FORMULATION OF SEQUENCES OF DIOPHANTINE 3-TUPLES THROUGH THE PAIR (3,6)

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ABSTRACT
This paper aims at formulating sequences of Diophantine 3-tuples through the pair (3,6)
KEY WORDS: Diophantine 3-tuple, sequence of Diophantine 3-tuples

INTRODUCTION
The problem of constructing the sets with property that product of any two of its distinct elements is one less than a square has a very long history and such sets have been studied by Diophantus. A set of m distinct positive integers \( \{a_1, a_2, a_3, \ldots, a_m\} \) is said to have the property \( D(n), n \in \mathbb{Z} - \{0\} \) if \( a_i a_j + n \) is a perfect square for all \( 1 \leq i < j \leq m \) or \( 1 \leq j < i \leq m \) and such a set is called a Diophantine m-tuple with property \( D(n) \).

Many Mathematicians considered the construction of different formulations of diophantine triples with the property \( D(n) \) for any arbitrary integer \( n \) [1] and also, for any linear polynomials in \( n \). In this context, one may refer [2-13] for an extensive review of various problems on diophantine triples.

This paper concerns with the construction of sequences of diophantine 3-tuples \((a, b, c)\) such that the product of any two elements of the set added by \((-2), (-9), (-14), (-17), D(k^2 + 8k - 2), D(k^2 - 8k - 2)\) in turn is a perfect square.

Sequence: 1
Let \( a = 6, \ c_0 = 3 \)

It is observed that
\[ ac_0 - 2 = 16, \text{ a perfect square} \]

Therefore, the pair \( (a, c_0) \) represents diophantine 2-tuple with the property \( D(-2) \).

Let \( c_1 \) be any non-zero polynomial such that

\[
\begin{align*}
ac_1 - 2 &= p^2 \\
c_0c_1 - 2 &= q^2
\end{align*}
\]

(1)

(2)

Eliminating \( c_1 \) between (1) and (2), we have

\[
c_0p^2 - aq^2 = (c_0 - a)(-2)
\]

(3)

Introducing the linear transformations

\[
p = X + aT, \quad q = X + c_0T
\]

(4)

in (3) and simplifying, we get

\[
X^2 = ac_0T^2 - 2
\]

which is satisfied by \( T = 1, X = 4 \)

In view of (4) and (1), it is seen that

\[
c_1 = 17
\]

Note that \( (a, c_0, c_1) \) represents diophantine 3-tuple with property \( D(-2) \)

Taking \( (a, c_1) \) and employing the above procedure, it is seen that the triple \( (a, c_1, c_2) \) where

\[
c_2 = 43
\]

exhibits diophantine 3-tuple with property \( D(-2) \)

Taking \( (a, c_2) \) and employing the above procedure, it is seen that the triple \( (a, c_2, c_3) \) where

\[
c_3 = 81
\]

exhibits diophantine 3-tuple with property \( D(-2) \)

Taking \( (a, c_3) \) and employing the above procedure, it is seen that the triple \( (a, c_3, c_4) \) where

\[
c_4 = 131
\]
exhibits diophantine 3-tuple with property $D(-2)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_{s+1} = 6s^2 - 4s + 1, \ s = 1, 2, 3,...$$

**Sequence: 2**

Let $a = 6, \ c_0 = 3$

It is observed that $ac_0 - 9 = 9$, a perfect square

Therefore, the pair $(a, c_0)$ represents diophantine 2-tuple with the property $D(-9)$.

Let $c_1$ be any non-zero polynomial such that

$$ac_1 - 9 = p^2 \tag{5}$$
$$c_0c_1 - 9 = q^2 \tag{6}$$

Eliminating $c_1$ between (5) and (6), we have

$$c_0p^2 - aq^2 = (c_0 - a)(-9) \tag{7}$$

Introducing the linear transformations

$$p = X + aT, \ q = X + c_0T \tag{8}$$

in (7) and simplifying we get

$$X^2 = ac_0 T^2 - 9$$

which is satisfied by $T = 1, \ X = 3$

In view of (8) and (5), it is seen that

$$c_1 = 15$$

Note that $(a, c_0, c_1)$ represents diophantine 3-tuple with property $D(-9)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$$c_2 = 39$$
exhibits diophantine 3-tuple with property $D(-9)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$$c_3 = 75$$

exhibits diophantine 3-tuple with property $D(-9)$

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

$$c_4 = 123$$

exhibits diophantine 3-tuple with property $D(-9)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_{s+1} = 6s^2 - 6s + 3, \ s = 1, 2, 3, \ldots$$

**Sequence: 3**

Let $a = 6, \ c_0 = 3$

It is observed that

$$ac_0 - 14 = 4, \text{ a perfect square}$$

Therefore, the pair $(a, c_0)$ represents diophantine 2-tuple with the property $D(-14)$.

Let $c_1$ be any non-zero polynomial such that

$$ac_1 - 14 = p^2 \tag{9}$$

$$c_0c_1 - 14 = q^2 \tag{10}$$

Eliminating $c_1$ between (9) and (10), we have

$$c_0p^2 - aq^2 = (c_0 - a)(-14) \tag{11}$$

Introducing the linear transformations

$$p = X + aT, \ q = X + c_0T \tag{12}$$

in (11) and simplifying we get

$$X^2 = ac_0T^2 - 14$$
which is satisfied by $T = 1, X = 2$

In view of (12) and (9), it is seen that

$$c_1 = 13$$

Note that $(a, c_0, c_1)$ represents diophantine 3-tuple with property $D(-14)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$$c_2 = 35$$

exhibits diophantine 3-tuple with property $D(-14)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$$c_3 = 69$$

exhibits diophantine 3-tuple with property $D(-14)$

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

$$c_4 = 115$$

exhibits diophantine 3-tuple with property $D(-14)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_{s+1} = 6s^2 - 8s + 5, \ s = 1, 2, 3,...$$

**Sequence: 4**

Let $a = 6, \ c_0 = 3$

It is observed that

$$ac_0 - 17 = 1, \ \text{a perfect square}$$

Therefore, the pair $(a, c_0)$ represents diophantine 2-tuple with the property $D(-17)$.

Let $c_1$ be any non-zero polynomial such that

$$ac_1 - 17 = p^2 \quad (13)$$

$$c_0c_1 - 17 = q^2 \quad (14)$$
Eliminating $c_1$ between (13) and (14), we have

$$c_0 p^2 - a q^2 = (c_0 - a)(-17) \quad (15)$$

Introducing the linear transformations

$$p = X + a T, \quad q = X + c_0 T \quad (16)$$

in (15) and simplifying we get

$$X^2 = ac_0 T^2 - 17$$

which is satisfied by $T = 1, X = 1$

In view of (16) and (13), it is seen that

$$c_1 = 11$$

Note that $(a, c_0, c_1)$ represents diophantine 3-tuple with property $D(-17)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

$$c_2 = 31$$

exhibits diophantine 3-tuple with property $D(-17)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

$$c_3 = 63$$

exhibits diophantine 3-tuple with property $D(-17)$

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

$$c_4 = 107$$

exhibits diophantine 3-tuple with property $D(-17)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_{s+1} = 6s^2 - 10s + 7, \quad s = 1, 2, 3, ....$$

**Sequence: 5**

Let $a = 6, \quad c_0 = 3$
It is observed that 
\[ ac_0 + k^2 + 8k - 2 = (k + 4)^2, \] a perfect square

Therefore, the pair \( (a, c_0) \) represents diophantine 2-tuple with the property \( D(k^2 + 8k - 2) \).

Let \( c_1 \) be any non-zero polynomial such that
\[ ac_1 + k^2 + 8k - 2 = p^2 \quad (17) \]
\[ c_0c_1 + k^2 + 8k - 2 = q^2 \quad (18) \]

Eliminating \( c_1 \) between (17) and (18), we have
\[ c_0p^2 - aq^2 = (c_0 - a)(k^2 + 8k - 2) \quad (19) \]

Introducing the linear transformations
\[ p = X + aT, \quad q = X + c_0T \quad (20) \]
in (19) and simplifying we get
\[ X^2 = ac_0T^2 + k^2 + 8k - 2 \]
which is satisfied by \( T = 1, \ X = k + 4 \)

In view of (20) and (17), it is seen that
\[ c_1 = 2k + 17 \]

Note that \( (a, c_0, c_1) \) represents diophantine 3-tuple with property \( D(k^2 + 8k - 2) \)

Taking \( (a, c_1) \) and employing the above procedure, it is seen that the triple \( (a, c_1, c_2) \) where
\[ c_2 = 4k + 43 \]

exhibits diophantine 3-tuple with property \( D(k^2 + 8k - 2) \)

Taking \( (a, c_2) \) and employing the above procedure, it is seen that the triple \( (a, c_2, c_3) \) where
\[ c_3 = 6k + 81 \]

exhibits diophantine 3-tuple with property \( D(k^2 + 8k - 2) \)

Taking \( (a, c_3) \) and employing the above procedure, it is seen that the triple \( (a, c_3, c_4) \) where
\[ c_4 = 8k + 131 \]

exhibits diophantine 3-tuple with property \( D(k^2 + 8k - 2) \)

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by \( (a, c_s, c_{s+1}) \) where

\[ c_{s+1} = 2(s-1)k + 6s^2 - 4s + 1, \ s = 1, 2, 3, ... \]

**Sequence: 6**

Let \( a = 6, \ c_0 = 3 \)

It is observed that

\[ ac_0 + k^2 - 8k - 2 = (k - 4)^2, \text{ a perfect square} \]

Therefore, the pair \( (a, c_0) \) represents diophantine 2-tuple with the property \( D(k^2 - 8k - 2) \).

Let \( c_1 \) be any non-zero polynomial such that

\[ ac_1 + k^2 - 8k - 2 = p^2 \tag{21} \]

\[ c_0c_1 + k^2 - 8k - 2 = q^2 \tag{22} \]

Eliminating \( c_1 \) between (21) and (22), we have

\[ c_0p^2 - aq^2 = (c_0 - a)(k^2 - 8k - 2) \tag{23} \]

Introducing the linear transformations

\[ p = X + aT, \ q = X + c_0T \tag{24} \]

in (23) and simplifying we get

\[ X^2 = ac_0T^2 + k^2 - 8k - 2 \]

which is satisfied by \( T = 1, X = k - 4 \)

In view of (24) and (21), it is seen that

\[ c_1 = 2k + 1 \]

Note that \( (a, c_0, c_1) \) represents diophantine 3-tuple with property \( D(k^2 - 8k - 2) \)

Taking \( (a, c_1) \) and employing the above procedure, it is seen that the triple \( (a, c_1, c_2) \) where
\[c_2 = 4k + 11\]

exhibits diophantine 3-tuple with property \(D(k^2 - 8k - 2)\)

Taking \((a, c_2)\) and employing the above procedure, it is seen that the triple \((a, c_2, c_3)\) where

\[c_3 = 6k + 33\]

exhibits diophantine 3-tuple with property \(D(k^2 - 8k - 2)\)

Taking \((a, c_3)\) and employing the above procedure, it is seen that the triple \((a, c_3, c_4)\) where

\[c_4 = 8k + 67\]

exhibits diophantine 3-tuple with property \(D(k^2 - 8k - 2)\)

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by \((a, c_s, c_{s+1})\) where

\[c_{s+1} = 2(s - 1)k + 6s^2 - 20s + 17, \ s = 1, 2, 3, \ldots\]

It is noted that, in each of the above sequences, the following relations are observed:

- The triple \((c_s, c_{s+1} + 6, c_{s+2})\) forms an arithmetic progression.

**Sequence: 7**

Let \(a = 3, \ c_0 = 6\)

It is observed that

\[ac_0 - 2 = 16, \text{ a perfect square}\]

Therefore, the pair \((a, c_0)\) represents diophantine 2-tuple with the property \(D(-2)\).

Let \(c_1\) be any non-zero polynomial such that

\[ac_1 - 2 = p^2\]

(25)

\[c_0c_1 - 2 = q^2\]

(26)

Eliminating \(c_1\) between (25) and (26), we have

\[c_0p^2 - aq^2 = (c_0 - a)(-2)\]

(27)

Introducing the linear transformations
\[ p = X + aT, \quad q = X + c_0T \]  

(28)

in (27) and simplifying, we get

\[ X^2 = ac_0T^2 - 2 \]

which is satisfied by \( T = 1, \ X = 4 \)

In view of (28) and (25), it is seen that

\[ c_1 = 17 \]

Note that \((a, c_0, c_1)\) represents diophantine 3-tuple with property \( D(-2) \)

Taking \((a, c_1)\) and employing the above procedure, it is seen that the triple \((a, c_1, c_2)\) where

\[ c_2 = 34 \]

exhibits diophantine 3-tuple with property \( D(-2) \)

Taking \((a, c_2)\) and employing the above procedure, it is seen that the triple \((a, c_2, c_3)\) where

\[ c_3 = 57 \]

exhibits diophantine 3-tuple with property \( D(-2) \)

Taking \((a, c_3)\) and employing the above procedure, it is seen that the triple \((a, c_3, c_4)\) where

\[ c_4 = 86 \]

exhibits diophantine 3-tuple with property \( D(-2) \)

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by \((a, c_s, c_{s+1})\) where

\[ c_{s+1} = 3s^2 + 2s + 1, \ s = 1, 2, 3, \ldots \]

**Sequence: 8**

Let \( a = 3, \ c_0 = 6 \)

It is observed that

\[ ac_0 - 9 = 9, \ \text{a perfect square} \]

Therefore, the pair \((a, c_0)\) represents diophantine 2-tuple with the property \( D(-9) \).
Let $c_1$ be any non-zero polynomial such that

\[ ac_1 - 9 = p^2 \tag{29} \]

\[ c_0c_1 - 9 = q^2 \tag{30} \]

Eliminating $c_1$ between (29) and (30), we have

\[ c_0p^2 - aq^2 = (c_0 - a)(-9) \tag{31} \]

Introducing the linear transformations

\[ p = X + aT, \; q = X + c_0T \tag{32} \]

in (7) and simplifying we get

\[ X^2 = ac_0T^2 - 9 \]

which is satisfied by $T = 1, \; X = 3$

In view of (32) and (29), it is seen that

\[ c_1 = 15 \]

Note that $(a, c_0, c_1)$ represents diophantine 3-tuple with property $D(-9)$

Taking $(a, c_1)$ and employing the above procedure, it is seen that the triple $(a, c_1, c_2)$ where

\[ c_2 = 30 \]

exhibits diophantine 3-tuple with property $D(-9)$

Taking $(a, c_2)$ and employing the above procedure, it is seen that the triple $(a, c_2, c_3)$ where

\[ c_3 = 51 \]

exhibits diophantine 3-tuple with property $D(-9)$

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

\[ c_4 = 78 \]

exhibits diophantine 3-tuple with property $D(-9)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where
Sequence: 9

Let \( a = 3, \ c_0 = 6 \)

It is observed that
\[
ac_0 - 14 = 4, \text{ a perfect square}
\]

Therefore, the pair \((a, c_0)\) represents diophantine 2-tuple with the property \( D(-14) \).

Let \( c_1 \) be any non-zero polynomial such that
\[
ac_1 - 14 = p^2 \tag{33}
\]
\[
c_0c_1 - 14 = q^2 \tag{34}
\]

Eliminating \( c_1 \) between (33) and (34), we have
\[
c_0p^2 - aq^2 = (c_0 - a)(-14) \tag{35}
\]

Introducing the linear transformations
\[
p = X + aT, \quad q = X + c_0T \tag{36}
\]

in (35) and simplifying we get
\[
X^2 = ac_0T^2 - 14
\]

which is satisfied by \( T = 1, X = 2 \)

In view of (36) and (33), it is seen that
\[
c_1 = 13
\]

Note that \((a, c_0, c_1)\) represents diophantine 3-tuple with property \( D(-14) \).

Taking \((a, c_1)\) and employing the above procedure, it is seen that the triple \((a, c_1, c_2)\) where
\[
c_2 = 26
\]

exhibits diophantine 3-tuple with property \( D(-14) \).

Taking \((a, c_2)\) and employing the above procedure, it is seen that the triple \((a, c_2, c_3)\) where
\[
c_3 = 45
\]
exhibits diophantine 3-tuple with property $D(-14)$

Taking $(a, c_3)$ and employing the above procedure, it is seen that the triple $(a, c_3, c_4)$ where

$$c_4 = 70$$

exhibits diophantine 3-tuple with property $D(-14)$

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by $(a, c_s, c_{s+1})$ where

$$c_{s+1} = 3s^2 - 2s + 5, \ s = 1, 2, 3, ...$$

**Sequence: 10**

Let $a = 3, \ c_0 = 6$

It is observed that

$$ac_0 - 17 = 1, \ \text{a perfect square}$$

Therefore, the pair $(a, c_0)$ represents diophantine 2-tuple with the property $D(-17)$.

Let $c_1$ be any non-zero polynomial such that

$$ac_1 - 17 = p^2 \quad (37)$$

$$c_0c_1 - 17 = q^2 \quad (38)$$

Eliminating $c_1$ between (37) and (38), we have

$$c_0p^2 - aq^2 = (c_0 - a)(-17) \quad (39)$$

Introducing the linear transformations

$$p = X + aT, \ q = X + c_0T \quad (40)$$

in (39) and simplifying we get

$$X^2 = ac_0T^2 - 17$$

which is satisfied by $T = 1, X = 1$

In view of (40) and (37), it is seen that

$$c_1 = 11$$
Note that \( (a, c_0, c_1) \) represents diophantine 3-tuple with property \( D(-17) \)

Taking \( (a, c_1) \) and employing the above procedure, it is seen that the triple \( (a, c_1, c_2) \) where
\[
c_2 = 22
\]

exhibits diophantine 3-tuple with property \( D(-17) \)

Taking \( (a, c_2) \) and employing the above procedure, it is seen that the triple \( (a, c_2, c_3) \) where
\[
c_3 = 39
\]

exhibits diophantine 3-tuple with property \( D(-17) \)

Taking \( (a, c_3) \) and employing the above procedure, it is seen that the triple \( (a, c_3, c_4) \) where
\[
c_4 = 62
\]

exhibits diophantine 3-tuple with property \( D(-17) \)

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by \( (a, c_s, c_{s+1}) \) where
\[
c_{s+1} = 3s^2 - 4s + 7 , \ s = 1, 2, 3, \ldots
\]

Sequence: 11

Let \( a = 3, \ c_0 = 6 \)

It is observed that
\[
a c_0 + k^2 + 8k - 2 = (k + 4)^2, \ a \text{ perfect square}
\]

Therefore, the pair \( (a, c_0) \) represents diophantine 2-tuple with the property \( D(k^2 + 8k - 2) \).

Let \( c_1 \) be any non-zero polynomial such that
\[
a c_1 + k^2 + 8k - 2 = p^2 \quad (41)
\]
\[
c_0 c_1 + k^2 + 8k - 2 = q^2 \quad (42)
\]

Eliminating \( c_1 \) between (41) and (42), we have
\[
c_0 p^2 - a q^2 = (c_0 - a) (k^2 + 8k - 2) \quad (43)
\]

Introducing the linear transformations
\[ p = X + aT, \quad q = X + c_0T \]  \hspace{1cm} (44)

in (43) and simplifying we get

\[ X^2 = ac_0T^2 + k^2 + 8k - 2 \]

which is satisfied by \( T = 1, \quad X = k + 4 \)

In view of (44) and (41), it is seen that

\[ c_1 = 2k + 17 \]

Note that \((a, c_0, c_1)\) represents diophantine 3-tuple with property \( D(k^2 + 8k - 2) \)

Taking \((a, c_1)\) and employing the above procedure, it is seen that the triple \((a, c_1, c_2)\) where

\[ c_2 = 4k + 34 \]

exhibits diophantine 3-tuple with property \( D(k^2 + 8k - 2) \)

Taking \((a, c_2)\) and employing the above procedure, it is seen that the triple \((a, c_2, c_3)\) where

\[ c_3 = 6k + 57 \]

exhibits diophantine 3-tuple with property \( D(k^2 + 8k - 2) \)

Taking \((a, c_3)\) and employing the above procedure, it is seen that the triple \((a, c_3, c_4)\) where

\[ c_4 = 8k + 86 \]

exhibits diophantine 3-tuple with property \( D(k^2 + 8k - 2) \)

The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by \((a, c_s, c_{s+1})\) where

\[ c_{s+1} = 2(s - 1)k + 3s^2 + 2s + 1, \quad s = 1, 2, 3, ... \]

**Sequence: 12**

Let \( a = 3, \quad c_0 = 6 \)

It is observed that

\[ ac_0 + k^2 - 8k - 2 = (k - 4)^2, \quad \text{a perfect square} \]

Therefore, the pair \((a, c_0)\) represents diophantine 2-tuple with the property \( D(k^2 - 8k - 2) \).
Let \( c_1 \) be any non-zero polynomial such that
\[
ac_1 + k^2 - 8k - 2 = p^2 \tag{45}
\]
\[
c_0c_1 + k^2 - 8k - 2 = q^2 \tag{46}
\]
Eliminating \( c_1 \) between (45) and (46), we have
\[
c_0p^2 - aq^2 = (c_0 - a)(k^2 - 8k - 2) \tag{47}
\]
Introducing the linear transformations
\[
p = X + aT, \quad q = X + c_0T \tag{48}
\]
in (47) and simplifying we get
\[
X^2 = ac_0T^2 + k^2 - 8k - 2
\]
which is satisfied by \( T = 1, \) \( X = k - 4 \)
In view of (48) and (45), it is seen that
\[
c_1 = 2k + 1
\]
Note that \((a, c_0, c_1)\) represents diophantine 3-tuple with property \( D(k^2 - 8k - 2) \)
Taking \((a, c_1)\) and employing the above procedure, it is seen that the triple \((a, c_1, c_2)\) where
\[
c_2 = 4k + 2
\]
exhibits diophantine 3-tuple with property \( D(k^2 - 8k - 2) \)
Taking \((a, c_2)\) and employing the above procedure, it is seen that the triple \((a, c_2, c_3)\) where
\[
c_3 = 6k + 9
\]
exhibits diophantine 3-tuple with property \( D(k^2 - 8k - 2) \)
Taking \((a, c_3)\) and employing the above procedure, it is seen that the triple \((a, c_3, c_4)\) where
\[
c_4 = 8k + 22
\]
exhibits diophantine 3-tuple with property \( D(k^2 - 8k - 2) \)
The repetition of the above process leads to the generation of sequence of diophantine 3-tuples whose general form is given by \((a, c_s, c_{s+1})\) where
\[ c_{s-1} = 2(s-1)k + 3s^2 - 14s + 17, \quad s = 1, 2, 3, \ldots \]

It is noted that, in each of the above sequences 7-12,

the triple \( (c_s, c_{s+1} + 3, c_{s+2}) \) forms an arithmetic progression.

In conclusion, one may attempt for obtaining sequences of higher order Diophantine tuples with suitable properties.

REFERENCES

9. M.A. Gopalan and V. Geetha, Formation of Diophantine Triples for Polygonal Numbers \( (t_{16,n} \text{ to } t_{25,n}) \) and Centered Polygonal Numbers \( (ct_{16,n} \text{ to } ct_{25,n}) \), IJITR, 2015, volume 3, Issue 1, 1837-1841