ISSN (Online): 2455-3662
EPRA International Journal of Multidisciplinary Research (IJMR) - Peer Reviewed Journal
Volume: 6 | Issue: 6 | June 2020 || Journal DOI: 10.36713/epra2013 || SJIF Impact Factor: 7.032 ||ISI Value: 1.188

RESPONSE OF LEGUERRE POLYNOMIAL VIA DINESH VERMA TRANSFORM (DVT)

Dinesh Verma
Associate Professor,
Yogananda College of Engineering & Technology, Jammu

Sanjay Kumar Verma
Assistant Teacher
Adarsh Inter College,
Jalesar (Etah), U.P.

ABSTRACT

The Dinesh Verma Transform (DVT) is a mathematical tool used in solving the differential equations. Dinesh Verma Transform (DVT) makes it easier to solve the problem in engineering application and make differential equations simple to solve. In this paper, we will Study on Applications of Dinesh Verma Transform (DVT) with Leguerre polynomial.

KEY WORDS: ***Dinesh Verma Transform (DVT), Leguerre Polynomial, Differential Equation.***

I. INTRODUCTION

The Dinesh Verma Transform (DVT) has been applied in different areas of science, engineering and technology [1], [2], [3] [4], [5], [6], [7]. The Dinesh Verma Transform (DVT) is applicable in so many fields and effectively solving linear differential equations. Ordinary linear differential equation with constant coefficient and variable coefficient can be easily solved by the Dinesh Verma Transform (DVT) without finding their general solutions [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18] [19], [20], [21], [22]. The Leguerre polynomial of nth order generally solved by adopting Laplace Transform, Elzaki Transform [23], [24], [25] [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36].. This paper we will Analyze the Dinesh Verma Transform (DVT) of Leguerre polynomial of nth order and the application of Dinesh Verma Transform (DVT) in solving the differential equations including Leguerre Polynomial.

1. BASIC DEFINITIONS

2.1 DEFINITION OF DINESH VERMA TRANSFORM (DVT)

Dr. Dinesh Verma recently introduced a novel transform and named it as **Dinesh Verma Transform (DVT).** Let f(t) is a well-defined function of real numbers t ≥ 0. The **Dinesh Verma Transform (DVT)** of f(t), denoted by $D\{\{f(t)\}$, is defined as

$$D\{\{f(t)\} = p^5 \int_0^\infty e^{-pt} f(t)dt = \bar{f}(p)$$

Provided that the integral is convergent, where $p$may be a real or complex parameter and D is the **Dinesh Verma Transform (DVT)** operator.

DINESH VERMA TRANSFORM OF ELEMENTARY FUNCTIONS

According to the definition of **Dinesh Verma transform (DVT)**,

$$\mathbf{D\{t^n\} = p^5 \int_0^\infty e^{-pt}\, t^n dt}$$

$$= p^5 \int_0^\infty e^{-z} \left(\frac{z}{p}\right)^n \frac{dz}{p} \quad , z = pt$$

$$= \frac{p^5}{p^{n+1}} \int_0^\infty e^{-z}\, (z)^n dz$$

Applying the definition of gamma function,

$$\text{D}\,\{y^n\} = \frac{p^5}{p^{n+1}} \lceil (n+1)$$

$$= \frac{1}{p^{n-4}} n!$$

$$= \frac{n!}{p^{n-4}}$$

Hence, $\mathbf{D\{t^n\} = \frac{n!}{p^{n-4}}}$

ISSN (Online): 2455-3662
EPRA International Journal of Multidisciplinary Research (IJMR) - Peer Reviewed Journal
Volume: 6 | Issue: 6 | June 2020 || Journal DOI: 10.36713/epra2013 || SJIF Impact Factor: 7.032 ||ISI Value: 1.188

Dinesh Verma Transform (DVT) of some elementary Functions

- $D\{t^n\} = \frac{n!}{p^{n-4}}$, $where\ n = 0,1,2,..$
- $D\{e^{at}\} = \frac{p^5}{p-a}$,
- $D\{sinat\} = \frac{ap^5}{p^2+a^2}$,
- $D\{cosat\} = \frac{p^6}{p^2+a^2}$,
- $D\{sinhat\} = \frac{ap^5}{p^2-a^2}$,
- $D\{coshat\} = \frac{p^6}{p^2-a^2}$.
- $D\{\delta(t)\} = p^4$

- The Inverse **Dinesh Verma Transform (DVT)** of some of the functions are given by
- $D^{-1}\left\{\frac{1}{p^{n-4}}\right\} = \frac{t^n}{n!}$, $where\ n = 0,1,2,..$
- $D^{-1}\left\{\frac{p^5}{p-a}\right\} = e^{at}$,
- $D^{-1}\left\{\frac{p^5}{p^2+a^2}\right\} = \frac{sinat}{a}$,
- $D^{-1}\left\{\frac{p^6}{p^2+a^2}\right\} = cosat$,
- $D^{-1}\left\{\frac{p^5}{p^2-a^2}\right\} = \frac{sinhat}{a}$,
- $D^{-1}\left\{\frac{p^6}{p^2-a^2}\right\} = coshat$,
- $D^{-1}\{p^4\} = \delta(t)$

DINESH VERMA TRANSFORM (DVT) OF DERIVATIVES

$$D\{f^{'}(t)\} = p\bar{f}(p) - p^5 f(0)$$

$$D\{f^{''}(t)\} = p^2\bar{f}(p) - p^6 f(0) - p^5 f^{'}(0)$$

$D\{f^{'''}(y)\} = p^3\bar{f}(p) - p^7 f(0) - p^6 f^{'}(0) - p^5 f^{''}(0)$And so on.

$D\{tf(t)\} = \frac{5}{p}\bar{f}(p) - \frac{d\bar{f}(p)}{dp}$,

$D\{tf^{'}(\mathrm{t})\} = \frac{5}{p}\left[p\bar{f}(p) - p^5\mathrm{f}(0)\right] - \frac{d}{dp}\left[p\bar{f}(p) - p^5\mathrm{f}(0)\right]$ and

$D\{tf^{''}(\mathrm{t})\} = \frac{5}{p}[p^2\bar{x}(p) - p^6 x(0) - p^5 x^{'}(0)] - \frac{d}{dp}[p^2\bar{x}(p) - p^6 x(0) - p^5 x^{'}(0)]$ And so on.

II. METHODOLOGY

Laguerre Polynomial

The Laguerre polynomial is defined as [4, 5]

$$L_n(u) = \frac{e^u}{n!}\frac{d^n}{du^n}(e^{-u}u^n)$$

We know that by the definition of Dinesh Verma Transform (DVT)

$$\mathrm{D}\{f(\mathrm{t})\} = \bar{f}(p) = p^5\int_0^\infty e^{-pt} f(\mathrm{t})dt.$$

Therefore,

$$D\{L_n(t)\} = p^5\int_0^\infty e^{-pt}\left\{\frac{e^t}{n!}\frac{d^n}{dt^n}\ (e^{-t}t^n)\right\}dt$$

$$= \frac{p^5}{n!}\int_0^\infty e^{-(p-1)t}\left\{\frac{d^n}{dt^n}\ (e^{-t}t^n)\right\}dt$$

$$= \frac{p^5}{n!}\left[(p-1)\int_0^\infty e^{-(p-1)t}\frac{d^{n-1}}{dt^{n-1}}(e^{-t}t^n)dt\right]$$

Integrating again,

$$\frac{p^5(p-1)^2}{n!}\int_0^\infty e^{-(p-1)t}\frac{d^{n-2}}{dt^{n-2}}(e^{-t}t^n)dt$$

Integrating n again,

$$= \frac{p^5(p-1)^n}{n!}\int_0^\infty e^{-(p-1)t}\ (e^{-t}t^n)dt$$

$$= \frac{(p-1)^n}{n!}\left[p^5\int_0^\infty e^{-pt}\ (t^n)dt\right]$$

$$= \frac{(p-1)^n}{n!}D\{t^n\}$$

But, By the definition of Dinesh Verma Transform (DVT)

$$\mathrm{D}\ \{\mathrm{F}\ (\mathrm{t})\} = p^5\int_0^\infty e^{-pt} f(\mathrm{t})dt.$$

Hence,

$$\frac{(p-1)^n}{n!}D\{t^n\} = \frac{(p-1)^n}{n!}.\frac{n!}{p^{n-4}}$$

Hence,

$$D\{L_n(t)\} = \frac{(p-1)^n}{p^{n-4}}$$

(A) Solve the differential equations

$$(D^2 + D)y = L_1(t)$$

with initial conditions

$y(0) = 0\ , y^{'}(0) = 1$

Solution:

Given equation can be written as

$$y^{''} + y^{'} = L_1(t)$$

Taking Dinesh Verma Transform (DVT) on sides

$$D\{y^{''}\} + D\{y^{'}\} = D\{L_1(t)\}$$

Because Leguerre polynomial of order 1 is

$$L_1\{t\} = \{1 - t\}$$

$$[p^2\bar{y}(p) - p^6 y(0) - p^5 y^{'}(0)]$$

$$+[p\bar{y}(p) -\ p^5 y(0)] = p^4 - p^3$$

Applying initial conditions, we get

$$[p^2\bar{y}(p) - p^5] + p\bar{y}(p) = p^4 - p^3$$

$$(p^2 + p)\bar{y}(p) = p^5 + p^4 - p^3$$

$$\bar{y}(p) = \frac{p^5 + p^4 - p^3}{(p^2 + p)}$$

Applying Inverse Dinesh Verma Transform (DVT), we get,

$$\boldsymbol{y = 2t - 1 + e^{-t} - \frac{t^2}{2}}$$

(B) Solve the differential equations
$(\boldsymbol{D^2 + d^2 D})\boldsymbol{y} = \boldsymbol{L_1(t)},$
with initial conditions

$$\boldsymbol{y(0) = 0\,, y'(0) = 0}$$

Solution
Given equation can be written as

$$y'' + d^2 y' = L_1(t)$$

Taking Dinesh Verma Transform (DVT) on sides

$$D\{y''\} + d^2 D\{y'\} = D\{L_1(t)\}$$

Because Leguerre polynomial of order 1 is

$$L_1\{t\} = \{1 - t\}$$

Now,

$$[p^2\bar{y}(p) - p^6 y(0) - p^5 y'(0)]$$

$$+d^2[p\bar{y}(p) - p^5 y(0)] = p^4 - p^3$$

Applying initial conditions, we get

$$[p^2 + pd^2]\bar{y}(p) = p^4 - p^3$$

$$\bar{y}(p) = \frac{p^4}{p(p + d^2)} - \frac{p^3}{p^2(p + d^2)}$$

Applying Inverse Dinesh Verma Transform (DVT), we get,

$$\boldsymbol{y = \left(\frac{1}{d^2} + \frac{1}{d^4}\right).t - \frac{t^2}{2d^2} + \left(\frac{1}{d^6} + \frac{1}{d^4}\right)(e^{-d^2 t} - 1)}$$

(C) Solve the differential equations

$$(\boldsymbol{D^2 + 4})\boldsymbol{y} = \boldsymbol{L_2(t)}$$

With initial conditions
$\boldsymbol{y(0) = 0\,, y'(0) = 1}$

Solution:
Given equation can be written as
$y'' + 4y = L_2(t)$
Taking Dinesh Verma Transform (DVT) on sides
$D\{y''\} + 4D\{y\} = D\{L_2(t)\}$
Because Leguerre polynomial of order 2 is
$L_2\{t\} = \frac{1}{2}\{2 - 4t + t^2\}$
Now,
$[p^2\bar{y}(p) - p^6 y(0) - p^5 y'(0)]$

$+4\bar{y}(p) = p^2(p-1)^2$
Applying initial conditions, we get

$$(p^2 + 4)\bar{y}(p) = p^2(p-1)^2 + p^5$$

$$\bar{y}(p) = \frac{p^2(p-1)^2}{(p^2+4)} + \frac{p^5}{(p^2+4)}$$

Applying inverse Dinesh Verma Transform

(DVT), we get,
$\boldsymbol{y = \frac{3}{16} + \frac{t^2}{8} - \frac{t}{2} - \frac{3}{16}cos2t + \frac{3}{4}sin2t}$

CONCLUSION

This paper has presented how to get the Dinesh Verma Transform (DVT) of Leguerre polynomial of nth order and the application of Dinesh Verma Transform (DVT) for solving the differential equations including Leguerre Polynomial.

REFERENCES

1. *Dinesh Verma, Putting Forward a Novel Integral Transform: Dinesh Verma Transform (DVT) and its Applications, International Journal of Scientific Research in Mathematical and Statistical Sciences, Volume -7, Issue-2, April-2020, pp: 139-145.*
2. *Rahul gupta, Rohit gupta and Dinesh Verma, Propounding a New Integral Transform: Gupta Transform with Applications in Science and Engineering, International Journal of scientific research in multidisciplinary studies (IJSRMS), Volume-6, Issue-3, March 2020, pp: 14-19.*
3. *Mohamed Elarabi Benattia , Kacem Belghaba, Application Of Aboodh Transform For Solving First Order Constant Coefficients Complex Equation, General Letters in Mathematics Vol. 6, No. 1, Mar 2019, pp.28-34*
4. *Dinesh Verma and Rohit Gupta, Laplace Transformation approach to infinite series, International Journal of Advance and Innovative Research, Volume 6, Issue 2 (XXXIII): April – June, 2019.*
5. *B.V.Ramana, Higher Engineering Mathematics.*
6. *Dr. B.S. Grewal, Higher Engineering Mathematics.*
7. *Shiferaw Geremew Gebede, Laplace transform of power series, impact: international journal of research in applied, natural and social sciences (impact: IJRANSS), Issn(p): 2347-4580; Issn (e): 2321-8851, vol. 5, Issue 3, mar 2017, 151-156.*
8. *Dinesh Verma, Rohit Gupta and Amit Pal Singh, Analysis of Integral Equations of convolution via Residue Theorem Approach, The International Journal of analytical and experimental modal, Volume-12, Issue-1, January 2020, 1565-1567.*
9. *Dinesh Verma, Analyzying Leguerre Polynomial by Aboodh Transform, ASIO Journal of Chemistry, Physics, Mathematics & Applied Sciences (ASIO-JCPMAS), Volume -4, Issue-1, 2020, ISSN: 2455-7064, PP:14-16.*
10. *Dinesh Verma and Rohit Gupta, Aplications of Elzaki Transform to Electrical Network Circuits with Delta*

Function, ASIO Journal of Chemistry, Physics, Mathematics & Applied Sciences (ASIO-JCPMAS), Volume -4, Issue-1, 2020, ISSN: 2455-7064, PP:21-23.

11. *Dinesh Verma and Rohit Gupta, Analyzying Boundary Value Problems in Physical Sciences via Elzaki Transform , by in ASIO Journal of Chemistry, Physics, Mathematics & Applied Sciences (ASIO-JCPMAS), Volume -4, Issue-1, 2020, ISSN: 2455-7064, PP:17-20.*
12. *Dinesh Verma, Elzaki Transform Approach to Differential Equatons with Leguerre Polynomial, International Research Journal of Modernization in Engineering Technology and Science (IRJMETS), Volume-2, Issue-3, March 2020, pp: 244-248.*
13. *Dinesh Verma, Elzaki Transform of some significant Infinite Power Series, International Journal of Advance Research and Innovative Ideas in Education (IJARIIE), Volume-6, Issue-1, February 2020, pp:1201-1209.*
14. *[14] Dinesh Verma, Aftab Alam , Analysis of Simultaneous differential Equations by Elzaki Transform Approach, Science ,Technology and Development Journal, Volume-9, Issue-1, January 2020, pp: 364-367.*
15. *Dinesh Verma, Applications of Laplace Transform to Differential Equations with Discontinuous Functions, New York Science Journal, Volume-13, Issue-5, May 2020, pp: 66-68.*
16. *Dinesh Verma, A Laplace Transformation approach to Simultaneous Linear Differential Equations, New York Science Journal" Volume-12, Issue-7, July 2019, pp: 58-61.*
17. *Dinesh Verma, A Useful technique for solving the differential equation with boundary values, Academia Arena" Volume-11, Issue-2, 2019, pp: 77-79.*
18. *Dinesh Verma, Relation between Beta and Gamma function by using Laplace Transformation, Researcher, Volume-10, Issue-7, 2018, pp: 72-74.*
19. *Dinesh Verma , An overview of some special functions, International Journal of Innovative Research in Technology (IJIRT), Volume-5, Issue-1, June 2018, pp: 656-659.*
20. *Dinesh Verma, Applications of Convolution Theorem, International Journal of Trend in Scientific Research and Development (IJTSRD), Volume-2, Issue-4, May-June 2018, pp: 981-984.*
21. *Dinesh Verma, Solving Fourier Integral Problem by Using Laplace Transformation, International Journal of Innovative Research in Technology (IJIRT), Volume-4, Issue-11, April 2018, pp:1786-1788.*
22. *Dinesh Verma, Applications of Laplace Transformation for solving Various Differential equations with variable co-efficient, International Journal for Innovative Research in Science and Technology (IJIRST),Volume-4, Issue-11, April 2018, pp: 124-127.*
23. *Dinesh Verma and Amit Pal Singh, Applications of Inverse Laplace Transformations, Compliance Engineering Journal, Volume-10, Issue-12, December 2019, ISSN 0898-3577; PP: 305-308.*
24. *Dinesh Verma and Rohit Gupta, A Laplace Transformation of Integral Equations of Convolution Type, International Journal of Scientific Research in Multidisciplinary Studies, Volume-5, Issue-9, September 2019, pp: 94-96.*
25. *Dinesh Verma and Amit Pal Singh, Solving Differential Equations Including Leguerre Polynomial via Laplace Transform, International Journal of Trend in scientific Research and Development (IJTSRD), Volume-4, Issue-2, February 2020, pp:1016-1019.*
26. *Dinesh Verma, Signification of Hyperbolic Functions and Relations, International Journal of Scientific Research & Development (IJSRD), Volume-07, Issue-5, May 2019, pp: 01-03.*
27. *H.R.Gupta, Dinesh verma, Effect of Heat and mass transfer on oscillatory MHD flow, Journal of applied mathematics and fluid mechanics, Volume- 3 November 2 (2011) ,PP: 165-172.*
28. *Dinesh Verma and Binay Kumar , Modeling for Maintence job cost-An Approach, International Journal for Technological Research in Engineering (IJTRE), Volume-2, Issue-7, March 2015, ISSN:No. 2347-4718.PP: 752-759.*
29. *Monika Kalra, Dinesh Verma, Effect of Constant Suction on Transient Free Convection Gelatinous Incompressible Flow Past a Perpendicular Plate With Cyclic Temperature Variation in Slip Flow Regime, International Journal of Innovative Technology and Exploring Engineering (IJITEE), volume-2, Issue-4, March (2013),PP:42-44.*
30. *Dinesh Verma, Monika Kalra, Free Convection MHD Flow Past a Vertical Plate With Constant Suction, International Journal of Innovative Technology and Exploring Engineering (IJITEE), volume-2, Issue-3, February (2013),PP: 154-157.*
31. *[31] Nitin Singh Sikarwar and Dinesh Verma, Micro Segmentation: Today's Success Formulae, International Journal of Operation Management and Services, vol. 2, November 1 (2012), PP: 1-6.*
32. *Nitin Singh Sikarwar, Dinesh verma, Faculty Stress Management, Global Journal of Management Science and Technology, Vol. 1, Issue 6 (July 2012), pp: 20-26.*
33. *[33] Dinesh Verma and Vineet Gupta, Uniform and non-uniform flow of common axis cylinder, International e journal of Mathematics and Engineering, Vol. I (IV) (2011), pp: 1141-1144.*
34. *Rohit Chopra, Arvind Dewangan, Dinesh Verma, Importance of Aerial Remote Sensing Photography, International e journal of Mathematics And Engineering, Vol.I (IV) (2010),pp:757-760.*
35. *Dinesh verma, .Vineet Gupta, Arvind dewangan, Solution of flow problems by stability, International e journal of Mathematics and Engineering, Vol. I (II) (2010), PP: 174-179.*
36. *Dinesh Verma, Empirical Study of Higher Order Diffeential Equations with Variable Coefficient by Dinesh Verma Transformation (DVT), ASIO Journal of Engineering & Technological Perspective Research (ASIO-JETPR), Volume -5, Issue-1, 2020, pp:04-07.*