SJIF Impact Factor 2022: 8.197| ISI I.F. Value: 1.241| Journal DOI: 10.36713/epra2016 ISSN: 2455-7838(Online)
EPRA International Journal of Research and Development (IJRD)
Volume: 7 | Issue: 10 | October 2022 - Peer Reviewed Journal

REGULAR (1, 2)*-GENERALIZED η-CLOSED SETS IN BITOPOLOGY

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ABSTRACT
In this paper, we introduce regular (1, 2)-generalized η-closed sets and obtain the relationships among some existing closed sets like (1, 2)*-semi- closed, (1, 2)*-α- closed and (1, 2)*-η-closed sets and their generalizations. Also we study some basic properties of (1, 2)*-rgη-open sets. Further, we introduce (1, 2)*-rgη-neighbourhood and discuss some properties of (1, 2)*-rgη-neighbourhood.*
KEYWORDS : *(1, 2)*-η-open, (1, 2)*-gη-closed, (1, 2)*-rgη-closed sets; (1, 2)*-rgη-neighbourhood*

1. INTRODUCTION

The study of bitopological spaces was first intiated by Kelly [**4**] in 1963. By using the topological notions, namely, semi-open, α-open and pre-open sets, many new bitopological sets are defined and studied by many topologists. In 2008, Ravi et al. [**8**] studied the notion of (1, 2)*-sets in bitopological spaces. In 2004, Ravi and Thivagar [7] studied the concept of stronger from of (1, 2)*-quatient mapping in bitopological spaces and introduced the concepts of (1, 2)*-semi-open and (1, 2)*-α-open sets in bitopological spaces. In 2010, K. Kayathri et al. [**3**] introduced and studied a new class of sets called regular (1, 2)*-g-closed sets and used it to obtain a new class of functions called (1, 2)*-rg-continuous, (1, 2)*-R-map, almost (1, 2)*-continuous and almost (1, 2)*-rg-closed functions in bitopological spaces. In 2015, D. Sreeja and P. Juane Sinthya [**11**] introduced (1, 2)*-rgα-closed sets. Some of its basic properties are studied. In 2022, H. Kumar [**5**] introduced the concept of (1, 2)*-η-open sets and (1, 2)*-η-neighbourhood and; studied their properties. Recently H. Kumar [**6**] introduced the concept of (1, 2)*-generalized η-closed sets and (1, 2)*-gη-neighbourhood and; investigated their properties.

2. PRELIMINARIES

Throughout the paper $(X, \Im_1, \Im_2)$, (Y, σ_1, σ_2) and $(Z, \wp_1, \wp_2)$ (or simply X, Y and Z) denote bitopological spaces.

Definition 2.1. Let S be a subset of X. Then S is said to be **$\Im_{1,2}$-open [7]** if $S = A \cup B$ where $A \in \Im_1$ and $B \in \Im_2$. The complement of a $\Im_{1,2}$-open set is **$\Im_{1,2}$-closed**.

Definition 2.2 [**7**]. Let S be a subset of X. Then
(i) the **$\Im_{1,2}$-closure** of S, denoted by $\Im_{1,2}$-cl(S), is defined as $\cap \{F : S \subset F$ and F is $\Im_{1,2}$-closed$\}$; (ii) the **$\Im_{1,2}$-interior** of S, denoted by $\Im_{1,2}$-int(S), is defined as $\cup \{F : F \subset S$ and F is $\Im_{1,2}$-open$\}$.

Note 2.3 [**7**]. Notice that $\Im_{1,2}$-open sets need not necessarily form a topology.

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Definition 2.4. A subset A of a bitopological space $(X, \Im_1, \Im_2)$ is called
(i) **regular $(1, 2)^*$-open** [7] if $A = \Im_{1,2}$-int $(\Im_{1,2}$-cl((A)).
(ii) **$(1, 2)^*$-semi-open** [7] if $A = \Im_{1,2}$-cl$(\Im_{1,2}$-int(A)),
(iii) **$(1, 2)^*$-α-open** [7] if $A \subset \Im_{1,2}$-int $(\Im_{1,2}$-cl$(\Im_{1,2}$-int (A))).
(iv) **$(1, 2)^*$-η-open** [5] if $A \subset \Im_{1,2}$ -int$(\Im_{1,2}$ -cl$(\Im_{1,2}$ -int)(A)) $\cup\ \Im_{1,2}$ -cl$(\Im_{1,2}$ -int)(A)).

The complement of a regular $(1, 2)^*$-open (resp. $(1, 2)^*$-semi-open, $(1, 2)^*$-α-open, $(1, 2)^*$-η-open) set is called **regular $(1, 2)^*$-closed** (resp. **$(1, 2)^*$-semi-closed, $(1, 2)^*$-α-closed, $(1, 2)^*$-η-closed**).

The **$(1, 2)^*$-semi-closure** (resp. **$(1, 2)^*$-α-closure, $(1, 2)^*$-η-closure**) of a subset A of X is denoted by **$(1, 2)^*$-s-cl(A)** (resp. **$(1, 2)^*$-α-cl(A), $(1, 2)^*$-η-cl(A)**), defined as the intersection of all $(1, 2)^*$-semi-closed. (resp. $(1, 2)^*$-α-closed, $(1, 2)^*$-η-closed) sets containing A.

The family of all regular $(1, 2)^*$-open (resp. regular $(1, 2)^*$-closed, $(1, 2)^*$-semi-open, $(1, 2)^*$-α-open, $(1, 2)^*$-η-open, $(1, 2)^*$-semi-closed, $(1, 2)^*$-α-closed, $(1, 2)^*$-η-closed) sets in X is denoted by $(1, 2)^*$-RO(X) (resp. $(1, 2)^*$-RC(X), $(1, 2)^*$-SO(X), $(1, 2)^*$-αO(X), $(1, 2)^*$-ηO(X), $(1, 2)^*$-SC(X), $(1, 2)^*$-αC(X), $(1, 2)^*$-ηC(X).

Remark 2.5. It is evident that any $\Im_{1,2}$-open set of X is an $(1, 2)^*$-α-open and each $(1, 2)^*$-α-open set of X is $(1, 2)^*$-semi-open but the converses are not true.

Remark 2.6. We have the following implications for the properties of subsets [**5**]:

regular $(1, 2)^*$-open $\Rightarrow$ $\Im_{1,2}$-open $\Rightarrow$ $(1, 2)^*$-α-open $\Rightarrow$ $(1, 2)^*$-semi-open $\Rightarrow$ $(1, 2)^*$-η-open

Where none of the implications is reversible.

3. $(1, 2)^*$-GENERALIZED η-CLOSED SETS IN BITOPOLOGICAL SPACES

Definition 3.1. A subset A of a bitopological space $(X, \Im_1, \Im_2)$ is called
(i) $(1, 2)^*$-generalized closed (briefly $(1, 2)^*$-g-closed) [**10**] if $\Im_{1,2}$-cl(A) $\subset$ U whenever $A \subset U$ and U is $\Im_{1,2}$-open in X.
(ii) regular $(1, 2)^*$- generalized closed (briefly $(1, 2)^*$-rg-closed) [**3**] if $\Im_{1,2}$-cl(A) $\subset$ U whenever $A \subset U$ and $U \in (1, 2)^*$-RO(X).
(iii) $(1, 2)^*$-weakly closed (briefly $(1, 2)^*$-w-closed) [**2**] if $\Im_{1,2}$-cl(A) $\subset$ U whenever $A \subset U$ and U is $(1, 2)^*$-semi-open in X.
(iv) $(1, 2)^*$-α-generalized closed (briefly $(1, 2)^*$-αg-closed) [**10**] if $(1, 2)^*$-α-cl(A) $\subset$ U whenever $A \subset U$ and U is $\Im_{1,2}$-open in X.
(v) regular $(1, 2)^*$-generalized α-closed (briefly $(1, 2)^*$-rgα-closed) [**11**] if $(1, 2)^*$-α-cl(A) $\subset$ U whenever $A \subset U$ and $U \in (1, 2)^*$-RO(X).
(vi) $(1, 2)^*$-generalized semi-closed (briefly $(1, 2)^*$-gs-closed) [**10**] if $(1, 2)^*$-s-cl(A) $\subset$ U whenever $A \subset U$ and U is $\Im_{1,2}$-open in X.
(vii) regular $(1, 2)^*$-generalized semi-closed (briefly $(1, 2)^*$-rgs-closed) [**10**] if $(1, 2)^*$-s-cl(A) $\subset$ U whenever $A \subset U$ and $U \in (1, 2)^*$-RO(X).

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(viii) $(1, 2)^*$-generalized η-closed (briefly $(1, 2)^*$-gη-closed) [**6**] if $(1, 2)^*$-η-cl(A) ⊂ U whenever A ⊂ U and U is $\Im_{1,2}$-open in X.
(ix) regular $(1, 2)^*$-generalized η-closed (briefly $(1, 2)^*$-rgη-closed) if $(1, 2)^*$-η-cl(A) ⊂ U whenever A ⊂ U and U ∈ $(1, 2)^*$-RO(X).

The complement of a $(1, 2)^*$-g-closed (resp. $(1, 2)^*$-rg-closed, $(1, 2)^*$-w-closed, $(1, 2)^*$-αg-closed, $(1, 2)^*$-rgα-closed, $(1, 2)^*$-gs-closed, $(1, 2)^*$-rgs-closed, $(1, 2)^*$-gη-closed) set is called $(1, 2)^*$-g-open (resp. $(1, 2)^*$-rg-open, $(1, 2)^*$-w-open, $(1, 2)^*$-rgα-open, $(1, 2)^*$-αg-open, $(1, 2)^*$-gs-open, $(1, 2)^*$-rgs-open, $(1, 2)^*$-gη-open).

We denote the set of all $(1, 2)^*$-rgη-closed sets in $(X, \Im_1, \Im_2)$ by $(1, 2)^*$-rgη-C(X).

Theorem 3.2. Every $\Im_{1,2}$-closed set is rgη-closed.
Proof. Let A be any $\Im_{1,2}$-closed set in $(X, \Im_1, \Im_2)$ and A ⊂ U, where U ∈ $(1, 2)^*$-RO(X). So $(1, 2)^*$-cl(A) = A . Since every $\Im_{1,2}$-closed set is $(1, 2)^*$-η-closed, so $(1, 2)^*$-η-cl(A) ⊂ $(1, 2)^*$-cl(A) = A. Therefore, $(1, 2)^*$-η-cl(A) ⊂ A ⊂ U. Hence A is $(1, 2)^*$-rgη-closed set.

Theorem 3.3. Every $(1, 2)^*$-g-closed set is $(1, 2)^*$-rgη-closed.
Proof. Let A be any $(1, 2)^*$-g-closed set in $(X, \Im_1, \Im_2)$ then $(1, 2)^*$-cl(A) ⊂ U whenever A ⊂ U, where U ∈ $(1, 2)^*$-RO(X), since every regular $(1 , 2)^*$-open set is $\Im_{1,2}$-open. So $(1, 2)^*$-η-cl(A) ⊂ $(1, 2)^*$-cl(A) ⊂ U. Therefore $(1, 2)^*$-η-cl(A) ⊂ U. Hence A is $(1, 2)^*$-rgη-closed set.

Theorem 3.4. Every $(1, 2)^*$-rg-closed set is $(1, 2)^*$-rgη-closed.
Proof. Let A be any $(1, 2)^*$-g-closed set in $(X, \Im_1, \Im_2)$ then $(1, 2)^*$-cl(A) ⊂ U whenever A ⊂ U, where U ∈ $(1, 2)^*$-RO(X). So $(1, 2)^*$-η-cl(A) ⊂ $(1, 2)^*$-cl(A) ⊂ U. Therefore $(1, 2)^*$-η-cl(A) ⊂ U. Hence A is $(1, 2)^*$-rgη-closed set.

Theorem 3.5. Every $(1, 2)^*$-α-closed set is $(1, 2)^*$-rgη-closed.
Proof. Let A be any $(1, 2)^*$-α-closed set in $(X, \Im_1, \Im_2)$ and A ⊂ U, where U ∈ $(1, 2)^*$-RO(X). Since every $(1, 2)^*$-α-closed set is $(1, 2)^*$-η-closed, so $(1, 2)^*$-η-cl(A) ⊂ $(1, 2)^*$-α-cl(A) = A. Therefore $(1, 2)^*$-η-cl(A) ⊂ A ⊂ U. Hence A is $(1, 2)^*$-rgη-closed set.

Theorem 3.6. Every $(1, 2)^*$-αg-closed set is $(1, 2)^*$-rgη-closed.
Proof. Let A be any $(1, 2)^*$-αg-closed set in $(X, \Im_1, \Im_2)$ then $(1, 2)^*$-α-cl(A) ⊂ U whenever A ⊂ U, where U ∈ $(1, 2)^*$-RO(X), since every regular $(1 , 2)^*$-open set is $\Im_{1,2}$-open. Given that A is $(1, 2)^*$-αg-closed set such that $(1, 2)^*$-α-cl(A) ⊂ U. But we have $(1, 2)^*$-η-cl(A) ⊂ $(1, 2)^*$-α-cl(A) ⊂ U. Therefore $(1, 2)^*$-η-cl(A) ⊂ U. Hence A is $(1, 2)^*$-rgη-closed set.

Theorem 3.7. Every $(1, 2)^*$-rgα-closed set is $(1, 2)^*$-rgη-closed.
Proof. Let A be any $(1, 2)^*$-αg-closed set in $(X, \Im_1, \Im_2)$ then $(1, 2)^*$-α-cl(A) ⊂ U whenever A ⊂ U, where U ∈ $(1, 2)^*$-RO(X). Given that A is $(1, 2)^*$-αg-closed set such that $(1, 2)^*$-α-cl(A) ⊂ U. But we have $(1, 2)^*$-η-cl(A) ⊂ $(1, 2)^*$-α-cl(A) ⊂ U. Therefore $(1, 2)^*$-η-cl(A) ⊂ U. Hence A is $(1, 2)^*$-rgη-closed set.
Theorem 3.8. Every $(1, 2)^*$-semi-closed set is $(1, 2)^*$-rgη-closed.

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Proof. Let A be any $(1, 2)^*$-semi-closed set in $(X, \Im_1, \Im_2)$ and $A \subset U$, where $U \in (1, 2)^*$-RO(X). Since every $(1, 2)^*$-semi-closed set is $(1, 2)^*$-η-closed, so $(1, 2)^*$-η-cl(A) $\subset (1, 2)^*$-s-cl(A) = A. Therefore $(1, 2)^*$-η-cl(A) $\subset A \subset$ U. Hence A is $(1, 2)^*$-rgη-closed set.

Theorem 3.9. Every $(1, 2)^*$-gs-closed set is $(1, 2)^*$-rgη-closed.
Proof. Let A be any $(1, 2)^*$-gs-closed set in $(X, \Im_1, \Im_2)$ then $(1, 2)^*$-s-cl(A) $\subset$ U whenever $A \subset U$, where $U \in (1, 2)^*$-RO(X), since every regular $(1 , 2)^*$-open set is $\Im_{1,2}$-open. Given that A is $(1, 2)^*$-gs-closed set such that $(1, 2)^*$-s-cl(A) $\subset$ U. But we have $(1, 2)^*$-η-cl(A) $\subset (1, 2)^*$-s-cl(A) $\subset$ U. Therefore $(1, 2)^*$-η-cl(A) $\subset$ U. Hence A is $(1, 2)^*$-rgη-closed set.

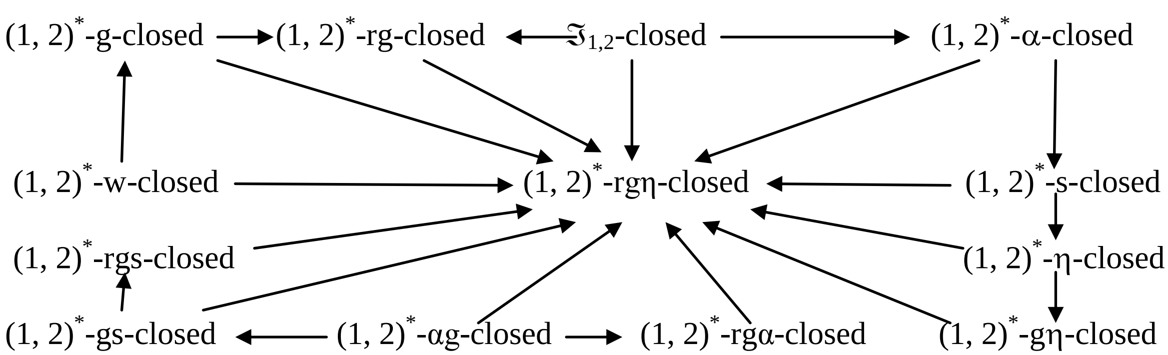
Theorem 3.10. Every $(1, 2)^*$-rgs-closed set is $(1, 2)^*$-rgη-closed.
Proof. Let A be any $(1, 2)^*$-rgs-closed set in $(X, \Im_1, \Im_2)$ then $(1, 2)^*$-s-cl(A) $\subset$ U whenever $A \subset U$, where $U \in (1, 2)^*$-RO(X). Given that A is $(1, 2)^*$-gs-closed set such that $(1, 2)^*$-s-cl(A) $\subset$ U. But we have $(1, 2)^*$-η-cl(A) $\subset (1, 2)^*$-s-cl(A) $\subset$ U. Therefore $(1, 2)^*$-η-cl(A) $\subset$ U. Hence A is $(1, 2)^*$-rgη-closed set.

Theorem 3.11. Every $(1, 2)^*$-η-closed set is $(1, 2)^*$-rgη-closed.
Proof. Let A be any $(1, 2)^*$-η-closed set in $(X, \Im_1, \Im_2)$ and $A \subset U$, where $U \in (1, 2)^*$-RO(X). Since A is $(1, 2)^*$-η-closed. Therefore $(1, 2)^*$-η-cl(A) = $A \subset U$. Hence A is $(1, 2)^*$-rgη-closed set.

Theorem 3.12. Every $(1, 2)^*$-gη-closed set is $(1, 2)^*$-rgη-closed.
Proof. Let A be any $(1, 2)^*$-gη-closed set in $(X, \Im_1, \Im_2)$ then $(1, 2)^*$-η-cl(A) $\subset$ U whenever $A \subset U$, where $U \in (1, 2)^*$-RO(X), since every regular $(1 , 2)^*$-open set is $\Im_{1,2}$-open. Given that A is $(1, 2)^*$-gη-closed set such that $(1, 2)^*$-η-cl(A) $\subset$ U. Therefore $(1, 2)^*$-η-cl(A) $\subset$ U. Hence A is $(1, 2)^*$-rgη-closed set.

Theorem 3.13. Every $(1, 2)^*$-w-closed set is $(1, 2)^*$-rgη-closed.
Proof. Let A be any $(1, 2)^*$-w-closed set in $(X, \Im_1, \Im_2)$ then $(1, 2)^*$-cl(A) $\subset$ U whenever $A \subset U$, where $U \in (1, 2)^*$-RO(X), since every regular $(1 , 2)^*$-open set is $(1 , 2)^*$-semi-open. So $(1, 2)^*$-η-cl(A) $\subset (1, 2)^*$-cl(A) $\subset$ U. Therefore $(1, 2)^*$-η-cl(A) $\subset$ U. Hence A is $(1, 2)^*$-rgη-closed set.

Remark 3.14. We have the following implications for the properties of subsets:



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Where none of the implications is reversible as can be seen from the following examples:

Example 3.15. Let X = {a, b, c, d} with $\Im_1$ = {ϕ, X, {a}, {b}, {a, b}, {b, c, d}} and $\Im_2$ = {ϕ, X, {c}, {a, c, d}}. Then

(i) $\Im_{1,2}$-closed sets : ϕ, X, {a}, {b}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.

(ii) $(1, 2)^*$-g-closed sets : ϕ, X, {a}, {b}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.

(iii) $(1, 2)^*$-rg-closed sets : ϕ, X, {a}, {b}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.

(iv) $(1, 2)^*$-α-closed sets : ϕ, X, {a}, {b}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.

(v) $(1, 2)^*$-αg-closed sets : ϕ, X, {a}, {b}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.

(vi) $(1, 2)^*$-rgα-closed sets : ϕ, X, {a}, {b}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.

(vii) $(1, 2)^*$-semi-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.

(viii) $(1, 2)^*$-gs-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.

(ix) $(1, 2)^*$-rgs-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.

(x) $(1, 2)^*$-η-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.

(xi) $(1, 2)^*$-gη-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.

(xii) $(1, 2)^*$-rgη-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.

(xiii) $(1, 2)^*$-w-closed sets : ϕ, X, {a}, {b}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.

Example 3.16. Let X = {a, b, c} with $\Im_1$ = {ϕ, X, {b}} and $\Im_2$ = {ϕ, X, {c}}. Then

(i) $\Im_{1,2}$-closed sets : ϕ, X, {a}, {a, b}, {a, c}.

(ii) $(1, 2)^*$-g-closed sets : ϕ, X, {a}, {a, b}, {a, c}.

(iii) $(1, 2)^*$-rg-closed sets : ϕ, X, {a}, {a, b}, {a, c}, {b, c}.

(iv) $(1, 2)^*$-α-closed sets : ϕ, X, {a}, {a, b}, {a, c}.

(v) $(1, 2)^*$-αg-closed sets : ϕ, X, {a}, {a, b}, {a, c}.

(vi) $(1, 2)^*$-rgα-closed sets : ϕ, X, {a}, {a, b}, {a, c}, {b, c}.

(vii) $(1, 2)^*$-semi-closed sets : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}.

(viii) $(1, 2)^*$-gs-closed sets : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}.

(ix) $(1, 2)^*$-rgs-closed sets : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}.

(x) $(1, 2)^*$-η-closed sets : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}.

(xi) $(1, 2)^*$-gη-closed sets : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}.

(xii) $(1, 2)^*$-rgη-closed sets : ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}.

(xiii) $(1, 2)^*$-w-closed sets : ϕ, X, {a}, {a, b}, {a, c}.

Example 3.17. Let X = {a, b, c, d} with $\Im_1$ = {ϕ, X, {a}} and $\Im_2$ = {ϕ, X, {b}, {a, b, c}}. Then

(i) $\Im_{1,2}$-closed sets : ϕ, X, {d}, {c, d}, {a, c, d}, {b, c, d}.

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(ii) $(1, 2)^*$-g-closed sets : ϕ, X, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(iii) $(1, 2)^*$-rg-closed sets : ϕ, X, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(iv) $(1, 2)^*$-α-closed sets : ϕ, X, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}.
(v) $(1, 2)^*$-αg-closed sets : ϕ, X, {c}, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(vi) $(1, 2)^*$-rgα-closed sets : ϕ, X, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(vii) $(1, 2)^*$-semi-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, c, d}, {b, c, d}.
(viii) $(1, 2)^*$-gs-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(ix) $(1, 2)^*$-rgs-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(x) $(1, 2)^*$-η-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, c, d}, {b, c, d}.
(xi) $(1, 2)^*$-gη-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.
(xii) $(1, 2)^*$-rgη-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(xiii) $(1, 2)^*$-w-closed sets : ϕ, X, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}.

Example 3.18. Let X = {a, b, c, d} with $\Im_1$ = {ϕ, X, {a}, {b}, {a, b}, {a, b, c}} and $\Im_2$ = {ϕ, X, {a, b, d}}. Then
(i) $\Im_{1,2}$-closed sets : ϕ, X, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}.
(ii) $(1, 2)^*$-g-closed sets : ϕ, X, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}.
(iii) $(1, 2)^*$-rg-closed sets : ϕ, X, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(iv) $(1, 2)^*$-α-closed sets : ϕ, X, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}.
(v) $(1, 2)^*$-αg-closed sets : ϕ, X, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}.
(vi) $(1, 2)^*$-rgα-closed sets : ϕ, X, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(vii) $(1, 2)^*$-semi-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, c, d}, {b, c, d}.
(viii) $(1, 2)^*$-gs-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, c, d}, {b, c, d}.
(ix) $(1, 2)^*$-rgs-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(x) $(1, 2)^*$-η-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, c, d}, {b, c, d}.
(xi) $(1, 2)^*$-gη-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, c, d}, {b, c, d}.
(xii) $(1, 2)^*$-rgη-closed sets : ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}.
(xiii) $(1, 2)^*$-w-closed sets : ϕ, X, {c}, {d}, {c, d}, {a, c, d}, {b, c, d}.

4. CHARACTERIZATIONS OF (1, 2)*-GENERALIZED η-CLOSED SETS

Theorem 4.1. The union of two $(1, 2)^*$-rgη-closed subsets of (X, $\Im_1$, $\Im_2$) is also $(1, 2)^*$-rgη-closed subset of (X, $\Im_1$, $\Im_2$).

SJIF Impact Factor 2022: 8.197| ISI I.F. Value: 1.241| Journal DOI: 10.36713/epra2016 ISSN: 2455-7838(Online)
EPRA International Journal of Research and Development (IJRD)
Volume: 7 | Issue: 10 | October 2022 - Peer Reviewed Journal

Proof. Assume that A and B are $(1, 2)^*$-rgη-closed set in $(X, \Im_1, \Im_2)$. Let U be regular $(1, 2)^*$-open set in $(X, \Im_1, \Im_2)$ such that $A \cup B \subset U$, then $A \subset U$ and $B \subset U$. Since A and B are $(1, 2)^*$-rgη-closed such that $(1, 2)^*$-η-cl(A) $\subset U$ and $(1, 2)^*$-η-cl(B) $\subset U$. Hence $(1, 2)^*$-η-cl($A \cup B$) = $(1, 2)^*$-η-cl(A) $\cup$ $(1, 2)^*$-η-cl(B) $\subset U$. That is $(1, 2)^*$-η-cl($A \cup B$) $\subset U$. Therefore $A \cup B$ is $(1, 2)^*$-rgη-closed set in $(X, \Im_1, \Im_2)$.

Theorem 4.2. The intersection of two $(1, 2)^*$-rgη-closed-sets in $(X, \Im_1, \Im_2)$ is also a $(1, 2)^*$-rgη-closed set in $(X, \Im_1, \Im_2)$.
Proof. Easy to proof.

Theorem 4.3. If a subset A is $(1, 2)^*$-rgη-closed, then $(1, 2)^*$-η-cl(A) − A does not contain any non-empty regular $(1, 2)^*$-closed set.
Proof. Suppose that A is $(1, 2)^*$-rgη-closed. Let F be a regular $(1, 2)^*$-closed subset of $(1, 2)^*$-η-cl(A) − A. Then $F \subset [(1, 2)^*\text{-}\eta\text{-cl}(A) \cap (X - A)]$ and so $A \subset [X - F]$. But A is $(1, 2)^*$-rgη-closed. Therefore $(1, 2)^*$-η-cl(A) $\subset [X - F]$. Consequently, $F \subset [X - (1, 2)^*\text{-}\eta\text{-cl}(A)]$. We already have $F \subset (1, 2)^*$-η-cl(A). Hence $F \subset [(1, 2)^*\text{-}\eta\text{-cl}(A) \cap X - (1, 2)^*\text{-}\eta\text{-cl}(A)] = \phi$. Thus $F = \phi$. Therefore $(1, 2)^*$-η-cl(A) − A contains no non-empty regular $(1, 2)^*$-closed set.

Example 4.4. The converse of Theorem 4.3 is not true.
Refer to **Example 3.18**. Let A = {a, b, c}. We have that $(1, 2)^*$-η-cl(A) − A = X − {a, b, c} = {d} does not contain any non-empty regular $(1, 2)^*$ -closed set. However, A is $(1, 2)^*$-rgη-closed in X.

Theorem 4.5. Let A be $(1, 2)^*$ -rgη-closed set. Then A is regular $(1, 2)^*$ -closed if and only if $[(1, 2)^*\text{-cl}((1, 2)^*\text{-int}(A)) - A]$ is regular $(1, 2)^*$-closed.

Proof. Let A be a $(1, 2)^*$ -rgη-closed. If A is regular $(1, 2)^*$ -closed, then $[(1, 2)^*\text{-cl}((1, 2)^*\text{-int}(A)) - A] = \phi$. We know ϕ is always regular $(1, 2)^*$-closed. Therefore $[(1, 2)^*\text{-cl}((1, 2)^*\text{-int}(A)) - A]$ is regular $(1, 2)^*$-closed.

Conversely, suppose that $[(1, 2)^*\text{-cl}((1, 2)^*\text{-int}(A)) - A]$ is regular $(1, 2)^*$-closed. Since A is $(1, 2)^*$-rgη-closed, $[(1, 2)^*\text{-cl}(A) - A]$ contains the regular $(1, 2)^*$-closed set $[(1, 2)^*\text{-cl}((1, 2)^*\text{-int}(A)) - A]$. By **Theorem 4.3**, $[(1, 2)^*\text{-cl}((1, 2)^*\text{-int}(A)) \setminus A] = \phi$. Hence $(1, 2)^*\text{-cl}((1, 2)^*\text{-int}(A)) = A$. Therefore A is regular $(1, 2)^*$-closed.

Remark 4.6. The converse of **Theorem 4.4** is not true as per the following example.

Example 4.7. Let X = {a, b, c, d, e} with $\Im_1 = \{\phi, X, \{a, b\}, \{a, b, c, d\}\}$ and $\Im_2 = \{\phi, X, \{c, d\}, \{a, b, c, d\}\}$. If we consider A = {a, c}, then $(1, 2)^*$-η-cl(A) − A = X − {a, c} = {b, d, e} does not contain any non-empty regular $(1, 2)^*$-closed set. However A is $(1, 2)^*$-rgη-closed.
Theorem 4.8. For an element $x \in (X, \Im_1, \Im_2)$, the set $(X, \Im_1, \Im_2) - \{x\}$ is $(1, 2)^*$-rgη-closed or regular $(1, 2)^*$-open.
Proof. Suppose $(X, \Im_1, \Im_2) - \{x\}$ is not regular $(1, 2)^*$-open set. Then $(X, \Im_1, \Im_2)$ is the only regular $(1, 2)^*$-open set containing $(X, \Im_1, \Im_2) - \{x\}$. This implies $(1, 2)^*$-η-cl($(X, \Im_1, \Im_2) - \{x\}$) $\subset (X, \Im_1, \Im_2)$. Hence $(X, \Im_1, \Im_2) - \{x\}$ is $(1, 2)^*$-rgη-closed set in $(X, \Im_1, \Im_2)$.

SJIF Impact Factor 2022: 8.197| ISI I.F. Value: 1.241| Journal DOI: 10.36713/epra2016 ISSN: 2455-7838(Online)
EPRA International Journal of Research and Development (IJRD)
Volume: 7 | Issue: 10 | October 2022 - Peer Reviewed Journal

Theorem 4.9. Let A be a $(1, 2)^*$-rgη-closed subset of X. If $A \subset B \subset (1, 2)^*$-η-cl(A), then B is also $(1, 2)^*$-rgη-closed in X.
Proof. Let $U \in (1, 2)^*$-rgη-O(X) with $B \subset U$. Then $A \subset U$. Since A is $(1, 2)^*$-rgη-closed, $(1, 2)^*$-η-cl(A) $\subset$ U. Also, since $B \subset (1, 2)^*$-η-cl(A), $(1, 2)^*$-η-cl(B) $\subset (1, 2)^*$-η-cl(A) $\subset$ U. Hence B is also $(1, 2)^*$-rgη-closed subset of X.

Remark 4.10. The converse of the **Theorem 4.9** need not be true in general. Consider the bitopological space $(X, \Im_1, \Im_2)$ where X = {a, b, c, d, e} with topology $\Im_1 = \{\phi, \{a, b\}, \{a, b, c, d\}, X\}$, $\Im_2 = \{\phi, \{c, d\}, \{a, b, c, d\}, X\}$, Let A = {b} and B = {b, c}. Then A and B are $(1, 2)^*$-rgη-closed sets in $(X, \Im_1, \Im_2)$ but $A \subset B$ is not subset in $(1, 2)^*$-η-cl(A) = {a, b}.

Theorem 4.11. Let A be a $(1, 2)^*$-rgη-closed in $(X, \Im_1, \Im_2)$. Then A is $(1, 2)^*$- η-closed if and only if $(1, 2)^*$-η-cl(A) – A is a regular $(1, 2)^*$-open.
Proof. Suppose A is a $(1, 2)^*$-η-closed in $(X, \Im_1, \Im_2)$. Then $(1, 2)^*$- η-cl(A) = A and so $(1, 2)^*$-η cl(A) – A = ϕ, which is regular $(1, 2)^*$-open in $(X, \Im_1, \Im_2)$.

Conversely, suppose $(1, 2)^*$-η-cl(A) – A is a regular $(1, 2)^*$-open set in $(X, \Im_1, \Im_2)$. Since A is $(1, 2)^*$-rgη-closed, by **Theorem 4.3** $(1, 2)^*$-η-cl(A) – A does not contain any nonempty regular $(1, 2)^*$-open in $(X, \Im_1, \Im_2)$. Then $(1, 2)^*$-η-cl(A) – A = ϕ. Hence A is (1, 2)*-η-closed set in $(X, \Im_1, \Im_2)$.

Theorem 4.12. If A is regular $(1, 2)^*$-open and $(1, 2)^*$-rgη-closed, then A is $(1, 2)^*$-rgη-closed set in $(X, \Im_1, \Im_2)$.
Proof. Let U be any regular $(1, 2)^*$-open set in $(X, \Im_1, \Im_2)$ such that $A \subset U$. Since A is regular $(1, 2)^*$-open and $(1, 2)^*$-rgη-closed, we have $(1, 2)^*$-η-cl(A) $\subset$ A. Then $(1, 2)^*$-η-cl(A) $\subset A \subset U$. Hence A is $(1, 2)^*$rgη-closed set in $(X, \Im_1, \Im_2)$.

Theorem 4.13. If a subset A of bitopological space $(X, \Im_1, \Im_2)$ is both regular $(1, 2)^*$-open and $(1, 2)^*$-rgη-closed, then it is $(1, 2)^*$-η-closed.
Proof. Suppose a subset A of bitopological space $(X, \Im_1, \Im_2)$ is both regular $(1, 2)^*$-open and $(1, 2)^*$-rgη-closed. Now $A \subset A$. Then $(1, 2)^*$-η-cl(A) $\subset$ A. Hence A is (1, 2)*-η-closed.

Corollary 4.14. Let A be regular $(1, 2)^*$-open and $(1, 2)^*$-rgη-closed subset in $(X, \Im_1, \Im_2)$. Suppose that F is $(1, 2)^*$-η-closed set in $(X, \Im_1, \Im_2)$. Then $A \cap F$ is an $(1, 2)^*$-rgη-closed set in $(X, \Im_1, \Im_2)$.
Proof. Let A be a regular $(1, 2)^*$-open and $(1, 2)^*$-rgη-closed subset in $(X, \Im_1, \Im_2)$ and F be closed. By **Theorem 4.13**, A is $(1, 2)^*$-η-closed. So $A \cap F$ is a $(1, 2)^*$-η-closed and hence $A \cap F$ is $(1, 2)^*$-rgη-closed set in $(X, \Im_1, \Im_2)$.
Theorem 4.15 [6]. If A is an open and S is $(1, 2)^*$-η-open in bitopological space $(X, \Im_1, \Im_2)$, then $A \cap S$ is $(1, 2)^*$-η-open in $(X, \Im_1, \Im_2)$.

Theorem 4.16. If A is both open and $(1, 2)^*$-g-closed set in $(X, \Im_1, \Im_2)$, then it is $(1, 2)^*$-rgη-closed set in $(X, \Im_1, \Im_2)$.

SJIF Impact Factor 2022: 8.197| ISI I.F. Value: 1.241| Journal DOI: 10.36713/epra2016 ISSN: 2455-7838(Online)
EPRA International Journal of Research and Development (IJRD)
Volume: 7 | Issue: 10 | October 2022 - Peer Reviewed Journal

Proof. Let A be an open and $(1, 2)^*$-g-closed set in $(X, \Im_1, \Im_2)$. Let $A \subset U$ and let U be a regular $(1, 2)^*$-open set in $(X, \Im_1, \Im_2)$. Now $A \subset A$. By hypothesis $(1, 2)^*$-η-cl(A) ⊂ A. That is $(1, 2)^*$-η-cl(A) ⊂ U. Thus A is $(1, 2)^*$-rgη-closed in $(X, \Im_1, \Im_2)$.

5. $(1, 2)^*$-RGη-OPEN SETS AND $(1, 2)^*$-RGη-NEIGHBORHOOD

In this section, we introduce $(1, 2)^*$-rgη-open sets in bitopological spaces and study some basic properties of $(1, 2)^*$-rgη-open sets. Also, we introduce $(1, 2)^*$-rgη-neighborhood (shortly $(1, 2)^*$-gη-nbhd in bitopological spaces by using the notion of $(1, 2)^*$-rgη-open sets. We prove that every nbhd of x in $(X, \Im_1, \Im_2)$ is $(1, 2)^*$-rgη-nbhd of x but not conversely.

Definition 5.1. A subset A in $(X, \Im_1, \Im_2)$ is called regular $(1, 2)^*$-generalized η-open (briefly, $(1, 2)^*$-rgη-open) in $(X, \Im_1, \Im_2)$ if A^c is $(1, 2)^*$-rgη-closed in $(X, \Im_1, \Im_2)$. We denote the family of all $(1, 2)^*$-rgη-open sets in X by $(1, 2)^*$-rgη-O(X).

Theorem 5.2. A set A is $(1, 2)^*$-rgη-open if and only if the following condition holds:

$$F \subset (1, 2)^*\text{-}\eta\text{-int}(A) \text{ whenever } F \text{ is regular } (1, 2)^*\text{-closed and } F \subset A.$$

Proof. Suppose the condition holds. Put [X − A] = B. Suppose that $B \subset U$ where $U \in (1, 2)^*$-R-O(X). Now $X - A \subset U$ implies $F = [X - U] \subset A$ and F is regular $(1, 2)^*$ -closed, which implies $F \subset (1, 2)^*$-η-int(A). Also $F \subset (1, 2)^*$-η-int(A) implies $[X - (1, 2)^*\text{-}\eta\text{-int}(A)] \subset [X - F] = U$. This implies $[X - ((1, 2)^*\text{-}\eta\text{-int}(X - B))] \subset U$. Therefore $[X - ((1, 2)^*\text{-}\eta\text{-int}(X - B))] \subset U$ or equivalently $(1, 2)^*$-η-cl(B) ⊂ U. Thus B is $(1, 2)^*$-rgη-closed. Hence A is $(1, 2)^*$-rgη-open.

Conversely, suppose that A is $(1, 2)^*$-rgη-open, $F \subset A$ and F is regular $(1, 2)^*$-closed. Then [X − F] is regular $(1, 2)^*$-open. Then $(X - A) \subset (X - F)$. Hence $(1, 2)^*$-η-cl(X − A) ⊂ (X − F) because (X − A) is $(1, 2)^*$-rgη-closed. Therefore $F \subset (X - (1, 2)^*\text{-}\eta\text{-cl}(X - A)) = (1, 2)^*\text{-}\eta\text{-int}(A)$.

Definition 5.3. Let $(X, \Im_1, \Im_2)$ be a bitopological space and let $x \in (X, \Im_1, \Im_2)$. A subset N of $(X, \Im_1, \Im_2)$ is said to be a **$(1, 2)^*$-rgη-nbhd** of x iff there exists a $(1, 2)^*$-rgη-open set G such that $x \in G \subset N$.

Definition 5.4. A subset N of a bitopological space $(X, \Im_1, \Im_2)$, is called a **$(1, 2)^*$-rgη-nbhd** of $A \subset (X, \Im_1, \Im_2)$ iff there exists a $(1, 2)^*$-rgη-open set G such that $A \subset G \subset N$.

Theorem 5.5. Every nbhd N of $x \in (X, \Im_1, \Im_2)$ is a $(1, 2)^*$-rgη-nbhd of $(X, \Im_1, \Im_2)$.
Proof. Let N be a nbhd of point $x \in (X, \Im_1, \Im_2)$. To prove that N is a $(1, 2)^*$-rgη-nbhd of x. By definition of nbhd, there exists an open set G such that $x \in G \subset N$. As every open set is $(1, 2)^*$-rgη-open set G such that $x \in G \subset N$. Hence N is $(1, 2)^*$-rgη-nbhd of x.
Remark 5.6. In general, a $(1, 2)^*$-rgη-nbhd N of $x \in (X, \Im_1, \Im_2)$ need not be a nbhd of x in $(X, \Im_1, \Im_2)$, as seen from the following example.

Example 5.7. Let X = {a, b, c, d} with topology $\Im_1$ = {ϕ, {a}, {b}, {a, b}, {a, b, c}, X} and $\Im_2$ = {ϕ, {a, b, d}, X} Then $(1, 2)^*$-rgη-O(X) = {ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a,

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Volume: 7 | Issue: 10 | October 2022 - Peer Reviewed Journal

b, d}, {a, c, d}, {b, c, d}}. The set {b, c} is $(1, 2)^*$-rgη-nbhd of the point b, there exists an $(1, 2)^*$-rgη-open set {b} is such that b ∈ {b} ⊂ {b, c}. However, the set {b, c} is not a nbhd of the point b, since no open set G exists such that b ∈ G ⊂ {a, c}.

Theorem 5.8. If a subset N of a space $(X, \Im_1, \Im_2)$ is $(1, 2)^*$-rgη-open, then N is a $(1, 2)^*$-rgη-nbhd of each of its points.
Proof. Suppose N is $(1, 2)^*$-rgη-open. Let x ∈ N. We claim that N is $(1, 2)^*$-rgη-nbhd of x. For N is a $(1, 2)^*$-rgη-open set such that x ∈ N ⊂ N. Since x is an arbitrary point of N, it follows that N is a $(1, 2)^*$-rgη-nbhd of each of its points.

Definition 5.9. Let x be a point in a space $(X, \Im_1, \Im_2)$. The set of all $(1, 2)^*$-rgη-nbhd of x is called the **$(1, 2)^*$-rgη-nbhd system** at x, and is denoted by $(1, 2)^*$-rgη-N(x).

Theorem 5.10. Let $(X, \Im_1, \Im_2)$ be a bitopological space and for each $x \in (X, \Im_1, \Im_2)$. Let $(1, 2)^*$- rgη-N(x) be the collection of all $(1, 2)^*$-rgη-nbhds of x. Then we have the following results.
(i) $\vee$ $x \in (X, \Im_1, \Im_2)$, $(1, 2)^*$-rgη-N(x) ≠ ϕ.
(ii) N ∈ $(1, 2)^*$-rgη-N(x) ⇒ x∈ N.
(iii) N ∈ $(1, 2)^*$-rgη-N(x), M ⊃ N ⇒ M ∈ $(1, 2)^*$-rgη-N(x).
(iv) N ∈ $(1, 2)^*$-rgη-N(x), M ∈ $(1, 2)^*$-rgη-N(x) ⇒ N ∩ M ∈ $(1, 2)^*$-rgη-N(x).
(v) N ∈ $(1, 2)^*$-rgη-N(x) ⇒ there exists M ∈ $(1, 2)^*$-rgη-N(x) such that M ⊂ N and M ∈ $(1, 2)^*$-rgη-N(y) for every y ∈ M.

Proof. (i) Since $(X, \Im_1, \Im_2)$ is a $(1, 2)^*$-rgη-open set, it is a $(1, 2)^*$-rgη-nbhd of every $x \in (X, \Im_1, \Im_2)$. Hence there exists at least one $(1, 2)^*$-rgη-nbhd (namely - $(X, \Im_1, \Im_2)$) for each $x \in (X, \Im_1, \Im_2)$. Hence$(1, 2)^*$-rgη-N(x) = ϕ for every $x \in (X, \Im_1, \Im_2)$.

(ii) If N ∈ $(1, 2)^*$-rgη-N(x), then N is a $(1, 2)^*$-rgη-nbhd of x. So by definition of $(1, 2)^*$-rgη-nbhd, x ∈ N.

(iii) Let N ∈ $(1, 2)^*$-rgη-N(x) and M ⊃ N. Then there is a $(1, 2)^*$-rgη-open set G such that x ∈ G ⊂ N. Since N ⊂ M, x ∈ G ⊂ M and so M is $(1, 2)^*$-rgη-nbhd of x. Hence M ∈ $(1, 2)^*$-rgη-N (x).

(iv) Let N ∈ $(1, 2)^*$-rgη-N(x) and M ∈ $(1, 2)^*$-rgη-N(x). Then by definition of $(1, 2)^*$-rgη-nbhd. Hence $x \in G_1 \cap G_2 \subset N \cap M$ ⇒ (1). Since $G_1 \cap G_2$ is a $(1, 2)^*$-rgη-open set, (being the intersection of two $(1, 2)^*$-rgη-open sets), it follows from (1) that N ∩ M is a $(1, 2)^*$-rgη-nbhd of x. Hence N ∩ M ∈ $(1, 2)^*$-rgη-N(x).

(v) If N ∈ $(1, 2)^*$-rgη-N(x), then there exists a $(1, 2)^*$-rgη-open set M such that x ∈ M ⊂ N. Since M is a $(1, 2)^*$-rgη-open set, it is $(1, 2)^*$-rgη-nbhd of each of its points. Therefore M ∈ $(1, 2)^*$-rgη-N(y) for every y∈ M.

6. CONCLUSION

In this paper, we introduce regular $(1, 2)^*$-generalized η-closed sets and obtain the relationships among some existing closed sets like $(1, 2)^*$-semi- closed, $(1, 2)^*$-α- closed and $(1, 2)^*$-η- closed sets and their generalizations. Also we study some basic properties of $(1, 2)^*$-rgη-open sets. Further, we introduce $(1, 2)^*$-rgη-neighbourhood and discuss some properties of $(1, 2)^*$-rgη-neighbourhood. The regular $(1, 2)^*$-generalized η-closed sets can be used to derive a new decomposition of unity, closed map and open map, homeomorphism, closure and interior and new separation axioms. This idea can be extended to ordered topological and fuzzy topological spaces.

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