Volume: 5 | Issue: 9 | September 2019 || SJIF Impact Factor: 5.614 ||ISI I.F Value: 1.188 ISSN (Online): 2455-3662

EPRA International Journal of
Multidisciplinary Research (IJMR) *Peer Reviewed Journal*

# INVESTIGATING THE DEPENDENCY STRUCTURE OF UNEMPLOYMENT RATE WITH REGARD TO EXPORT AND IMPORT USING COPULA METHOD

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## ABSTRACT

***Unemployment is one of the biggest economic problems of countries. Exports and imports are also the main factors affecting the unemployment rate. The emergence of the relationship structure between these factors and unemployment will provide a better understanding of the problem of unemployment. In this study, the dependency structure of monthly export and import amounts with unemployment rate was investigated using the copula method and modeled with a two-dimensional copula. The copula function, which characterizes the dependence, was determined by looking at the AIC value from the appropriate copula models and the results were evaluated.***

**KEYWORDS:** ***Copula, Unemployment, Export, Import***

## 1. INTRODUCTION

Unemployment is the most important economic problem in the globalizing world. In order for countries to meet their growth targets, they need to solve this problem or keep it to a minimum. For this reason, studies are carried out in order to determine the social and economic causes of unemployment. Countries are trying to reduce the unemployment rate by benefiting from the findings of the studies.

The direct examination of the relationship between the factors affecting unemployment and unemployment may not give us the structure of the relationship. Blanchard and Katz (1997) developed the concept of natural unemployment rate, and said that pure growth in production had no effect on unemployment in the long run, but indirectly in the short run. Yılmaz (2005) investigated whether there is a causal relationship between unemployment rate and inflation and determined that there is no causality. Göktaş and Öznur (2010), using the main components analysis and regression analysis methods in order to determine the variables affecting the unemployment rate, it was found that the variables determined to have an effect on the unemployment rate were the amount of exports and imports. Ümit and Karataş (2018) found a causality relationship from growth to unemployment using autoregressive method. Akcan (2018), using the X-12-ARIMA and VAR methods, determined that the

variable that best describes the amount of change in unemployment rate is the real exchange rate.

Import is defined as the process of purchasing various goods or services from foreign countries. Export is the process by which a country sells goods or services to foreign countries. In this study, the dependency structure of the import and export amount with the unemployment rate is examined. In this context, the copula method was used to determine the dependency structure. Gaussian and Joe copula models were found to be the most suitable models for revealing the dependency structure.

## 2. MATERIAL AND METHOD

Copulas reveal the dependency structure between random variables. Copulas are a multivariate, special function that correlates the common distribution function of a random variable vector with the margins of that distribution. The main purpose of the copula function is to obtain the multivariate distribution, which is the most appropriate for the observed data, by revealing the dependency structure. In mathematical terms, copulas are functions that have uniform distribution on univariate margins [0,1] and connect multivariate distributions to their univariate margins.

Copulas does not require any assumptions for marginal probability distributions and allows modeling of nonlinear dependence between dependent and independent variables. Therefore, copulas have an important place in practice since they allow working independently of the scale in terms of dependence and are the starting point for building distribution families with two or more variables (Nelsen, 2003).

Two-dimensional copulas with $I = [0,1]$ are defined as follows. $C: I^2 \to I$

$(u, v) \to C(u, v)$

- For $\forall\, u, v \in I$, $C(u, 0) = C(0, v) = 0$ ie a function with a basis
- For $\forall\, u, v \in I$, $C(u, 1) = u$ and $C(1, v) = v$
- For $\forall\, u_1, u_2, v_1, v_2 \in I$, $u_1 \leq u_2$ and $v_1 \leq v_2$;

$$C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0 \qquad (\mathbf{2.1})$$

In other words, a 2-incremental function provides the conditions $C(u, v)$ function is a two-dimensional copula (Nelsen, 2006).

### 2.1. Sklar Theorem

Let $H$ be a joint distribution function of two variables with margins $F$ and $G$. So, for $\forall\, x, y \in \bar{R}$, there is a $C$ copula with $H(x, y) = C(F(x), G(y))$.

If $F$ and $G$ are continuous, $C$ is unique. Otherwise, the $C$, $F$ and $G$ functions are uniquely defined on the cartesian product of the value sets. Conversely, if $C$ is a copula and $F$ and $G$ are marginal distribution functions, then $H$ is a joint distribution function with margins $F$ and $G$. $F^{(-1)}$ and $G^{(-1)}$ are the inverse of the $F$ and $G$ distribution functions, respectively, and if $F$ and G are continuous, for $\forall\, (u, v)$, $C(u, v) = H(F^{(-1)}(u), G^{(-1)}(v))$. This result allows us to create copulas from joint distribution functions (Sklar, 1959).

### 2.2. Gaussian Copula

The Gaussian copula is expressed as follows with the dependency parameter, $\theta$.

$$C(u, v) = \Phi_G(\Phi^{-1}(u), \Phi^{-1}(v); \theta)$$
$$= \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\theta^2}} * \left(\frac{-(s^2 - 2\theta st + t^2)}{2(1-\theta^2)}\right) dsdt \qquad (\mathbf{2.2})$$

Where $\Phi$ is the standard normal distribution function, $\Phi_G(u, v)$ is the standard normal distribution function with two variables. In the Gaussian copula, the $\theta$ dependence parameter is a Pearson correlation measure and is limited to $[-1,1]$. For $\theta \to 0$, the independence becomes the hood. When $\theta \to -1$ and $\theta \to 1$, Frechet reaches the lower limit and Frechet upper limit, respectively. It is a flexible copula model because it allows equally positive and negative dependence (Trivedi and Zimmer, 2007).

### 2.3. Joe Copula

The Joe copula is expressed as follows with the dependency parameter, $\theta$.

$$C(u,v) = 1 - [(1-u)^{\theta} + (1-v)^{\theta} - (1-u)^{\theta}(1-v)^{\theta}]^{1/\theta}\ ,\ 1 \le \theta < \infty \qquad (\mathbf{2.3})$$

When $\theta \to 1^{+}$, the independence gate $C(u,v) = \pi(u,v) = uv$ is reached. Joe exhibits strong right tail addiction to the copula.

The generator function for Joe copula is expressed as follows.

$$\varphi(t) = -ln\big(1-(1-t)^{\theta}\big) \qquad (\mathbf{2.4})$$

### 2.4. Copula Estimation and Selection

In maximum likelihood method, the parameters of the predicted multivariate distribution, the parameters of the marginal distributions and the parameters related to the copula, which characterize the dependency structure, are estimated simultaneously (Joe, 1997).

Let data matrix $S = \{x_{1t}, x_{2t}, \dots, x_{nt}\}_{t=1}^{T}$ be the sample. In this case, the log-likelihood function is expressed as follows.

$$l(\theta) = \sum_{t=1}^{T} ln\{c\big(F_1(x_{1t}), F_2(x_{2t}), \dots, F_n(x_{nt})\big)\} + \sum_{t=1}^{T}\sum_{j=1}^{n} ln\, f_j\big(x_{jt}\big) \qquad (\mathbf{2.5})$$

Here, $\theta$ is the vector of parameters for the copula and marginals. If the marginal distributions and the copula function are known, the above log-likelihood function is written and the maximum likelihood estimator is found by $\hat{\theta}_{MLE} = maxl(\theta)$ (Cherubini et al, 2004).

Different copula functions exhibit different dependency patterns. Therefore, when it is desired to investigate the dependency structure, several copula models are estimated and the most appropriate data is selected. For example, if the data exhibits right-tail dependence, the selected copula function should allow modeling of that dependency. If a data context exhibits both positive and negative dependence, the choice of copula that allows only positive dependence is restrictive. In the experimental studies, it is the most effective method to discover the dependence structure by trying more than one copula model (Trivedi and Zimmer, 2005).

The maximum likelihood method is used to determine the most suitable copulas according to the AIC value used.

$$AIC = -2\ln(L) + 2k \qquad (\mathbf{2.6})$$

Here, $\ln(L)$ is the maximized log-likelihood value, $k$ is the number of parameters. The model with the smallest AIC value of the copula is the model that best demonstrates dependence (Akaike, 1974).

## 3. APPLICATION

In this study, Turkey's monthly export and the import quantity between January 2005-May 2019, and the unemployment rate has been studied. Export quantity, import quantity and unemployment rate are handled as random variables, and the dependency structure between them is determined by appropriate copula models. The parameters characterizing the dependency structure are obtained by maximum likelihood method. In order to determine the most suitable one of the obtained copula models, the AIC value is examined and the model with the lowest AIC value is selected. The binary dependency structures are interpreted according to the selected copula model.

Appropriate copula models selected according to the dependency structure between export quantity and unemployment rate. Maximum likelihood (loglik) and AIC values of these models are given in Table 1. Since Joe Copula model has minimum AIC value, it is the best model for the dependence between export quantity and unemployment. The parameter value of this family is obtained as 1.067985. The amount of exports and unemployment rate exhibits weak right-tail dependence. This means that these two variables are more prone to increase together than decrease.

**Table 1: Appropriate Copula families and parameter values of export quantity and unemployment rate**

| Copula | Loglik | AIC | $\widehat{\boldsymbol{\theta}}$ |
|---|---|---|---|
| Gaussian | 0.2 | 1.6 | 0.051 |
| Student's t | 0.15 | 3.7 | 0.049, 30 |
| Clayton | 0.01 | 1.98 | 0.014 |
| Gumbel | 0.43 | 1.14 | 1.042 |
| Frank | 0.09 | 1.83 | 0.195 |
| Joe | 0.54 | **0.92** | **1.068** |

Appropriate copula models selected according to the dependency structure between import quantity and unemployment rate. Maximum likelihood (loglik) and AIC values of these models are given in Table 2. Since Gaussian copula model has a minimum AIC, it is best model for the dependence between the amount of imports and the unemployment rate. The parameter value of this family is obtained as -0.31141. There is a low negative dependence between the amount of imports and the unemployment rate. This means that one of

these variables tends to increase, while the other tends to decrease. As a result, one of these variables increases and the other decreases.

**Table 2: Appropriate Copula families and parameter values of import quantity and unemployment rate**

| Copula | Loglik | AIC | $\widehat{\theta}$ |
|---|---|---|---|
| Gaussian | 7.98 | **-13.96** | **-0.311** |
| Student's t | 7.88 | -11.76 | -0.309,30 |
| Frank | 7.23 | -12.47 | -1.819 |

## 4. CONCLUSİON

In this study, the dependency structure of the export and import quantities, which are known to have an effect on the unemployment rate, was investigated. Since the nonlinear structure of dependence cannot be understood by using linear methods, the copula method was used. Among the different models available in the copula method, the one that was suitable for the data was chosen according to the AIC values. Joe copula model was selected between export and unemployment rate and Gaussian copula model was selected between import and unemployment rate. When the dependency structure was examined according to the current models, it was seen that the export and unemployment rate tends to increase together. On the other hand, it was observed that the import and unemployment rate had a negatively low linear relationship.

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