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DETERMINING OF DEPENDENCY BETWEEN EXCHANGE RATES THROUGH COPULA – GARCH MODEL

Emre Yıldırım[1]
[1] Department of Statistics, Ondokuz Mayis University, Samsun, Turkey.

Mehmet Ali Cengiz[2]
[2] Professor, Department of Statistics, Ondokuz Mayis University, Samsun, Turkey.

ABSTRACT

Copula is one of crucial methods used in modelling dependency between financial assets. In financial investigations, techniques based on multivariate normality are often utilized. However, it is difficult to hold this assumption in actual data sets. Since satisfying multivariate normality for copula is not necessary, it has been widely used as effective tool in modelling dependency recently. Copula provides rather flexibility for modelling dependency due to the fact that both it has a whole range of models and it allows for modelling different dependency measures such as tail dependency. Copula that coordinates with univariate GARCH model outperforms other techniques in modelling the relation among variables. In this paper dynamic dependency between six exchange rates against U.S. dollar which have the most traded in world market is desired to deal with. Modelling of dependence structure between exchange rates is rather significant in terms of leading investors to right investments. For this purpose, Copula-GARCH model is used to model dependency between exchange rates. Challenges such as autocorrelation and heteroscedasticiy confronted in financial returns are handled by AR-GARCH (Autocorrelation – Generalized Autocorrelation Conditional Heteroscedasticity) model. Then by obtaining inputs for copula, parameter estimation is performed by means of ML (Maximum Likelihood) and the best fitted copula is determined by way of information criterion such as AIC and BIC. It is found out that the parameters of copula changes over time and it is determined that time varying Student's t copula models better dynamic dependency between returns of exchange rates. In consequence of the study it is understood that copula can model dynamic dependency between financial assets without being satisfied heavy assumptions which are necessary in classical methods and estimation of dependency between exchange rates can be performed through this method.

KEYWORDS: *dependency, copula, GARCH model, exchange rate*

1. INTRODUCTİON

Modelling of dependence structure between variables is one of issues investigated commonly in many disciplines such as statistics and econometrics. Determining dependency accurately allows for performing robust estimation for interest variables. Distributions of variables represent key point in specifying relations. For instance, ordinary correlation coefficient provides accurate dependence structure if distributions of interest variables are normally distributed. However, it is difficult to hold the assumption in actual data sets. Financial returns are insufficient to satisfy the assumption of multivariate normality since they are often skewed and heavy tailed. Copula is a multivariate distribution function whose margins are uniformly distributed. While there is wide range of univariate distributions, the number of multivariate distributions is very few. On the other hand, existing multivariate distributions are not sufficient in modelling of all actual data due to heavy assumptions. For example, degree of freedoms of each margins are not necessary to be equal for Student's t copula while multivariate Student's t distribution needs to satisfy the assumption which each margins have the same degree of freedom. Another advantage of modelling via copula is when familiar multivariate distributions are inadequate for modelling dependency, different multivariate distributions can be constructed by using copula. Many of them offer efficient results even if all of them do not give possibility of flexible usage. According to Sklar's theorem (1959) which unearths the presence of copulas any n-dimensional joint distribution function can be decomposed to n-margins and a copula. Critical point in modelling multivariate distribution is reverse of Sklar's theorem. This indicates that joint distributions for any two variables can be generated via copula irrespective of marginal distributions of interest variables. Additionally this theorem allows for increasing the number of multivariate parametric distributions (Patton, 2006). Recently applications of copula in literature have fairly increased due to its existing and improving features. A great amount of searches was carried out in many field such as industry, agriculture, health and economic by using copula. Sriboonchitta et al. (2013) established relationship between conditional volatility of agriculture price and agricultural production index. Basher et al. (2014) investigated dependence structure for seven stocks in GCC in pairs. Hu (2006) dealt with dependency in financial markets via mixed copula. Moreover, Manner and Reznikova (2012) performed detailed investigation on time varying copula and Cerqueti and Lupi (2016) considerably contributed to theory of stochastic dependency. For applications of copulas in finance Yıldırım and Cengiz (2016), Rodriguez (2007), Genest et al. (2009), Wang et al. (2009), Mendez and Souza (2004) and Dias and Embrechts (2004) can be investigated. Nelsen (2006) and Joe (1997) can be viewed for elaborate information on copula. In this paper modelling of dependence structures for currencies which have the most traded in world economic markets is aimed. Empirical results obtained from this study indicate that returns of financial time series are not normally distributed. In this case using classical methods can give misleading results and thus it can increase the risk of investments. Copula method is used to model dependency between exchange rate since returns of exchange rate are not normally distributed and it is not necessary to satisfy any assumption for marginal distributions. Autocorrelation and heteroscadesticity that are often encountered in financial time series are achieved by AR – GARCH models. After filtering for each univariete marginal, standardized residuals acquired from AR – GARCH model are transformed into $[0,1]$ interval of uniform distribution by ECDF (Empirical Cumulative Distribution Function) and thus inputs which are necessary for copula are obtained. Then parameters of copula are estimated by means of ML (maximum likelihood) before modelling dependency between related exchange rates.

Other sections in study are organized as follows: In section 2, theoretical background of econometric models as well as copula models. In section 3, description of data is presented. While empirical findings are exhibited in section 4, recommendations and results gained from this paper are summarized in section 5.

2. THEORETICAL BACKGROUND

2.1. Econometric model

Two main problems to be solved in financial time series are autocorrelation and heteroscedasticity. In autocorrelated and heteroscedastic series since features of estimator are not satisfied, results acquired can fairly be misleading. For this purpose autocorrelation and heteroscedasticity problems can be handled by AR (Autocorrelation) and GARCH (Generalized Autocorrelation Conditional Heteroscedasticity), respectively. GARCH model which was introduced by Bolerslev (1986) is crucial tool in analysis of financial time series and it is used to model the change in variance of data. AR (1) – GARCH (1,1) model used in this search is stated as follows:

$$r_{i,t} = \alpha + \beta r_{i,t-1} + e_{i,t} \quad (1)$$

$$e_{i,t} = h_{i,t}\varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim St \quad (2)$$

$$h_{i,t} = \omega_{i,t} + \delta e_{i,t-1}^2 + \gamma h_{i,t-1} \quad (3)$$

Where (α, β) and (ω, δ, γ) are parameters of AR (1) and GARCH (1,1) models respectively and $\omega > 0, \delta > 0, \gamma > 0$.

2.2. Theory of copula

Sklar's theorem which reveals the presence of copula provides a link between joint distribution function and copula. According to the theorem, there exists C

copula for p-dimensional H joint distribution with $H_i,\ i = 1, \dots, p$ marginals:

$$H(x_1, \dots, x_p) = C(H_1(x_1), \dots, H_p(x_p)) \qquad (4)$$

Note that copula C is unique if marginals are continuous. Reverse of Sklar's theorem is crucial to construct multivariate distributions and it can be demonstrated as follows:

$$C(u_1, \dots, u_p) = H\left(H_1^{-1}(u_1), \dots, H_p^{-1}(u_p)\right) \qquad (5)$$

Here,

$$u_i = H_i(x_i), \quad i = 1, \dots, p \qquad (6)$$

If distribution function H is $p-$ times differentiable, joint density function can be written as follows:

$$h(x) = \frac{\partial^p}{\partial_{x_1} \dots \partial_{x_p}} H(x) \quad = \prod_{i=1}^{p} h_i(x_i) \frac{\partial^p}{\partial_{u_1} \dots \partial_{u_p}} C\left(H_1(x_1), \dots, H_p(x_p)\right)$$

$$h(x) = \prod_{i=1}^{p} h_i(x_i) c\left(H_1(x_1), \dots, H_p(x_p)\right) \quad (7)$$

$x = (x_1, \dots, x_p)$ and joint density function h can be rewritten as noted below using Eq. (6):

$$h(x) = \prod_{i=1}^{p} h_i(x_i) c(u_1, \dots, u_p) \qquad (8)$$

Where, c represents density function of copula. As it is understood from Eq. (8) it is not essential that marginals and copula that constructs any joint distribution come from the same family.

2.3. Copula functions

Copula which is used to create multivariate distributions is split into two as static and time varying. In this section elliptical copulas which compose of Gaussian and Student's t are investigated. Firstly static copulas and then time varying copulas are presented with fundamental characteristics. For comprehensive information on static copula, Nelsen (2006) and Joe (1997) can be view while detailed information on dynamic copula can be obtained by Patton (2006).

2.3.1. Static copula

In this study only Gaussian and Student's t copula are investigated although there are a variety of static copulas. Bivariate Gaussian copula can be stated as follow: $C_{Gaussian}(u_1, u_2, \rho)$

$$= \int_{-\infty}^{\phi^{-1}(u_1)} \int_{-\infty}^{\phi^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\rho^2}}\, x\, \exp\left(-\frac{x_1^2 - 2\rho x_1 x_2 + x_2^2}{2(1-\rho^2)}\right) dx_1 dx_2$$

$$= \phi_\rho(\phi^{-1}(u_1), \phi^{-1}(u_2); \rho) \qquad (9)$$

Where, u_1 and u_2 are cumulative distribution function of residuals and ρ is Pearson correlation coefficient. Besides ϕ^{-1} is inverse cumulative distribution of standardized Gaussian. Bivariate Student's t copula from elliptical copulas can be defined as noted below:

$C_{Student's\,T}(u_1, u_2, \rho)$

$$= \int_{-\infty}^{T_v^{-1}(u_1)} \int_{-\infty}^{T_v^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\rho^2}} \left[1 + \frac{x_1^2 - 2\rho x_1 x_2 + x_2^2}{v(1-\rho^2)}\right]^{\frac{v+2}{2}}$$

$$= T_v(T_v^{-1}(u_1), T_v^{-1}(u_2); v, \rho) \qquad (10)$$

T is Student's t distribution with Pearson correlation ρ and degree of freedom v. T_v^{-1} is inverse cumulative distribution of standardized Student's t. main difference between Student's t and Gaussian copula is that Student's t copula can model tail dependency (Sriboonchitta et al., 2013).

2.3.2. Time varying and DCC copulas

Time varying and DCC copula are some of methods used when parameters of copula change over time. Time varying and DCC copula which was introduced by Engle (2002) compatibly work with Gaussian and Student's t copulas.

Time varying copulas can be view as dynamic generalization of Pearson correlation coefficient or Kendal's tau. It is assumed that time varying copula always follows ARMA (Autoregressive Moving Average) process (Manner and Reznikova, 2012). Parameter ρ_t of Gaussian copula is defined as follows:

$$\rho_t = (1 - \alpha - \beta).R + \alpha.r_{t-1} + \beta\rho_{t-1} \quad (11)$$

Where R is sample correlation. DCC model introduced by Engle (2002) is used for dynamic conditional correlation. According to the model, while parameter $\rho_t$of Gaussian and Student's t evolves over time and degree of freedom for Student's t is static. DDC model work as follows. First marginal distribution F_t is used to obtain transformed $u_t = F_t(\varepsilon_t)$ values which are uniformly distributed [0,1]. Then, reverse of standard normal or standard t distribution is applied to get $Y_t = F^{-1}(u_t)$ values that come from Gaussian or Student's t family. It is assumed that correlation matrix R for Y_t changes over time as follows:

$$R_t = diag\{Q_t\}^{-1/2} Q_t diag\{Q_t\}^{-1/2} \qquad (12)$$

Here,

$$Q_t = \bar{Q}_t(1 - \alpha - \beta) + \alpha Y_{t-1}.Y_{t-1}' + \beta.Q_{t-1} \qquad (13)$$

$\bar{Q}_t$ is sample covarience matrix of Y_t while α and β are parameters to be needed from observations and they are estimated via ML (Maximum Likelihood). Note that time varying and DCC models only support Gaussian and Student's t distributions.

2.4. Parameter estimations

Parameters of copula are estimated in two stages: marginal distributions and copula are estimated respectively. Joint density function h is stated as follows:

$$L(\varphi,\theta;x) = ml\big(h(\varphi;x)\big) + l\big(c(\theta;u)\big)$$

Here,

$$ml\big(h(\varphi;x)\big) = \sum_{i=1}^{p} \log\big(h_i(\varphi_i;x_i)\big) \qquad (15)$$

$$ml\big(c(\theta;u)\big) = \log\Big(c\big(\theta;H_1(x_1),\dots H_p(x_p)\big)\Big) \quad (16)$$

Parameter estimations can be performed by optimizing likelihood functions given in Eq. (15) and Eq. (16).

3. DATA AND DESCRIPTIVE STATISTICS

3.1. Data

In this paper for the sake of application, daily exchange rates from financial time series are interested in. For this purpose, daily values of six exchange rates against U.S. dollar that have the most traded in world economic markets constitute data sets.

Actual series of interest variables is presented in Fig. 1. These values which are extracted from worldbank compose of Euro (EURO), Japanese yen (YEN), British pound (GBP), Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF). While starting date of data sets is 21 April 2010, ending date is 6 August 2017. Return series are investigated Instead of actual series to satisfy the stationary of interest exchange rates. Returns series for all variables demonstrated in Fig. 2. In analysis stage, geometric returns of exchange rates are calculated and values obtained multiply by 100. Empirical studies for proposed model are executed by transformed data sets and a set of important results are found out. Geometric return is evaluated as follows:

$$r_{i,t} = \log\left(\frac{p_{i,t}}{p_{i,t-1}}\right) \qquad (17)$$

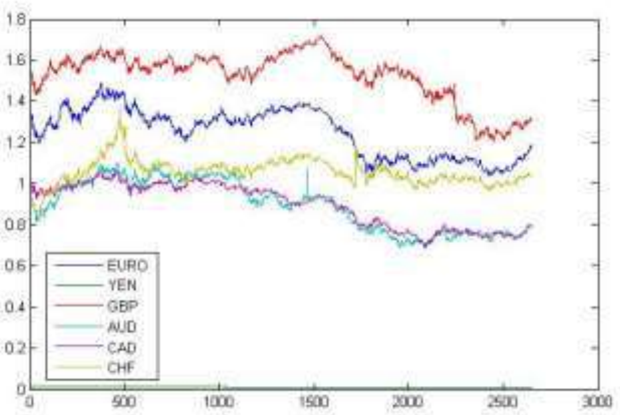


Figure 1: Actual values for each exchange rate against U.S. dollar

Descriptive statistics for values of related exchange rates are demonstrated in Table 1. Means of returns are so close to zero and they are smaller than standard deviation. This implies that interest exchange rates have high volatility.

Table 1: Descriptive statistics of returns of exchange rates

	Mean	Std. Dev.	Skewness	Kurtosis
EURO/USD	-0.0050	0.4950	-0.0238	6.1850
YEN/USD	-0.0065	0.5143	-0.0774	9.2750
GBP/USD	-0.0062	0.4755	-1.8132	33.3406
AUD/USD	-0.0061	0.7316	-0.1016	139.2746
CAD/USD	-0.0090	0.4465	-0.1642	7.6111
CHF/USD	0.0035	0.6138	2.1694	93.2029

When statistics of returns are compared to each other, it is seen that Swiss franc (CHF) has highest mean while standard deviation for Australian dollar (AUD) is higher than others.

Thus it refers that the most volatile exchange rate is Australian dollar. However, it is understood that these exchange rates exhibit slightly skewed and excess kurtosis. This indicates that returns of exchange rate are not normally distributed. It is understood from Fig. 2 that changes of exchange rate returns are rather high between 2011 and 2012.

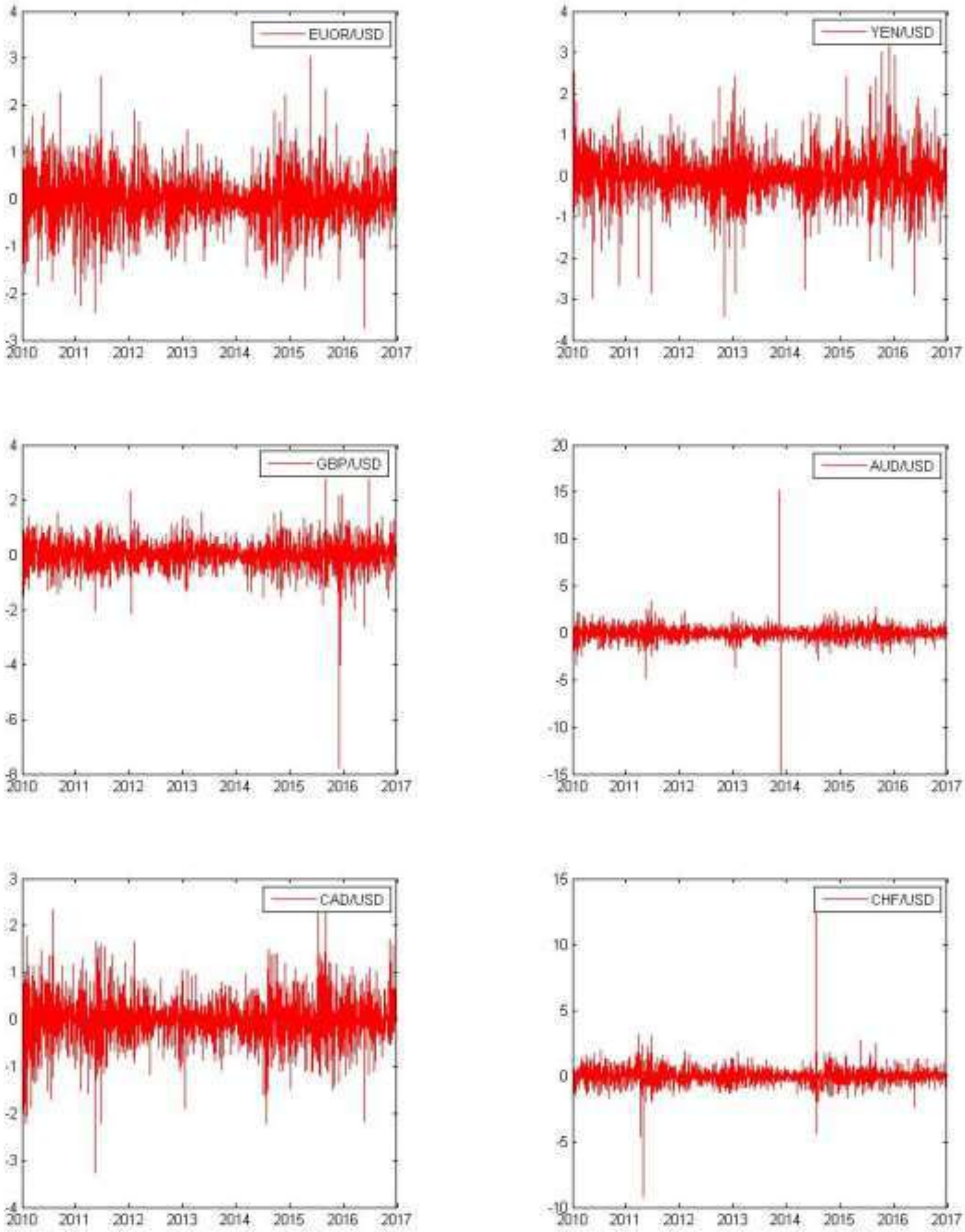


Figure 2: Return series for each exchange rate

Underlying tests for each return series are implemented and results are presented in Table 2. It is specified that all of return series of exchange rates against U.S. dollar are not normally distributed as a result of Jorque Bera test. Furthermore, it can be found out that normal distribution is insufficient to model tails of sample distributions when viewed in appendix in Fig. A2. Then, it is understood that all return series are stationary via Augmented Dickey Fuller (ADF) test. It is determined that apart from GBP/USD exchange rate returns of others have heteroscedasticity by Engle's ARCH test and excluding EURO/USD and YEN/USD other exchange rates are autocorrelated through Ljung – Box test. Therefore, return series of interest exchange rates are modelled via AR – GARCH approach.

Table 2: Tests for exchange rates returns

	Jorque Bera	ADF	ARCH LM	Ljung-Box
EURO/USD	0.0000	0.0000	0.0010	0.8902
YEN/USD	0.0000	0.0000	0.0015	0.5328
GBP/USD	0.0000	0.0000	0.1227	0.0000
AUD/USD	0.0000	0.0000	0.0000	0.0000
CAD/USD	0.0000	0.0000	0.0000	0.0157
CHF/USD	0.0000	0.0000	0.0000	0.0000

4. EMPIRICAL FINDINGS

4.1. Estimations for marginal distributions

The aim of this paper is to model dependence structure between exchange rates. For this purpose copula which is a significant tool in modelling dependency can be used irrespective of distributions of interest variables. Before copula parameters marginal distributions for six exchange rate are estimated, AR (1) – GARCH (1, 1) model which overcomes problems such as autocorrelation and heteroscedasticity encountered in financial time series is used. Since distributions for return series are not normally distributed, other distributions are searched for marginal distributions. İt is determined that Student's t is the best fitted distributions for residuals of GARCH model via information criterion such as AIC and BIC. Results of autocorrelation for standardized residuals obtained from GARCH model is presented in appendix in Fig. A1 and parameter estimates for each return series are given in Table 3.

Table 3: Parameter estimates of marginal distributions

	α	β	ω	δ	γ	ν	AIC	BIC
EURO	0.0019* (0.006)	0.0032* (0.012)	0.0006 (0.001)	0.0221 (0.004)	0.9779 (0.006)	3.1640 (0.179)	-3157.51	-3192.80
YEN	-0.0028* (0.006)	-0.0117* (0.017)	0.0041 (0.003)	0.0306 (0.006)	0.9694 (0.011)	2.5101 (0.102)	-3151.11	-3186.40
GBR	0.0043 (0.007)	0.0169 (0.021)	0.0015 (0.001)	0.0265* (0.007)	0.9735 (0.009)	2.8571 (0.173)	-2733.37	-2768.66
AUD	0.0079 (0.009)	-0.0291 (0.016)	0.0535 (0.023)	0.0981 (0.039)	0.8698 (0.031)	2.4461 (0.136)	-4354.06	-4389.35
CAD	-0.0033 (0.005)	-0.0227 (0.018)	0.0015 (0.001)	0.0301 (0.006)	0.9699 (0.008)	2.8809 (0.159)	-2538.42	-2573.72
CHF	0.0031 (0.006)	0.0175 (0.014)	0.0022 (0.001)	0.0205 (0.007)	0.9795 (0.006)	2.5783 (0.124)	-3398.08	-3433.37

Note: *Asteriks(*) indicates that coefficients are not significant at the 0.05 level*

Standardized residuals are acquired after parameters of marginal distributions are estimated. These residuals are needed to transform to uniform interval [0,1] for estimation of copula parameters. Probability integral transformation is utilized to transform standardized residuals into unit interval and thus input variables for estimation of copula parameters are obtained.

Table 4: Results of parameter estimation for time varying copula

	ν	α	β	LL
Gaussian	-	0.0202	0.9692	337.35
Student's t	3.1871	0.0178	0.9801	445.32

Table 5: Results of parameter estimation for DCC copula

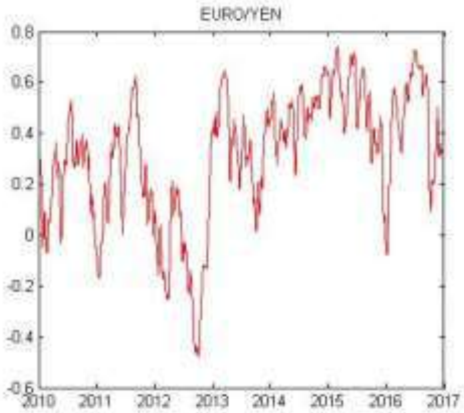
	ν	α	β	LL
Gaussian	-	0.0190	0.9723	345.88
Student's t	3.6718	0.0202	0.9767	444.43

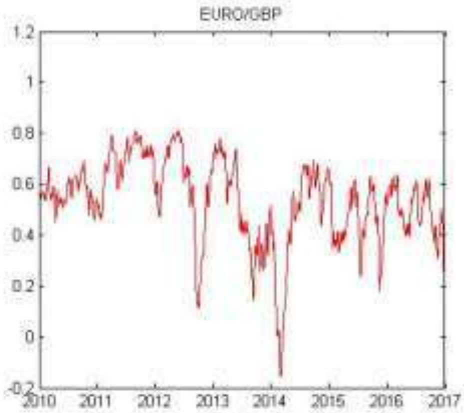
Selection of copula is rather critical in terms of modelling accurately the dependency. Copula parameter estimations and information criterions for interest exchange rates are shown in Table 4 and Table 5. The best fitted copula is selected via AIC and BIC and according to results it is determined that the best fitted copula for the values of exchange rates which are slightly skewed and heavy tailed is time varying Student's t copula. It is known that Student's t copula outperforms Gaussian copula in terms of modelling tails of distribution and results from this study confirm this case (Sriboonchitta et al., 2013). By using time varying Student's t copula, modelling dependency between Euro and other currencies against U.S. dollar in demonstrated in Fig. 3. It is possible to model dynamic dependency of other exchange rates against U.S. dollar via estimated results. Dynamic correlation is demonstrated in table 6.

Table 6: Dynamic correlation of exchange rates against U.S. dollar

	EURO	YEN	GBP	AUD	CAD	CHF
EURO	1	0.6910	0.8084	0.6454	0.5967	-0.1117
YEN	0.6910	1	0.5249	0.3019	0.5784	0.2257
GBP	0.8084	0.5249	1	0.8148	0.4328	0.1260
AUD	0.6454	0.3019	0.8148	1	0.6960	-0.2799
CAD	0.5967	0.5784	0.4328	0.6960	1	-0.4668
CHF	-0.1117	0.2257	0.1260	-0.2799	-0.4668	1

This table indicates that dependence structure between exchange rates is rather different from each other. Some of them are correlated negatively while other exchange rates exhibit positive correlation. Although relationship between Euro and most of exchange rate are considerably strong and positive, correlation between Euro and CHF are very low and negative. Moreover, interesting result obtained from this paper is CHF that is sixth of most traded exchange rate in the world market generally is correlated negatively with other exchange rates. This implies that when exchange rates except CHF depreciate, unlike these exchange rates, since CHF appreciate, for investors purchasing CHF can be significant investment to protect their money and to increase their gains. The mean return of CHF/U.S dollar parity is positive unlike other exchange rates and its standard deviation is moderate.





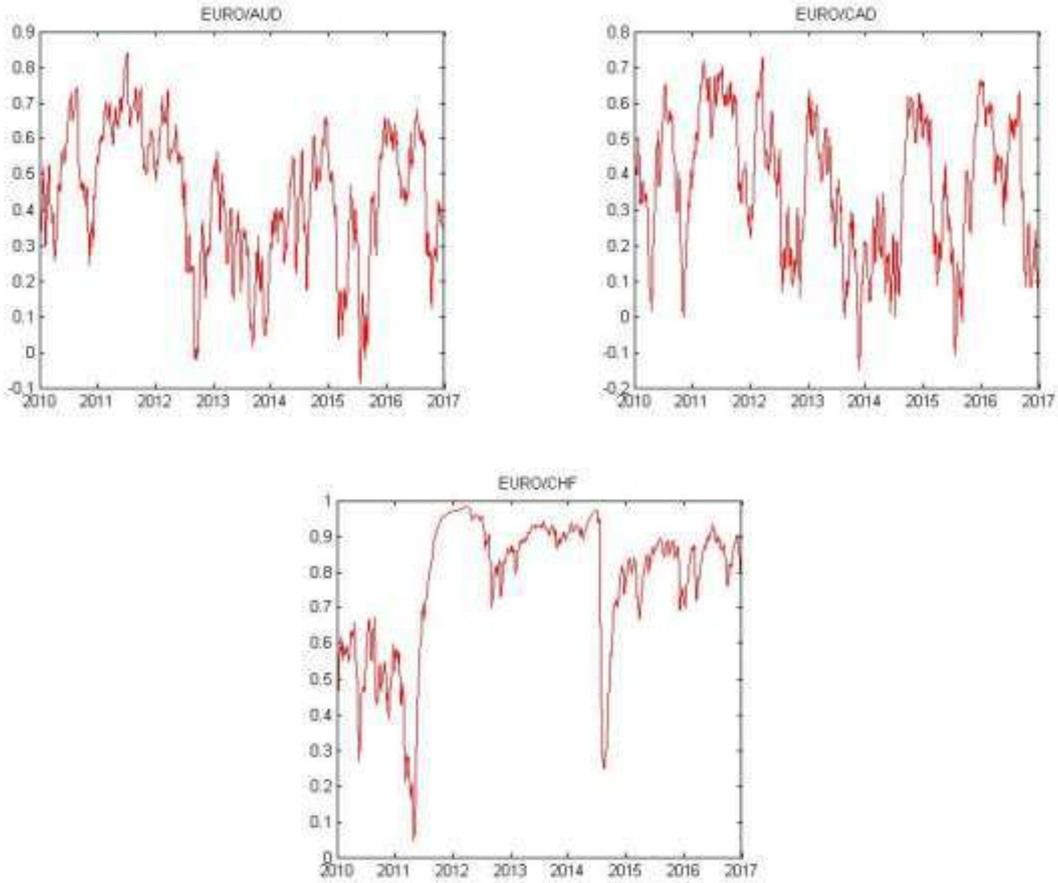


Figure 3: Dynamic correlation between Euro and other currencies against U.S. dollar

This also infers that CHF is almost reliable investment tool in case other exchange rates tend to decrease. On the other hand, the most related positively exchange rates are GBP/AUD and GBP/AUD, respectively while CAD/CHF is the most related negatively exchange rates.

5. RESULTS

One of the most important information for investors in financial markets is dependence structure between investment tools. Modelling dependency is crucial in terms of estimating possible loss and gains of investors. The purpose of this paper is to model dependency between values of six exchange rates against U.S. dollar which have the most traded in world economic markets. Since distributions for the values of exchange rates are not normally distributed, instead of classical technique alternative methods are investigated. For this purpose, copula approach which is used to model flexibility dependence structure between variable is investigated and thus some assumptions such as normality are not needed. On the other hand, it is understood that there are autocorrelation and heteroscedasticity problem and they are solved via AR – GARCH model. Input variable are obtained to estimate copula parameters after modelling marginal distributions. It is found out that dependency for exchange rates changes over time and the best fitted dynamic copula models are searched for dependence structure. According to empirical results, it is determined that the best fitted copula in modelling dependency between exchange rates is Student's t copula which is rather successful for modelling data sets with heavy tailed. By using this approach it is possible to model the dynamic correlation between related exchange rates. This contributes investors to determine right investments and to minimize their loss. In this paper, relations between exchange rates are investigated but other factors such as political decisions can affect this dependence and thus for future research the effect of different significant variable on exchange rate dependence can be taken into account.

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