly, derivation of the basic fluid mechanics equations requires calculus. For example, all hydraulic analysis programs, which aid in the design of storm drain and open channel systems, use calculus numerical methods to obtain the results. In hydrology, volume is calculated as the area under the curve of a plot of flow versus time and is accomplished using calculus.

Without calculation we cant proceed in structural Engineering . In structural engineering, calculus is used to determine the forces in complex configurations of structural elements. Structural analysis relating to seismic design requires calculus. In a soil structure context, calculations of bearing capacity and shear strength of soil are done using calculus, as is the determination of lateral earth pressure and slope stability in complex situations.

Many examples of the use of calculus are found in mechanical engineering, such as computing the surface area of complex objects to determine frictional forces, designing a pump according to flow rate and head, and calculating the power provided by a battery system. Newton's law of cooling is a governing differential equation in HVAC design that requires integration to solve.



METHODOLOGY (THE PROBLEM)

Suppose we take n measurements (in meters) across the river at points $L < x_1 < x_2 < \dots < x_n < R$ where L and R are the left and right edges of the river. At each point x_i we measure the depth d_i in meters and the velocity v_i in meters per second. So the depth and velocity at L and R is zero.

Because water flows faster at the surface of a river than at the bottom, there are different methods for taking a velocity measurement. We used two. For depths less than 0.5 m, we measured the velocity at 60% below the surface. For greater depths we took two measurements, one 20% below the surface and one 20% above the bottom and used the average of the two.

RIEMANN SUMS

In a first course in integral calculus, we learn how to construct definite integrals as Riemann sums. Many instructors will also apply this to volumes of revolution. Shown in the figure I've divided the section up into subsections (the vertical lines). Each subsection is a little trapezoid. What is the velocity of the water in each little trapezoid and the area of each little trapezoid? Using linear interpolation to interpolate the velocity $v(x)$ on the section, that is, for $x_L \leq x \leq x_R$ we have

$$v(x) = v_L + \frac{v_R - v_L}{x_R - x_L} \cdot (x - x_L)$$

Similarly we can approximate the depth $d(x)$ for $x_L \leq x \leq x_R$ by a straight line as shown in the figure

$$d(x) = d_L + \frac{d_R - d_L}{x_R - x_L} \cdot (x - x_L)$$

Now to approximate as a Riemann sum we take the depth at positions on the interval and multiply by the velocity at that position and the width of the interval. This is just the area of each subsection multiplied by a velocity. So this

$$Area = \int_{x_L}^{x_R} d(x) dx$$

just the definite integral for the area, but we are multiplying by velocity. Therefore the flow on the section is given by

$$Flow = \int_{x_L}^{x_R} v(x) \cdot d(x) dx$$

A few Examples :

A cylindrical tank with a radius of 20 centimeters and an arbitrary height is filling with water at a rate of 1.5 liters per second. What is the rate of change of the water level (its height)?

Explanation:

The first step we may take is to write out the formula for the volume of a cylinder:



$$V = \pi r^2 h$$

Where r represents the radius and h the height.

With this known we can find the rate change of volume with respect to height, by deriving these functions with respect to height:

$$dV = \pi r^2 dh$$

Since we're interested in the rate change of height, the dh term, let's isolate that on one side of the equation:

$$dh = dV / \pi r^2$$

dV , the rate change of volume, is given to us as 1.5 liters per second, or 1500 cm³/second. Plugging in our known values, we can thus solve for dh :

$dh = (1500 \text{ cm}^3 / \pi (20 \text{ cm})^2) = 154 \text{ cm/s}$
The volume v of water (in liters) in a pool at time t (in minutes) is defined by the equation $v(t) = 7t^2 + 5t - 12$. If Paul were to siphon water from the pool using an industrial-strength hose, what would be the rate of flow at $t = 2$ in liters per minute?

Possible Answers:

Explanation:

We can determine the rate of flow by taking the first derivative of the volume equation for the time provided.

Given

$$v(t) = 7t^2 + 5t - 12, \text{ we can apply the power rule,}$$

$$f(x) = x^n \rightarrow f'(x) = nx^{n-1}.$$

$$\text{Then } v'(t) = 14t + 5.$$

Therefore, at $t = 2$,

$$v'(2) = 14(2) + 5 = 28 + 5 = 33 \text{ liters per minute.}$$

The volume v of water (in liters) in a tank at time t (in minutes) is defined by the equation $v(t) = 4t^3 + 7t^2 + 10t$. What is the tank's rate of flow at $t = 1$ in liters per minute?

Explanation:

We can determine the rate of flow by taking the first derivative of the volume equation for the time provided.

Given,

$$v(t) = 4t^3 + 7t^2 + 10t, \text{ then using the power rule which states,}$$

$$f(x) = x^n \rightarrow f'(x) = nx^{n-1} \text{ thus } v'(t) = 12t^2 + 14t + 10.$$

Therefore, at $t = 1$,

$$v'(1) = 12(1)^2 + 14(1) + 10 = 12 + 14 + 10 = 36 \text{ liters per minute.}$$



CONCLUSION

The important point is that the forces are ALWAYS working on it. In contrast, we could think of the second hand on a clock that clicks to the next tic mark every second. This isn't changing continuously like the cork, but a force is acting on it at discrete instances, that is, once every second. Calculus is not designed for this type of situation, but for the first one. So based on the philosophy that there are forces acting on our "objects" (by this I mean whatever we want to consider: the cork's spatial location, it's velocity, it's density, or anything else), we need something that measures how they change. So if we have a cork in a river, it will be moved by the currents and we want some way of understanding how the current causes the position of the cork to change. This is something called a vector field. At every point in the river, it tells where the cork is going to go. Well, not quite, since after it moves a little bit, it is at another point and thus moves according to another vector (all a vector is is an arrow that tells me which direction to move and how fast). This is the magic of calculus. It lets us deal with what seems like a mess. Essentially you move in a straight line in the direction the vector says, but only for a really small amount of time, and then you are at a new point and move in a straight line according to the vector there for a small amount of time, and so on. This is called the flow of a vector field (because it is the way the water flows in the river). This is essentially all a differential equation is. It associates a vector (which says where to go) to each point in your space.

REFERENCES

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BIOGRAPHY



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