HEAT CONDUCTED THROUGH FINS OF VARYING CROSS-SECTIONS VIA ROHIT TRANSFORM

Neeraj Pandita  
Assistant Professor, Department of Mechanical Engineering, Yogananda College of Engineering and Technology, Jammu

Rohit Gupta  
Lecturer of Physics, Department of Applied Sciences, Yogananda College of Engineering and Technology, Jammu

ABSTRACT  
The conduction of heat takes place through the fins or spines from particle to particle due to temperature gradient in the direction of decreasing temperature. Heat is not lost equally by each element of the fin but is lost mostly near the base of the fin. Thus there would be wastage of the material if a uniform fin is used. These aspects demand the construction of the fins of varying cross-sections like triangular fin, hyperbolic fin and the parabolic fin. The triangular and parabolic fins of varying cross-sections are usually analyzed by ordinary calculus approach. The paper analyzes triangular and parabolic fins of varying cross-sections to find the rate of conduction of heat through them via the new integral transform called Rohit Transform.

INDEX TERMS: Triangular fin, Parabolic fin, Rohit Transform, Temperature Distribution, Rate of Conduction of Heat.

I. INTRODUCTION  
The conduction of heat takes place through the fins or spines from particle to particle due to temperature gradient in the direction of decreasing temperature [1, 2]. Fins or spines are the extended surfaces which are mostly used in the devices which exchange heat [3-8] like computer central processing unit, power plants, radiators, heat sinks, etc. The triangular and parabolic fins of varying cross-sections are usually analyzed by ordinary calculus approach [4-7]. The paper analyzes triangular and parabolic fins of varying cross-sections to find the rate of conduction of heat through them via the new integral transform called Rohit Transform. This Transform has been put forward by the author Rohit Gupta in recent years [8] and is not widely known. The Rohit Transform has been applied in science and engineering to solve most of the initial value problems in science and engineering [9-16]. The Rohit Transform comes out to be very effective tool to find the temperature distribution along a triangular fin, and a parabolic fin and hence the rate of conduction of heat through them.

Basics of Rohit Transform:  
The Rohit Transform [10, 11, 12] of \( g(y) \), \( y \geq 0 \) is defined as  
\[
R\{g(y)\} = r^3 \int_0^\infty e^{-ry} g(y)dy = G(r),
\]
provided that the integral is convergent, where \( r \) may be a real or complex parameter. The Rohit Transform of some derivatives [7, 13, 14, 15] of \( g(y) \) is given by  
\[
R\{g'(y)\} = rR\{g(y)\} - r^3 g(0)
\]
Or  
\[
R\{g''(y)\} = r^2 G(r) - r^4 g(0) - r^3 g'(0),
\]
\[
R\{yg(y)\} = \frac{3}{r} R\{g(y)\} - \frac{d}{dr} R\{g(y)\},
\]
II. MATERIAL AND METHOD  

Case I: Triangular Fin

The differential equation for analyzing a triangular fin [5, 8] (assuming that heat flow pertains to one-dimensional conduction of heat) is given by

\[ R(yg'(y)) = 2R(g(y)) - r \frac{d}{dr}R(g(y)), \]

\[ R(yg''(y)) = rR(g(y)) + r^2g(0) - r^2 \frac{d}{dr}R(g(y)) \]

where \( D = \sqrt{\frac{2ht}{ck}} \), \( L \) is the length of the fin between the base at \( x = L \) and the tip at \( x = 0 \), \( t \) is the thickness of the fin which increases uniformly from zero at the tip to \( t \) at the base, \( k \) is thermal conductivity, \( h \) is the coefficient of transfer of heat by convection, \( \theta(x) = T(x) - T_0 \), \( T_0 \) is the temperature of the environment of the fin and \( T_0 \) is the temperature at the base \( x = 0 \) of the fin.

Multiplying both sides of (1) by \( x \), we get

\[ x \theta''(x) + \theta'(x) - D^2 \theta(x) = 0 \ldots \ldots (2) \]

The Rohit Transform [10, 11, 12, 13] of (2) gives

\[ \left[ -p^2 \frac{d}{dp} \theta(p) + p^3 \theta(0) + p \theta'(p) \right] + p \theta(p) \right) - p^3 \theta(0) - D^2 \theta(p) = 0 \ldots \ldots (3) \]

Put \( \theta(0) = b \) and \( \theta'(0) = a \), and simplifying and rearranging (3), we get

\[ \frac{\theta'(p)}{\theta(p)} \left[ \frac{2}{p} - \frac{p^2}{p^2} \right] \ldots \ldots (4) \]

Integrating both sides of (4) w.r.t. \( p \) and simplifying, we get

\[ \log_e \theta(p) = [2 \log_e p + D^2 \frac{1}{2} + \log_e c] \ldots \ldots (5) \]

Simplifying (5), we get

\[ \theta(p) = cp^2 e^{(D^2 \frac{1}{2})} \]

Expanding the exponential term, we get

\[ \theta(p) = cp^2 \left[ 1 + \frac{D^2}{2!} + \frac{(D^2)^2}{3!} + \frac{(D^2)^3}{4!} \ldots \ldots \right] \]

\[ \text{or} \]

\[ \theta(p) = c \left[ p^2 + D^2 p + \frac{D^4}{2!} + \frac{D^6}{3!} p + \frac{D^8}{4!} p^2 \ldots \ldots \right] \ldots \ldots (6) \]

The inverse Rohit Transform [8] of (6) provides

\[ \theta(x) = c \left[ 1 + \frac{1}{4}(2D \sqrt{x})^2 + \frac{1}{2!} \left( \frac{2D \sqrt{x}}{2} \right)^4 \right] \]

\[ + \frac{1}{3!} \left( \frac{2D \sqrt{x}}{2} \right)^6 + \frac{1}{4!} \left( \frac{2D \sqrt{x}}{2} \right)^8 \ldots \ldots (7) \]

The modified Bessel function [17] of the first kind of order \( n \) and its first order derivative are given by

\[ I_n(z) = \sum_{r=0}^{\infty} \frac{1}{r!} \left( \frac{z}{2} \right)^{n+2r} \ldots \ldots (8) \]

Also \( \frac{d}{dx}(I_n(x)) = I_{n+1}(x) \frac{d}{dx}(x) \ldots \ldots (9) \]

Put \( z = 2D \sqrt{x} \) and \( n = 0 \), we get

\[ I_0(2D \sqrt{x}) = \sum_{r=0}^{\infty} \frac{1}{r!} \left( \frac{2D \sqrt{x}}{2} \right)^{2r} \ldots \ldots \]

Or

\[ I_0(2D \sqrt{x}) = 1 + \left( \frac{2D \sqrt{x}}{2} \right)^2 + \frac{1}{2!} \left( \frac{2D \sqrt{x}}{2} \right)^4 + \frac{1}{3!} \left( \frac{2D \sqrt{x}}{2} \right)^6 + \frac{1}{4!} \left( \frac{2D \sqrt{x}}{2} \right)^8 \ldots \ldots \]

Hence (7) can be rewritten as

\[ \theta(x) = cl_0(2D \sqrt{x}) \ldots \ldots (10) \]

To find the constant \( c \), at \( x = L \), \( \Theta(L) = \Theta_0 \), therefore

\[ c = \frac{\Theta_0}{l_0(2D \sqrt{L})} \]

Hence (10) can be written as

\[ \Theta(x) = \frac{\Theta_0}{l_0(2D \sqrt{L})} l_0(2D \sqrt{x}) \ldots \ldots (11) \]

The equation (11) gives the temperature distribution along the length of the triangular fin.
The heat conducted through the triangular is given by the Fourier’s Law [18, 19, 20] of heat conduction as

\[ H = kA \left( \Theta'(x) \right)_{x=L} = kbt \left( \Theta'(x) \right)_{x=L} \]

Using (11), we get

\[ H = kbt \frac{\theta_0}{\sqrt{L}} \int_0^L (2D\sqrt{L}) \left( \frac{d}{dx} \left( 2D\sqrt{x} \right) \right)_{x=L} \]

On simplifying, we get

\[ H = kbtD \frac{\theta_0}{\sqrt{L}} \int_0^L (2D\sqrt{L}) \ldots (12) \]

Put the value of D, we get

\[ H = b\sqrt{2kbt} \frac{\theta_0}{\sqrt{L}} \int_0^L (2D\sqrt{L}) \ldots \ldots (13) \]

This equation (13) gives the expression for the rate of conduction of heat through the triangular fin.

**Case II: Parabolic fin**

The differential equation for analyzing a parabolic fin [5, 9] (assuming that heat flow pertains to one dimensional conduction of heat) is given by

\[ x^2 \theta''(x) + 2x \theta'(x) - M^2 l^2 \theta(x) = 0 \ldots \ldots (14) \]

where \( M = \frac{1}{\sqrt{bk}} \), \( l \) is the length of the fin between the base at \( x = l \) and the tip at \( x = 0 \), \( t \) is the thickness of the fin which increases uniformly from zero at the tip to \( t \) at the base, \( k \) is thermal conductivity, \( h \) is the coefficient of transfer of heat by convection, \( \theta(x) = T(x) - T_0 \), \( T_0 \) is the temperature of the environment of the fin and \( T_0 \) is the temperature at the base \( x = 0 \) of the fin.

Substituting \( x = e^z \), the equation (14) can be rewritten into a form:

\[ \theta''(x) + \theta'(x) - M^2 l^2 \theta(x) = 0 \ldots \ldots (15) \]

The Rohit Transform [13, 14, 15, 16] of (15) gives

\[ [p^2 \theta(p) - p^4 \theta(0) - p^4 \theta'(0)] + p \theta(p) - p^3 \theta(0) - M^2 l^2 \theta(p) = 0 \ldots \ldots (16) \]

Put \( \theta(0) = P \) and \( \theta'(0) = Q \), and simplifying and rearranging (16), we get

\[ \theta(p) = \frac{p^3(p + 1)P + p^3Q}{p^2 + p - M^2 l^2} \]

Or

\[ \theta(p) = \frac{p^3(pP + (P + Q))}{(p - c_1)(p + c_2)} \ldots (17) \]

where \( c_1 = \frac{-1 + (1 + 4M^2l^2)^{1/2}}{2} \) and \( c_2 = \frac{-1 - (1 + 4M^2l^2)^{1/2}}{2} \).

This equation (17) can be rewritten as

\[ \theta(p) = \frac{c_1P + P + Q}{c_1 + c_2} \frac{p^3}{(p - c_1)} + \frac{c_2P + P + Q}{c_2 - c_1} \frac{p^3}{(p + c_2)} \ldots (18) \]

The inverse Rohit Transform [8] of (18) provides

\[ \Theta(x) = \frac{c_1P + P + Q}{c_1 + c_2} e^{c_1x} + \frac{c_2P + P + Q}{c_2 - c_1} e^{-c_2x} \ldots \ldots (19) \]

As \( \Theta(0) \) is finite \([4 - 7] \), therefore, the term \( \frac{c_2P + P + Q}{c_2 - c_1} e^{-c_2x} \) is equated to zero

i.e. \( \frac{c_2P + P + Q}{c_2 - c_1} x^{-c_2} = 0 \), which gives \( c_2P + P + Q = 0 \) or \( Q = -(c_2P + P) \).

From (19), we have

\[ \Theta(x) = \frac{c_1 - c_2}{c_1 + c_2} P x^c_1 \ldots \ldots (20) \]

To find the constant \( P \), at \( x = l \), \( \Theta(l) = \Theta_0 \), therefore from (20), \( P = \frac{c_1 + c_2}{c_1 - c_2} \Theta_0 l^{-c_1} \)

Hence (20) can be rewritten as

\[ \Theta(x) = \Theta_0 l^{-c_1} x^c_1 \]

Or

\[ \Theta(x) = \Theta_0 (x/l)^{c_1} \ldots \ldots (21) \]

The equation (21) gives the temperature distribution along the length of the parabolic fin.

The heat conducted through the parabolic fin is given by the Fourier’s Law of heat conduction [19, 20] as

\[ H = kA \left( \Theta'(x) \right)_{x=l} = kbt \left( \Theta'(x) \right)_{x=l} \]
Using equation (21), we get

\[ H = \frac{kbt \Theta_0 c_1}{l} \]

Or

\[ H = \frac{kbt \Theta_0}{2t} \left(1 + \frac{1 + 4M^2}{2} \right)^{1/2} \ldots (22) \]

This equation (22) gives the expression for the rate of conduction of heat through the parabolic fin.

### III. RESULT AND CONCLUSION

We have found the temperature distribution along the lengths of the triangular fin as well as parabolic fin and hence the rate of conduction of heat through them via Rohit Transform means. It is found that with the increase in the length of the triangular fin or parabolic fin, temperature increases and hence the rate of conduction of heat at any cross-section of the triangular fin or parabolic fin increases. The method has come out to be a very effective tool to find temperature distribution along the lengths of the triangular fin as well as parabolic fin and hence the rate of conduction of heat through them.

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